Differentialoperatoren

Der Nablaoperator:

$$\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$$

Der Gradient:

$$\operatorname{grad} U(x,y,z) = \vec{\nabla} U = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}\vec{j} + \frac{\partial U}{\partial z}\vec{k}$$

$$\operatorname{grad} U(\rho,\varphi,z) = \frac{\partial U}{\partial \rho}\vec{e}_r + \frac{1}{\rho}\frac{\partial U}{\partial \varphi}\vec{e}_\varphi + \frac{\partial U}{\partial z}\vec{k}$$

$$\operatorname{grad} U(r,\varphi,\theta) = \frac{\partial U}{\partial r}\vec{e}_r + \frac{1}{r\sin\theta}\frac{\partial U}{\partial \varphi}\vec{e}_\varphi + \frac{1}{r}\frac{\partial U}{\partial \vartheta}\vec{e}_\theta$$

Die Divergenz:

$$\operatorname{div} \vec{A}(x,y,z) = \vec{\nabla} * \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\operatorname{div} \vec{A}(\rho,\varphi,z) = \vec{\nabla} * \vec{A} = \frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Die Rotation:

$$\operatorname{rot}\vec{A}(x,y,z)=\vec{\nabla}\times\vec{A}=\begin{Vmatrix}\vec{i} & \vec{j} & \vec{k}\\ & \nabla & \\ A_x & A_y & A_z\end{Vmatrix}$$

$$\operatorname{rot}\vec{A}(\rho,\varphi,z)=\vec{\nabla}\times\vec{A}=\frac{1}{\rho}\begin{Vmatrix}\vec{e}_\rho & \vec{e}_\varphi & \vec{e}_z\\ \frac{\partial}{\partial\rho} & \frac{\partial}{\partial\varphi} & \frac{\partial}{\partial z}\\ A_\rho & \rho A_\varphi & A_z\end{Vmatrix}$$

Der Laplaceoperator:

$$\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}+\frac{\partial^2}{\partial z^2}$$

$$\Delta=\frac{1}{\rho}\frac{\partial}{\partial\rho}\left(\rho\frac{\partial}{\partial\rho}\right)+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}+\frac{\partial^2}{\partial z^2}$$

$$\Delta U=\operatorname{div}\operatorname{grad}U$$

$$\Delta\vec{A}=\operatorname{grad}\operatorname{div}\vec{A}-\operatorname{rot}\operatorname{rot}\vec{A}$$

$$\Delta\vec{A}=\Delta A_x\vec{i}+\Delta A_y\vec{j}+\Delta A_z\vec{k}$$