The world as a hologram: News from string theory

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Integrability in Gauge and String Theory 2015
Public Lecture

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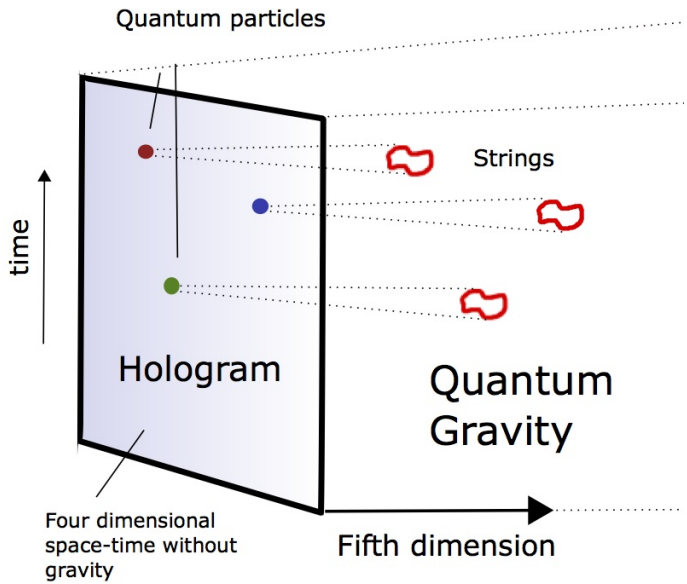
The problem

- Two keystones of fundamental physics:
  1. Einstein's theory of gravity [1915]
  2. Quantum theory: [1920-1930]

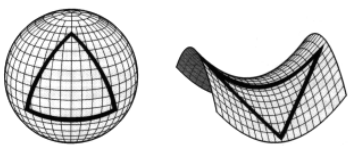
  $\Rightarrow$ Standard model of elementary particle physics [1950-75]
  Electromagnetic, weak and strong forces
- **Not known how to combine the two!** $\Rightarrow$ "unified theory"
- The most promising ansatz: **String theory** [since 1984]
- Both keystones are intimately connected

  $\Rightarrow$ Holographic principle of quantum gravity [since 1997]

The world as a holgram



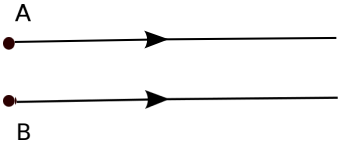
Gravity



Gravity according to Newton

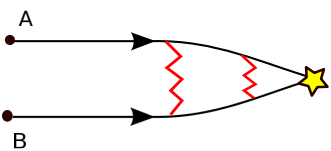
Sir Isaac Newton, 1642-1727

- Two bodies moving through empty space in the absence of forces



$$F_A = m_A \frac{d^2x}{dt^2} = 0 \quad \Rightarrow x(t) = vt + c$$

- As bodies have masses m_A und m_B there is an attractive gravitational force acting between them



$$F = G_N \frac{m_A \cdot m_B}{(x_A - x_B)^2}$$

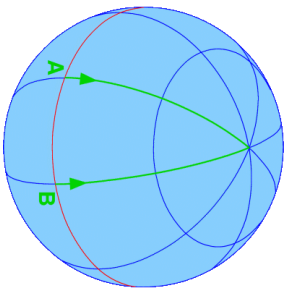
Newton's law of gravity

- Time is absolute: t "ticks" the same for A and B.
- Forces are mediated instantaneously
- Space is flat and infinitely extended (Euclidean): $\mathbb{R}^3$

Gravity according to Einstein

Albert Einstein, 1879-1955

- The attraction of two masses arises through the curvature of space (and time)



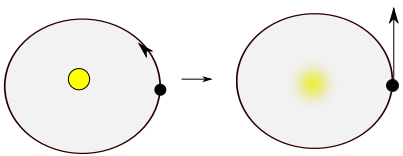
Bodies A and B continue to move on straight paths albeit in a curved space! There is no force acting between them.

- Gravity is a fictitious force similar to the centrifugal force known from everyday life.
- Space-time is dynamic: Gets curved by the matter moving through it and influences the motion of matter via its curvature. A highly coupled system.

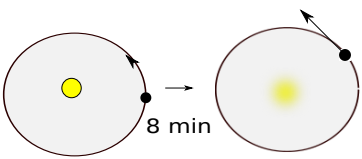
A Gedankenexperiment

Let us remove the sun from our solar system!

- Newton's World: The gravitational force acts instantaneous:
  The "removal" of the sun is immediately felt by the earth



- Einstein's World: Changes in the curvature of space and time propagate with c the velocity of light



- Gravitational waves $\hat{=}$ ripples of space-time!
  Predicted alrady in 1915, presently searched for in graviational wave detectors.

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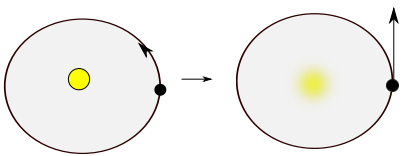

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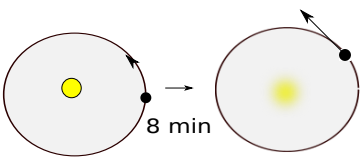

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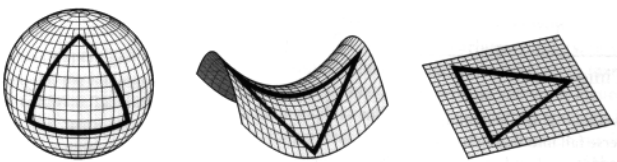
The theory of general relativity

- The principle:

  Space-time dictates the movement of matter, matter dictates the curvature of space-time.



- Einstein's field equations:

$$R_{\mu\nu} - \tfrac{1}{2}\, g_{\mu\nu}\, R + g_{\mu\nu}\, \Lambda = \tfrac{8\pi\, G_N}{c^4}\, T_{\mu\nu}$$

Geometry (Math) $=$ Matter (Physics)

Curvature R Cosmological constant Λ Energy-momentum tensor $T_{\mu\nu}$

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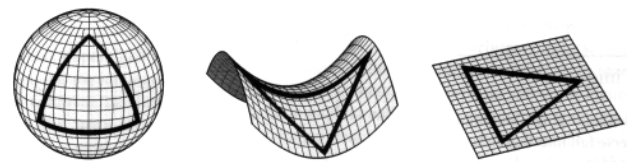

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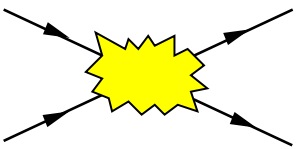
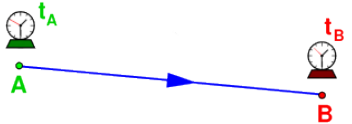
Quantum mechanics



A particle starts it motion from point A at time t_A. Will it reach the point B at t_b?

- Classical physics:
  Prediction (from Newton's law): Depending on its intitial velocity and the forces acting upon it, the particle will either reach B at t_b or not:



Deterministic prediction: yes/no

- Quantum physics:
  There is no definite answer to this question! Only the prediction for a probability of observing the particle at space-time point B at the time t_B is possible.

$$W_{(A,t_a)\to(B,t_b)} = 0.73$$

Quantum mechanics


M.Planck

W.Heisenberg

E.Schrödinger

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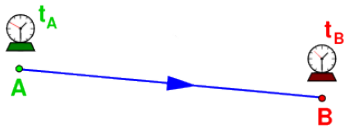

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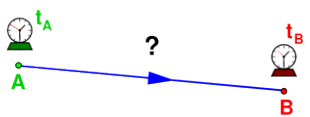


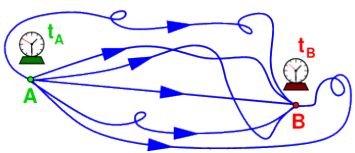
$$W_{(A,t_a)\to(B,t_b)} = 0.73$$

Feynman's path integral

Richard Feynman, 1918-1988

- Feynman's approach to quantum mechanics allows for the computation of this probability:

  Consider all possible paths from A to B. Every path is weigthed by a factor (the action) and the total probability follows from the sum of all possible paths



$$W_{(A,t_a)\to(B,t_b)} = \left| \sum_{\text{all paths}} e^{\text{action}/\hbar} \right|^2$$

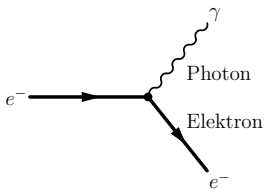
  The classical path is the path with minimal action $\Rightarrow$ typically gives a dominant contribution to W.

- Time is still an absolute quantity here $\Rightarrow$ "non-relativistic" quantum mechanics

Quantum field theory or relativistic quantum mechanics [1950s-70s]

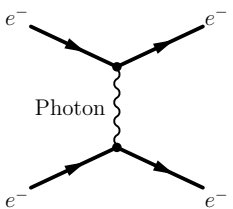
- Absolutely surprising effect of the constancy of the speed of light:

  Creation and annihilation of particles



  Photon radiation

- Forces are transmitted via the exchange of elementary particles

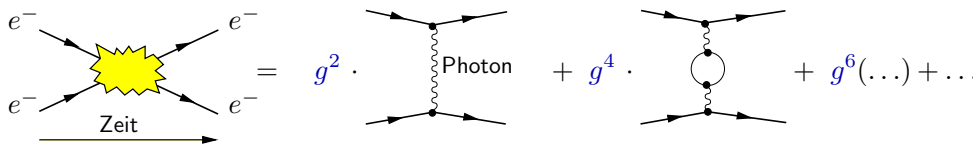


Carrier particle of the electromagnetic force: Photon ("light")

Quantum field theory

- Example: Quantum electrodynamics:

  Theoretical description of electrons and photons and their interactions
  (g: charge or "coupling constant")

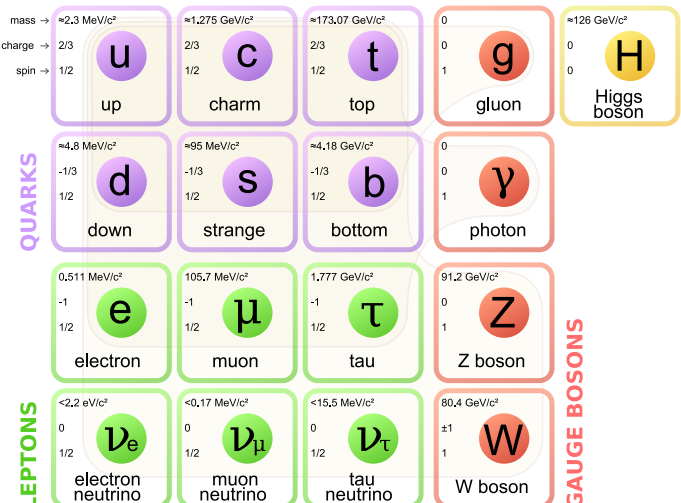


Scattering of electrons Perturbative series in $g \ll 1$

- Renormalization: $g \to g(E)$
- But what happens when $g \sim 1$? $\Rightarrow$ non-perturbative quantum field theory
- Three fundamental forces described via **gauge field theory** [1955,1971]

Standard model of particle physics

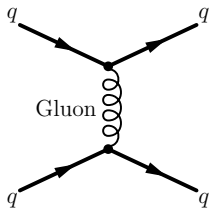
- Forces: Gauge bosons $\quad \mathbf{SU(N)}$ gauge fields $N \times N$ matrices
- Matter: Quarks & leptons
- Higgs boson: Scalar particle



The strong force: Quantum chromodynamics

- SU(3) gauge theory: gluons and quarks (q)

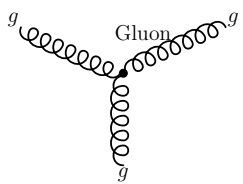
Interaction:



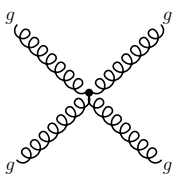
Responsible for the stability of the proton and nuclei

- Gluons have self interactions

$\mathbf{g} \sim$



$\mathbf{g^2} \sim$



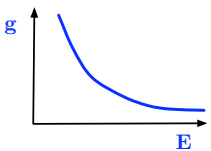
- Strong force: (at LHC energies)
  - $1000 = 10^3$ times stronger that electromagnetic force
  - $100000 = 10^5$ times stronger than the weak force
  - $10000\ldots000 = 10^{38}$ times stronger than gravity!!

Quantum chromodynamics: The inverse giant

"Jim Knopf", M. Ende

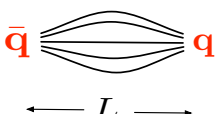
- Coupling strength depends on energy $\mathbf{g} \to \mathbf{g}(E)$

  [Gross, Wilczeck, Pollitzer]



- Low Energies ($\mathbf{g} \gg 1$): Confinement
  No free quarks and gluons are being observed.
- Instead: Bound states (hadrons)

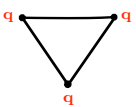
  Mesons:



- $\bar{q}q$-potential: $\boxed{V = \frac{1}{l_S^2} L}$ "color flux tube"

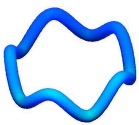
The strong force as a string theory

- Color flux tube reminds of a microscopic string: ['t Hooft, 1974]

Hadron		String picture
Meson (e.g. pion)	$\hat{=}$	
Glueball	$\hat{=}$	
Baryon (e.g. Proton)	$\hat{=}$	

- Vision: Strings are adequate description for strongly coupled gauge fields.
- But: Strings describe quantum gravity and the strong force does not contain gravity!
- Puzzle resolved by holographic principle...

Quantum gravity and string theory

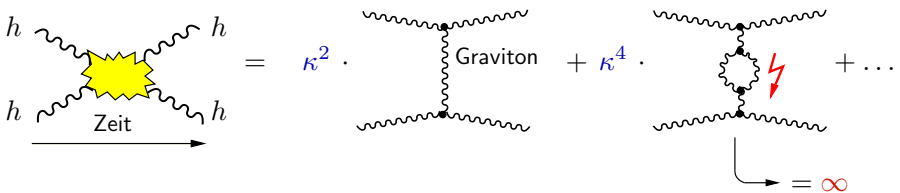


Gravity as a quantum theory?

- Treat Einstein's theory of gravity b y the rules of quantum field theory:

  Gravitons: Small curvature fluctuations about a given space-time structure

  Gedankenexperiment: Scattering of gravitons



- **NOT** "renormalizable" $\Rightarrow$ Forced to extend Einstein's theory

  Quantum field theory of gravity requires measurements of infinitely many empirically determined parameters

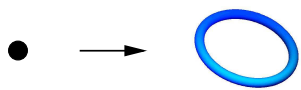
  Very limited predictability ...

- Coupling constant $\kappa = \sqrt{G_N}$ of gravitons extremely weak:
  Becomes relevant only at length scales $10^{-33}\mathrm{cm}$

String theory

How can we reach predictability within quantum gravity?

- **Simple idea:** Replace particles by extended 1d object: **"String"**



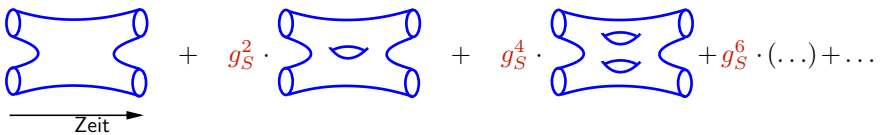
$$l_S \sim 10^{-33}\text{cm}$$

- Quantum mechanics of a relativistic string:

  Centre of mass movement + internal oscillations:

String interactions: Perturbative series

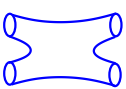
- Generalization of particles interactions: [1984-1995]



g_S: **string coupling constant**

There are no divergencies! Interaction is "soft" as not localized to a point

- Graviton scattering:


$\hat{=}$ Einstein's theory of gravity

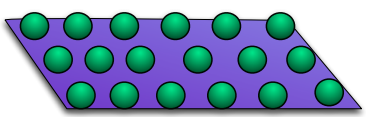
$g_S^2 \cdot$ 
$\hat{=}$ Quantum corrections to Einstein's theory

Properties and predictions of string theory

- Higher dimensions:

  As a quantum theory strings are only consistent in 9 space dimensions!

  $\Rightarrow$ 6 extra space dimensions



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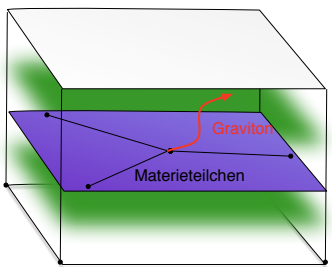


  Geometry of hidden 6 dimensions predicts particle spectrum 3 dimensional world

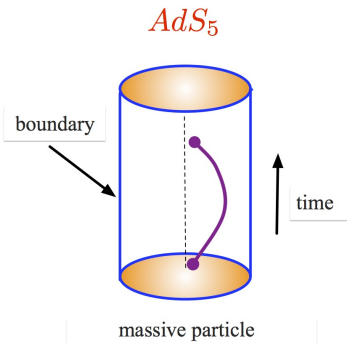
- Prediction of supersymmetry: bosons $\Leftrightarrow$ fermions
- Puzzle: There exist a gigantic number of possible compactifications from $9 \rightarrow 3$

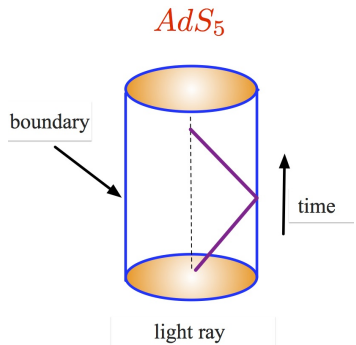
The world as a hologram

Source: Scientific american

Quantum gravity in negatively curved space-times

- Since 1997 revolutionary progress in our understanding of quantum gravity in anti-de-Sitter space (AdS_d) $\hat{=}$ constant neg. curvature [Willem de Sitter, 1872-1934]
- AdS_5 is (4+1)-dimensional space-time with a boundary: $\mathbb{R}^3 \times$ time

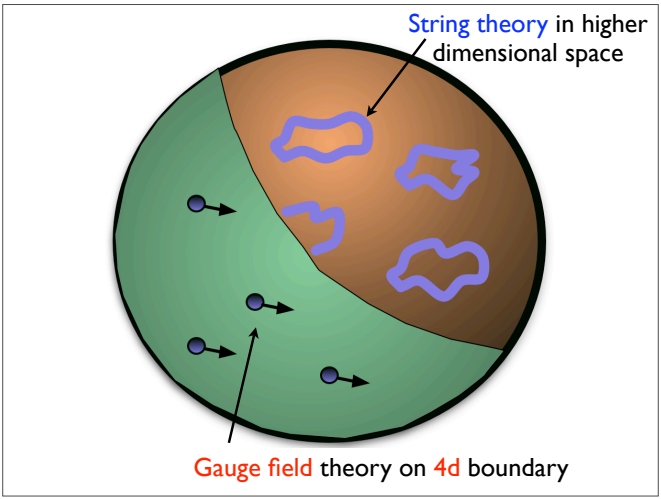




- "Gravity in a box"
- String theory well defined on $AdS_5 \times M_5$, e.g. choose $M_5 = S^5$ 5d-sphere.

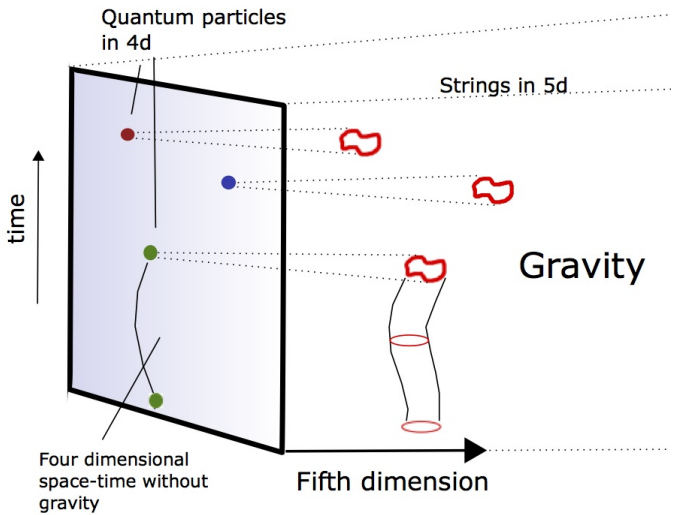
Maldacena's String-Gauge Duality [1997]

Holographic principle: Strings in the bulk of space-time (Anti-de-Sitter space), quantum particles (gluons) on the boundary



Two dual descriptions of **one** physical entity: Gauge theory $\hat{=}$ String theory in AdS

The world as a hologram: Resolution of dimensions



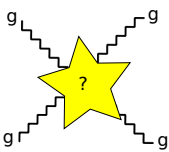
4d particle theory $\Leftrightarrow$ (5+5)d gravity theory $\Leftrightarrow$ 2d string theory

Integrability in gauge and string theory

Novel insights discussed at the IGST Conference allow computation of **exact** quantum properties in both theories.

$\Rightarrow$ No approximation in coupling constant g any longer!!

Example 1: Probability for the scattering of two gluons into two gluons in supersymemtric QCD



$=$

$$= \begin{cases} 8g^2 - \frac{8}{3}\pi^2\, g^4 + \frac{88}{45}\,\pi^4\, g^6 - 16(\frac{73}{630}\pi^6 + 4\,\zeta(3)^2)\, g^8 + \dots & \text{for} \quad g \ll 1 \\ 4g - \frac{3\log 2}{\pi} - \frac{K}{4\pi^2}\,\frac{1}{g - 3\log 2/4\pi} - \frac{27\,\zeta(3)}{2^9\,\pi^3}\,\frac{1}{g^2} - \dots & \text{for} \quad g \gg 1 \end{cases}$$

$\times$ (Simple functions of the involved momenta)

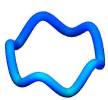
Exactly known function of g and gluon-momenta

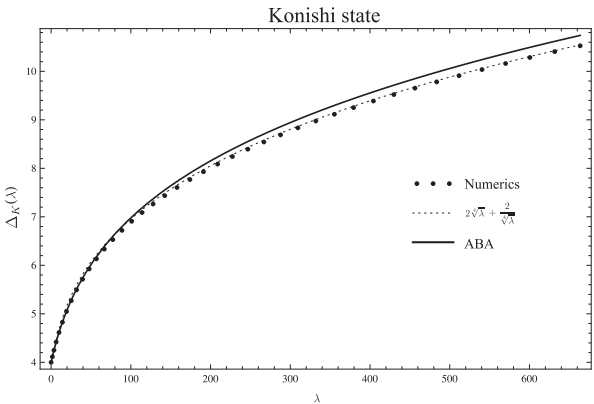
[Beisert,Eden,Staudacher]

Integrability in gauge and string theory

Example 2: Exact internal excitation energy of a closed string in AdS-space.

First excitation mode of the string



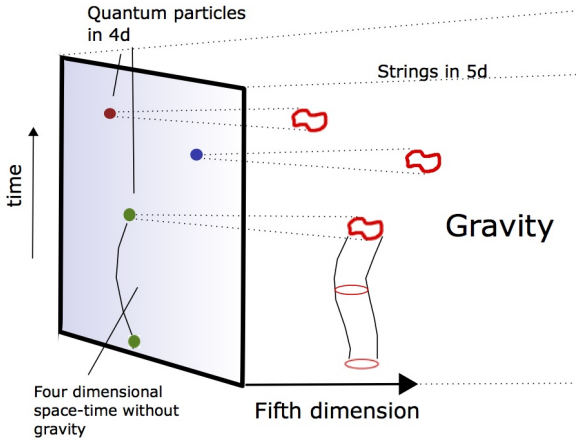


$$\Delta_K(\lambda) = \text{Energy stored in vibrating string}$$

$$\lambda = \left(\frac{\text{Curvature radius of AdS-space}}{\text{String length}}\right)^4 = g^2$$

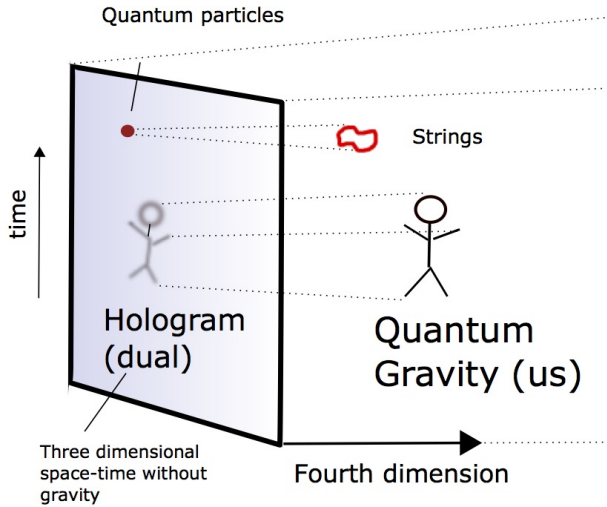
[Gromov,Kazakov,Vieira]

Summary



- Quantum gravity ($\hat{=}$ string theory) in (d+1)-dimensions is equivalent to quantum particle theory (gauge field theory) in d-dimensions w/o gravity
- Novel tools for solution of the quantum particle theory in 3 space dimensions

Outlook



- Prospect: Use (2+1)d particle theory to describe quantum gravity in (3+1)d
- Black holes! Our universe?

Thank you for your attention