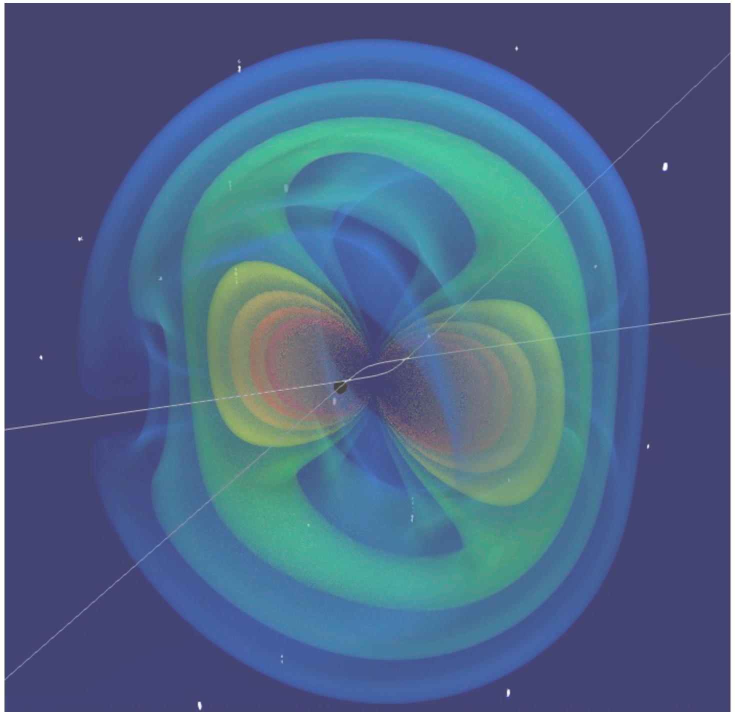
HIGH PRECISION GRAVITATIONAL WAVE PHYSICS FROM A WORLDLINE QUANTUM FIELD THEORY



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Based on joint work with

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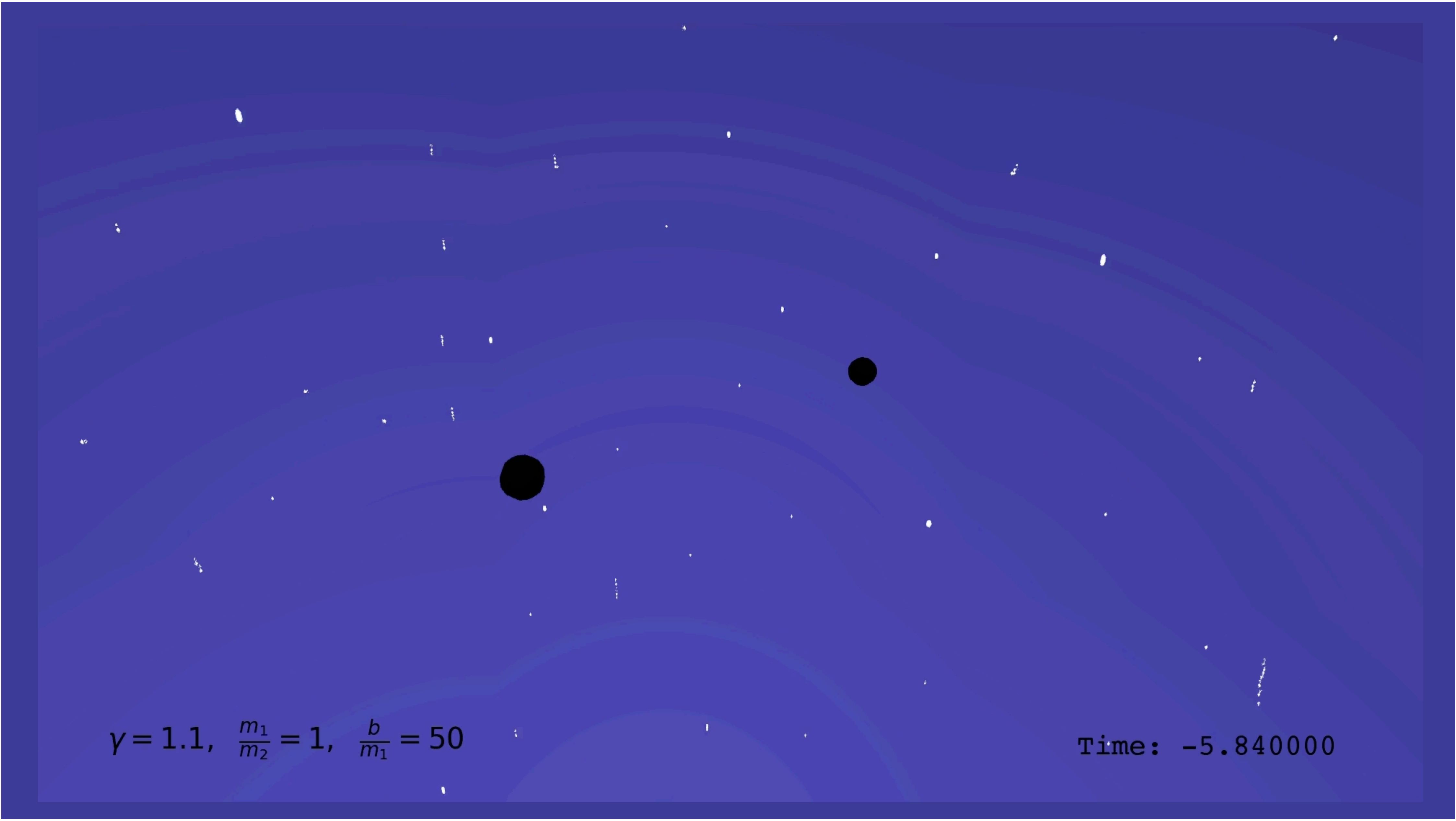
2010:02865, *JHEP* 02 (2021) 048
2101.12688, ***PRL*** 126 (2021) 20
2106.10256, ***PRL*** 128 (2022) 1
2109.04465, *JHEP 01 (2022) 027*
2201.07778, ***PRL*** 128 *(2022) 14*
2207.00569, *JHEP* 10 (2022) 128

RTG 2575:
Rethinking Quantum Field Theory

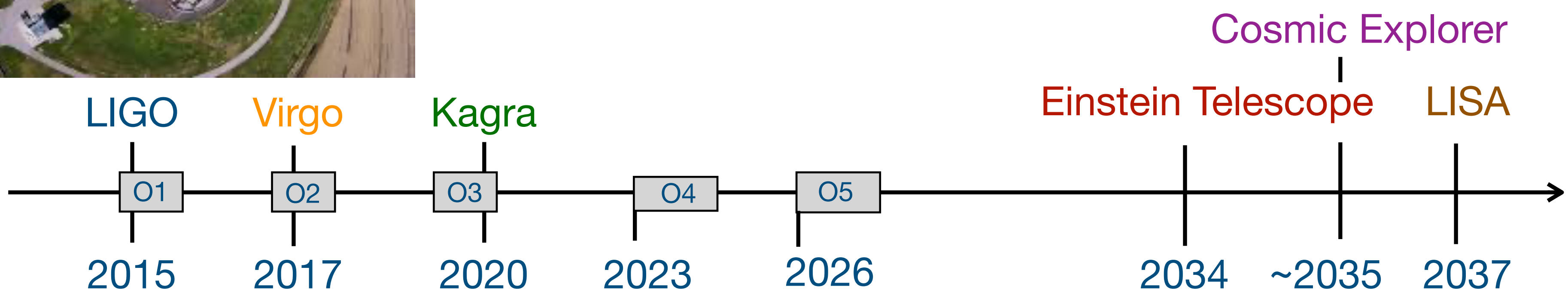

$\gamma = 1.1, \quad \frac{m_1}{m_2} = 1, \quad \frac{b}{m_1} = 50$
Time: -5.840000

ERA OF GRAVITATIONAL WAVE PHYSICS: NEED FOR HIGH-PRECISION PREDICTIONS

- Upcoming 3rd generation of gravitational wave observatories with 10^2 sensitivity increase
- Need for accurate waveform predictions well beyond state-of-the art

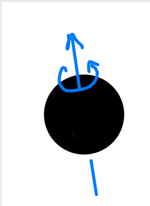
High-precision predictions necessary basis to study fundamental questions in physics:

- Is Einstein's theory correct?
- Black hole formation & population?
- Neutron star properties?
- Physics beyond the standard model?



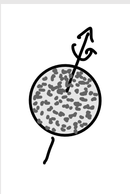
GRAVITATIONAL TWO-BODY PROBLEM

Black Hole



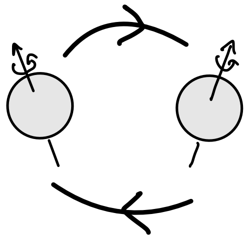
mass, spin

Neutron Star

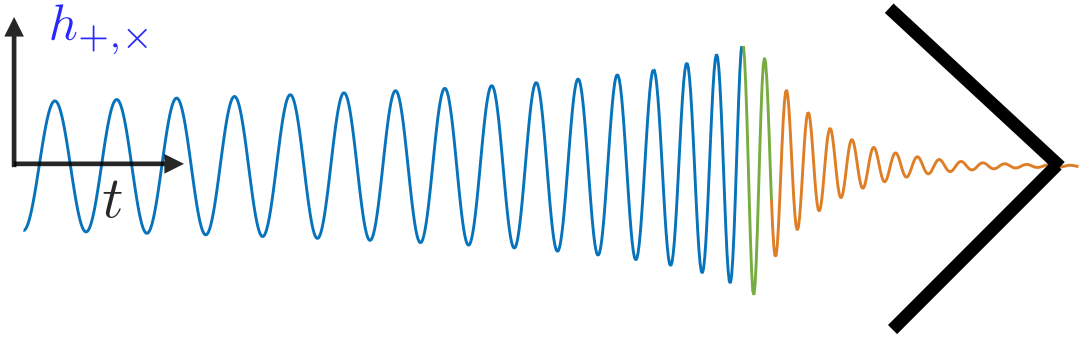


mass, spin, radius, tidal deformability

Black Hole/Neutron Star Binaries:



Bound state



inspiral

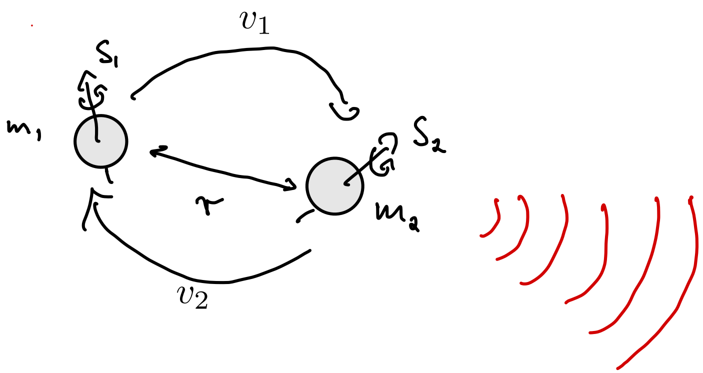
merger

- During **inspiral**: weak gravitational fields $\quad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\, h_{\mu\nu}$
- **Quantum** field theory formalism for **classical** two-body problem:

WORLDLINE QUANTUM FIELD THEORY

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 black holes or neutron stars:

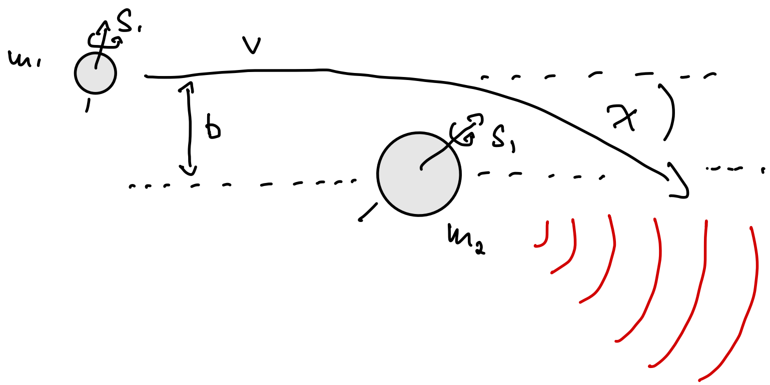
Virial-theorem: $\frac{GM}{r} \sim v^2$ $\quad (c = 1)$

post-Newtonian (PN) expansion in G & v^2

$$\boxed{\text{Weak field expansion:} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}}$$

$$\kappa = \sqrt{32\pi G}$$

Newton's constant

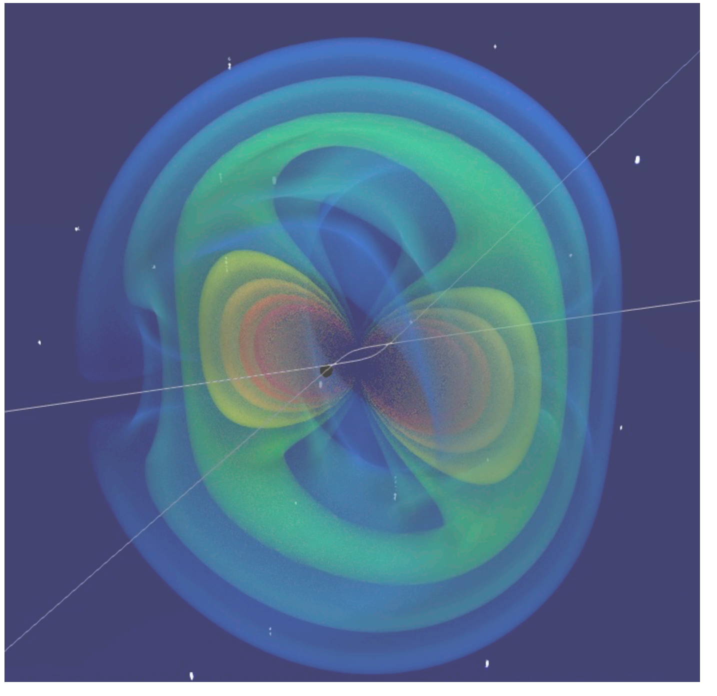


Scattering of 2 black holes or neutron stars:

Weak field (G), but exact in v^2

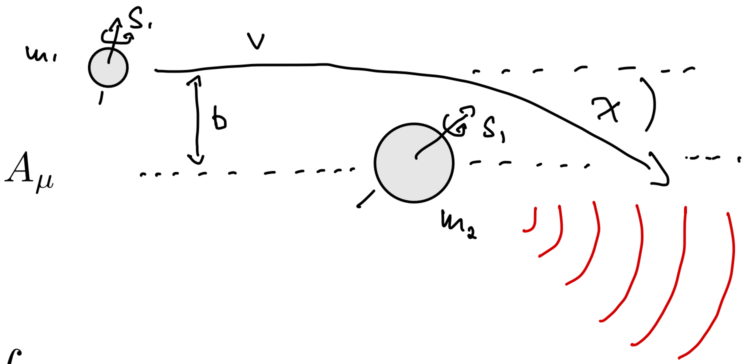
post-Minkowskian (PM) expansion

WORLDLINE QUANTUM FIELD THEORY



[Jakobsen,Mogull,JP,Sauer]

WARMUP: ELECTROMAGNETISM



Scattering of charged particles in Maxwell's theory

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu$$

$$\partial_\mu := \frac{\partial}{\partial x^\mu}$$

$$S = -\sum_{i=1}^{2} \int d\tau_i \left(m_i \sqrt{\eta_{\mu\nu} \dot{x}_i^\mu(\tau_i)\, \dot{x}_i^\nu(\tau_i)} - q_i\, A_\mu(x)\, \dot{x}_i^\mu(\tau_i) \right) - \frac{1}{16\pi} \int d^4x F_{\mu\nu}(x)\, F^{\mu\nu}(x) + S_{\text{g.f}}$$

Charged particles

Maxwell theory & gauge fixing

Equations of motion:

$$\partial_\mu F^{\mu\nu}(x) = 4\pi j^\nu(x)$$

Maxwell's eqs.

$$\frac{d^2 x_i^\mu}{ds^2} = \frac{q_i}{m_i} F^\mu{}_\nu \frac{dx_i^\nu}{ds}$$

Lorentz eqs.

Solve perturbatively in q_i

$$A_\mu(x) = \sum_{n=1}^{\infty} q^n\, A_\mu^{(n)}(x)$$

emitted radiation

$$x_i(\tau) = b_i^\mu + v_i^\mu \tau + \sum_{n=1}^{\infty} q^n z_i^{(n)\,\mu}(\tau)$$

straight line: „in“ state

deflections

Non trivial due to back reaction !

WARMUP: ELECTROMAGNETISM

1) Worldline simplification: Go to proper time gauge:

$$S_p = -m \int d\tau \sqrt{\eta_{\mu\nu}\dot{x}^\mu \dot{x}^\nu} \longrightarrow \tilde{S}_p = -\frac{m}{2}\int \left(e^{-1}\,\eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu + e\right)$$

Algebraic equations of motion for e: $\frac{\delta \tilde{S}}{\delta e} = 0 \longrightarrow e^2 = \eta_{\mu\nu}\dot{x}^\mu\dot{x}^\nu$

Plug this back into $\tilde{S}_p$ to find S_p

Proper time gauge: $e = 1 \leftrightarrow \dot{x}^2 = 1$

2) Space-time simplification: Go to Lorenz gauge: $\partial \cdot A = 0$

3) Gauge fixed theory:

$$\boxed{\tilde{S} = -\sum_{i=1}^{2}\int d\tau_i \left(\frac{m_i}{2}\dot{x}_i^2(\tau_i) - q_i\, A_\mu[x_i(\tau_i)]\,\dot{x}_i^\mu(\tau_i)\right) + \frac{1}{16\pi}\int d^4x A_\mu(x)\Box A^\mu(x)}$$

Worldline Space-time

TRICK: QED WITHOUT THE Q

$$\tilde{S} = -\sum_{i=1}^{2} \int d\tau_i \left(\frac{m_i}{2} \dot{x}_i^2(\tau_i) - q_i\, A_\mu[x_i(\tau_i)]\, \dot{x}_i^\mu(\tau_i) \right) + \frac{1}{16\pi} \int d^4x A_\mu(x) \Box A^\mu(x)$$

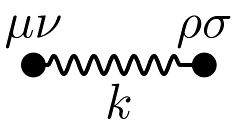
To set up efficient perturbation theory: Quantize and take $\hbar \to 0$ limit! $\quad x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z^\mu(\tau)$

$$\langle \mathcal{O} \rangle = \int D[A, z] \mathcal{O}\, e^{-\frac{i}{\hbar}\tilde{S}[A,z]} \quad \xrightarrow{\hbar \to 0} \quad \begin{aligned} \lim_{\hbar \to 0} \langle z^\mu(\tau) \rangle &= z^\mu_{\text{classical}}(\tau) \\ \lim_{\hbar \to 0} \langle A^\mu(x) \rangle &= A^\mu_{\text{classical}}(x) \end{aligned}$$

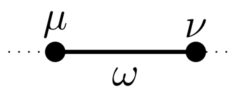
Tree-level one-point functions solve classical equations of motion

FEYNMAN DIAGRAMMATIC EXPANSION

Propagators:



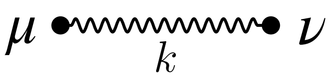
$$\langle A^\mu(k) A^\nu(-k) \rangle = i \frac{\eta_{\mu\nu}}{(k^0 + i0)^2 - \vec{k}^2}$$



$$\langle z^\mu(\omega) z^\nu(-\omega) \rangle = -\frac{i}{m} \frac{\eta_{\mu\nu}}{(\omega + i0)^2}$$

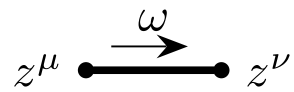
FEYNMAN DIAGRAMMATIC EXPANSION

- Propagators: Photon



$$\langle A^\mu(k) A^\nu(-k) \rangle = i \frac{\eta_{\mu\nu}}{(k^0 + i0)^2 - \vec{k}^2}$$

Worldline



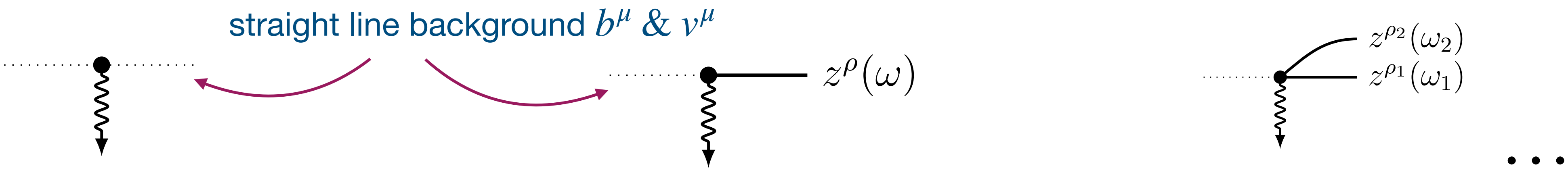
$$\langle z^\mu(\omega) z^\nu(-\omega) \rangle = -\frac{i}{m} \frac{\eta_{\mu\nu}}{(\omega + i0)^2}$$

- Vertices: Arise from only interaction term in action $\tilde{S}$:

$$q_i\, A_\mu[x_i(\tau_i)]\, \dot{x}_i^\mu(\tau_i)$$

$$A_\mu(x(\tau)) = \int_k e^{ik\cdot(b+v\tau+z(\tau))} A_\mu(-k) = \sum_{n=0}^{\infty} \frac{i^n}{n!} \int_k e^{ik\cdot(b+v\tau)} (k \cdot z(\tau))^n\, A_\mu(-k)$$

Worldline-Photon Vertices



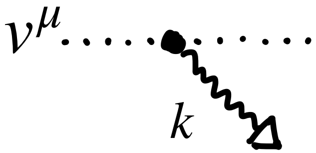
$\sim q\, e^{ik\cdot b} \delta(k \cdot v)\, v^\mu$

$\sim q\, e^{ik\cdot b} \delta(k \cdot v + \omega)\, (\omega \eta^{\mu\rho} + v^\mu k^\rho)$

arbitrary # of $z^{\rho_i}(\omega_i)$'s

EXAMPLES

- Single charge at rest: Coulomb potential



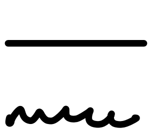
$$= \quad \langle A^\mu(k) \rangle = \frac{q\, v^\mu \delta(v \cdot k)}{k^2}\, e^{ik\cdot b}$$

Put particle at rest & origin:

$$v^\mu = (1, \vec{0}) \qquad b^\mu = 0 \qquad \Rightarrow \qquad \vec{A} = 0$$

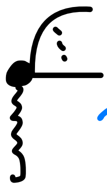
$$A_0(x) = \phi(x) = \int d^4k \langle A_0(k) \rangle = q \int d^3k \frac{1}{\vec{k}^2} e^{-i\vec{k}\cdot\vec{x}} = \frac{q}{4\pi|\vec{x}|}$$

- $\hbar$ counting:



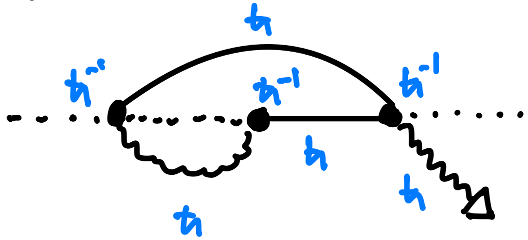
$\sim \hbar$

$\sim \hbar$



$\sim \frac{1}{\hbar}$

E.g:



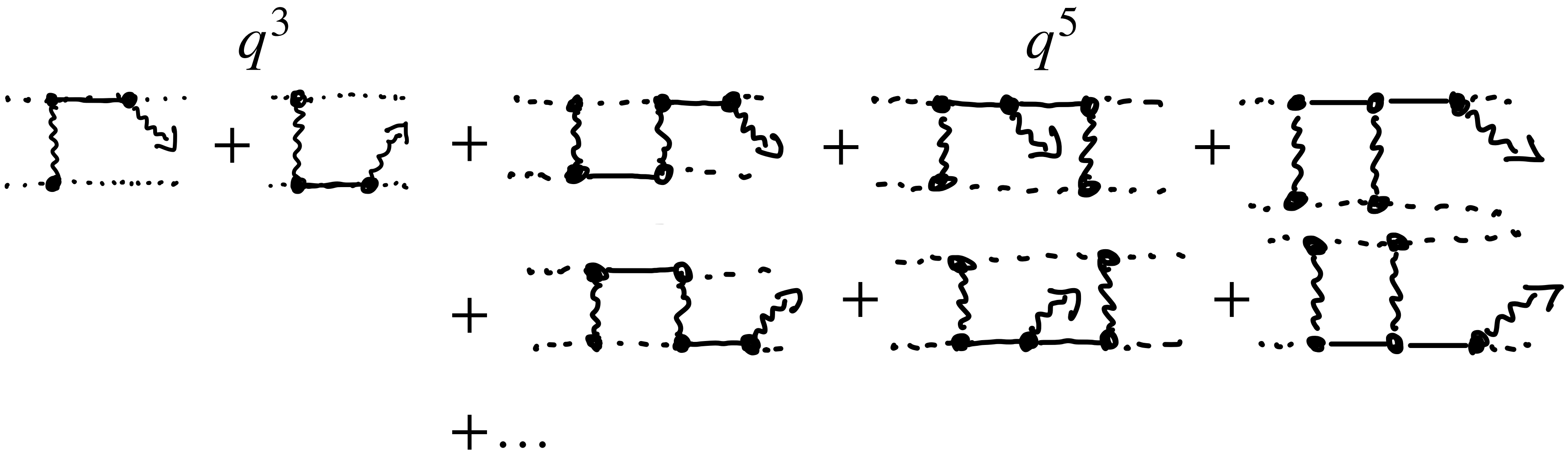
$\sim \hbar$

$\Rightarrow$ Loops are quantum effects!

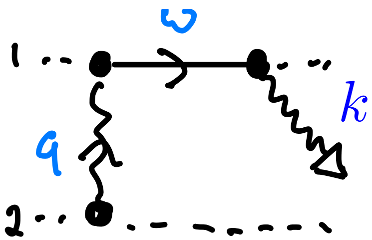
SCATTERING OF TWO CHARGES PARTICLES: BREMSSTRAHLUNG

- Two worldlines

$$\langle A^\mu(k)\rangle =$$



Bremsstrahlung

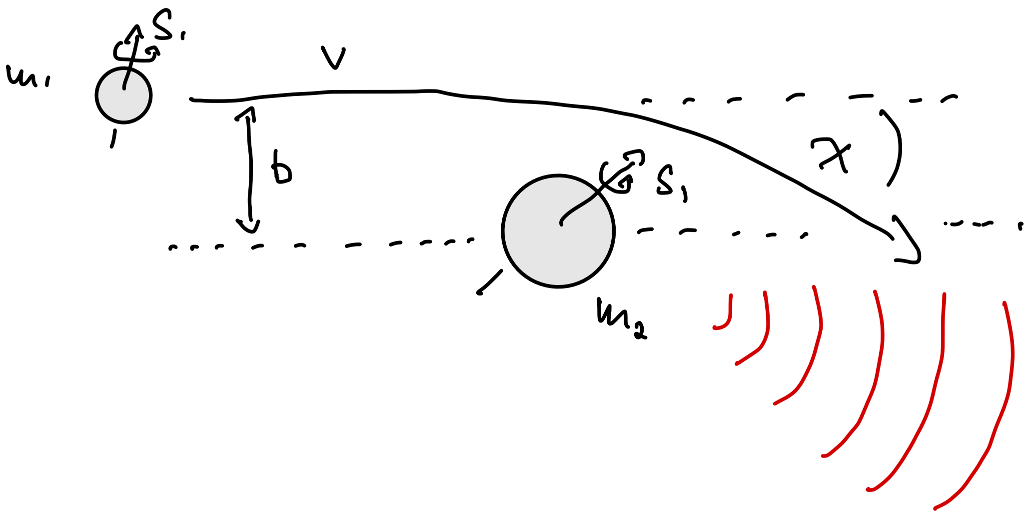


$$= \int_{q,\omega} \frac{\delta(v_2\cdot q)\,\delta(v_1\cdot q-\omega)\delta(\omega - v_1\cdot k)}{q^2\,\omega^2\,k^2} \times [\text{Numerator}]$$

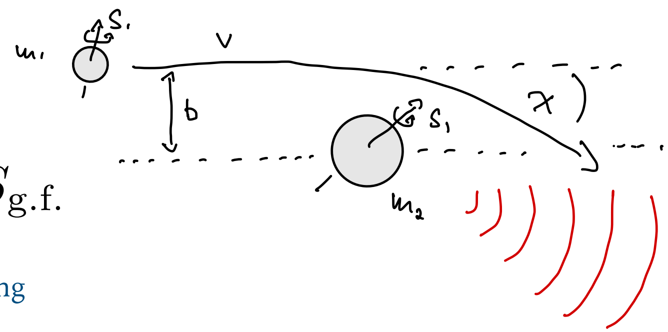
4+1 integrations
versus
3 delta-functions

⇒ Non-trivial integral due to hybrid (4d-1d) theory

THE GENERAL RELATIVISTIC 2-BODY PROBLEM



RELATIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH



Point-particle approximation for BHs (or NSs)

$$S = -\sum_{i=1}^{2} \int d\tau_i \sqrt{g_{\mu\nu} \dot{x}_i^{\mu}(\tau_i) \dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G} \int d^4x \sqrt{-g} R + S_{\text{g.f.}}$$

Point particle approximation — Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} = \frac{\kappa^2}{8} T_{\mu\nu} \qquad \ddot{x}_i^{\mu} + \Gamma^{\mu}{}_{\nu\rho} \dot{x}_i^{\nu} \dot{x}_i^{\rho} = 0$$

Einstein's eqs. — Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G} \sum_{n=0}^{\infty} G^n h_{\mu\nu}^{(n)}(x) \qquad x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\, \tau + \sum_{n=1}^{\infty} G^n z_i^{(n)\,\mu}(\tau)$$

emitted radiation — straight line: „in“ state — deflections

3) Construct observables

Far field waveform:

$$\lim_{r \to \infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t - r, \theta, \varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

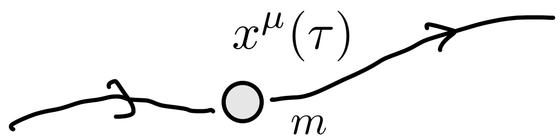
„Impulse“ (change in momentum):

$$\Delta p_i^{\mu} = m_i \dot{x}_i^{\mu} \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau\, \ddot{x}_i^{\mu}(\tau)$$

WORLDLINE QUANTUM FIELD THEORY

Mogull, **JP**, Steinhoff *JHEP* 02 (2021) 048

- Model Black Holes/Neutron Stars as a point particles



$$S_{\mathrm{BH/NS}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^\mu(\tau)\,\dot{x}^\nu(\tau) \quad + \text{[spin \& tidal effects]}$$

They interact through Einstein's gravity:

$$S = S_{\mathrm{BH/NS}} + \frac{1}{16\pi\, G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^\mu(\tau) = b_i^\mu + v_i^\mu\tau + z^\mu(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$

- Path integral quantisation perturbative in Newton's constant G but exact in velocity

$$\langle\mathcal{O}\rangle_{\mathrm{WQFT}} = \int D[h,z]\,\mathcal{O}\,e^{-\frac{i}{\hbar}S[z,h]} \quad \xrightarrow{\hbar \to 0}$$

Tree-level one-point functions $\langle h_{\mu\nu}\rangle$ and $\langle z^\mu\rangle$ solve classical equations of motion

$\Rightarrow$ Advanced quantum field theory technology for classical gravitational wave physics

WORLDLINE QUANTUM FIELD THEORY: PERTURBATIVE SETUP

$$S_{\mathrm{WQFT}} = -\frac{m}{2}\int d\tau\, g_{\mu\nu}\,\dot{x}^{\mu}(\tau)\,\dot{x}^{\nu}(\tau) + \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,R(g)$$

- Scattering scenario: $x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\tau + z^{\mu}(\tau)$ $\qquad g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}$
- Worldline propagators: $z^{\mu} \xrightarrow{\omega} z^{\nu}$ $\qquad \langle z^{\mu}(\omega) z^{\nu}(-\omega)\rangle = -\frac{i}{m}\frac{\eta_{\mu\nu}}{(\omega+i0)^2}$
- Perturbative (quantum) gravity:

$$\sqrt{-g}\,R(g) \xrightarrow[g_{\mu\nu} = \eta_{\mu\nu} + \sqrt{G}\,h_{\mu\nu}]{} \boxed{-\frac{1}{2}h_{\mu\nu}(P^{-1})^{\mu\nu;\rho\sigma}\Box h_{\rho\sigma} + \sqrt{G}[\partial^2 h^3] + \sqrt{G}^2[\partial^2 h^4] + \sqrt{G}^3[\partial^2 h^5] + \ldots}$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

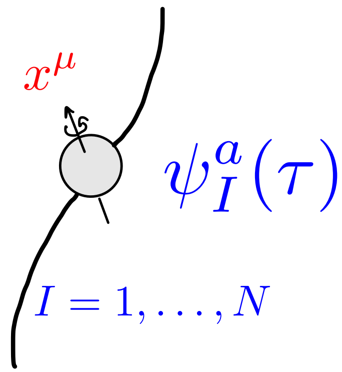
$\Rightarrow$ graviton propagator:

$$\underset{-}{\mu\nu} \overset{\rightarrow}{\underset{k}{\leadsto}} \underset{+}{\rho\sigma} = i\frac{P_{\mu\nu;\rho\sigma}}{(k^0+i0)^2 - \mathbf{k}^2}$$

N.B. need to take retarded propagator (in-in formalism)

PUTTING SPIN ON THE WORLD-LINE

[Jakobsen,Mogull,JP,Steinhoff]



- Hidden **supersymmetry** of spinning-black holes!

  Captures (Spin)^N interactions

- Presently only control spin-orbit & spin-spin interactions ($N = 2$)

$$S_{\mathrm{BH/NS}} = -m \int d\tau \Big[\tfrac{1}{2} g_{\mu\nu} \dot{x}^\mu \dot{x}^\nu + i \bar{\psi} D_\tau \psi + \tfrac{1}{2} R_{abcd} \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d + C_E R_{a\mu b\nu} \dot{x}^\mu \dot{x}^\nu \bar{\psi}^a \psi^b \, \bar{\psi} \cdot \psi \Big]$$

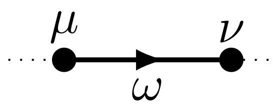
spin degrees of freedom neutron star term

Scattering scenario:

$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau)$$
$$\psi_i^a(\tau) = \Psi_i^a + \psi_i'^a(\tau)$$

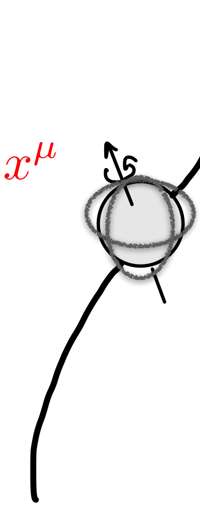
Spin tensor of BHs/NSs $\quad S_i^{ab} = -2im \bar{\psi}_i^{[a} \psi_i^{b]}$

Integrate out $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$ perturbatively!



$$\langle \psi^a(\omega) \bar{\psi}^b(-\omega) \rangle = \frac{-i\eta^{ab}}{m(\omega + i0)}$$

TIDAL INTERACTIONS



- First layer of tidal & finite size effects:

$$S_{\mathrm{tidal}} = m \int d\tau \left[c_{E^2} E_{\mu\nu} E^{\mu\nu} + c_{B^2} B_{\mu\nu} B^{\mu\nu} \right]$$

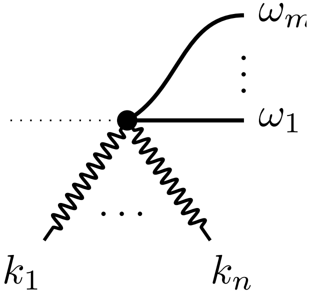
Electric and magnetic curvature:

$$E_{\mu\nu} := R_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta \qquad B_{\mu\nu} := R^*_{\mu\alpha\nu\beta} \dot{x}^\alpha \dot{x}^\beta$$

Wilson coefficients (or „Love numbers“): c_{E^2} & c_{B^2} (vanish for black holes)

WORLDLINE QUANTUM FIELD THEORY: VERTICES

- Worldline vertices: n-gravitons & m world-line fluctuations

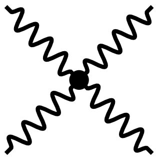
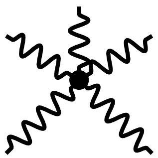
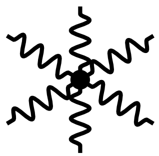
$V_{n|m} =$ 

$$= m\sqrt{G}^n \, e^{ib\cdot\sum_j k_j}\,\delta\left(v\cdot\sum_{j=1}^{n} k_j + \sum_{i=1}^{m}\omega_i\right)\times\begin{pmatrix}\text{polynomial in } \omega_i, k_j \\ \text{of degree } 2n+m \\ \text{depending on } v^\mu,\ S^{\mu\nu}\end{pmatrix}$$

+ C_E, c_{E^2}, c_{B^2} for neutron stars

Energy conservation on worldline

- „Bulk" graviton vertices:

 $\sim \sqrt{G}^2 k^2,$  $\sim \sqrt{G}^3 k^2$  $\sim \sqrt{G}^4 k^2, \ldots$

Four-momentum conservation in bulk

CLAIM:

$$\langle \mathcal{O} \rangle_{\mathrm{WQFT}} = \int D[h, z] \mathcal{O}\, e^{-\frac{i}{\hbar} S[z,h]} \xrightarrow{\hbar \to 0}$$

Tree-level one-point functions $\langle h_{\mu\nu} \rangle$ and $\langle z^{\mu} \rangle$ solve classical equations of motion

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

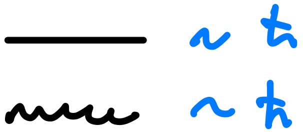
[Jakobsen]

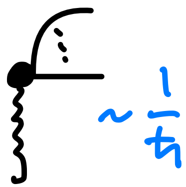
- Action: $S[\Phi_A]$ with fields $\Phi_A(x_A) = \{h_{\mu\nu}(x), z^\mu(\tau)\}$ and coordinates $x_A = \{x^\mu, \tau\}$

- Partition function in the presence of sources

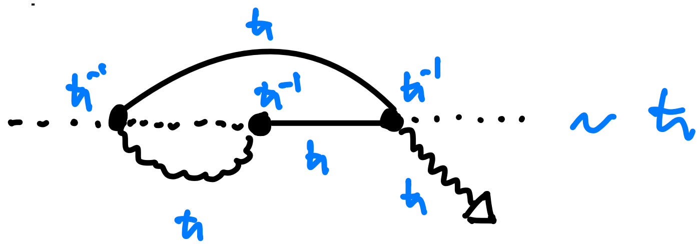
$$Z[J_A] = \int D[\Phi_A] \exp\left[\frac{i}{\hbar}\left(S[\Phi_A] + \sum_A \int dx_A J_A(x_A)\, \Phi_A(x_A)\right)\right]$$

- $\hbar$ counting:





E.g:



$\Rightarrow$ Loops are quantum effects!

- Scalings of connected n-point functions:

$$\langle \Phi_{A_1} \ldots \Phi_{A_n} \rangle_{\text{conn}} \sim \sum_L \hbar^{-1+n+L}$$ (L-loop connected n-point diagrams)

Well defined classical limit only for n=1 and L=0: **Tree-level one-point functions**

CLASSICAL DYNAMICS FROM ONE-POINT FUNCTIONS

[Jakobsen]

- Factorization $$\lim_{\hbar\to 0} \langle \Phi_{A_1}\Phi_{A_2}\dots\Phi_{A_n}\rangle_{\text{discon}} = \langle \Phi_{A_1}\rangle^{\text{tree}}_{\text{con}}\langle \Phi_{A_2}\rangle^{\text{tree}}_{\text{con}}\dots\langle \Phi_{A_n}\rangle^{\text{tree}}_{\text{con}}$$

- Consequence for Schwinger-Dyson equations:

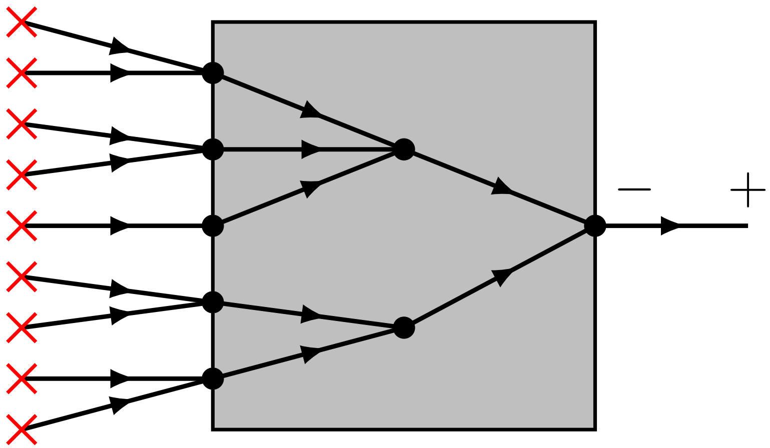
$$\left\langle \frac{\delta S[\Phi_A]}{\delta \Phi_A} \right\rangle = 0 \quad \xrightarrow{\hbar \to 0} \quad \boxed{\frac{\delta S[\langle \Phi_A\rangle_{\text{tree}}]}{\delta \Phi_A} = 0}$$

Tree-level one-point functions solve classical equations of motion

- Importantly $S[\Phi_A]$ must be independent of $\hbar$ (not the case in amplitudes approach - massive field!) -> Key advantage of WQFT approach (no „super classical“ terms)

- Need non-trivial background field configurations for non-vanishing one-point functions

THE IN-IN (SCHWINGER-KELDYSH) FORMALISM FOR WQFT



OR WHY RETARDED PROPAGATORS?

[Jakobsen,Mogull,JP,Sauer]

STANDARD PATH INTEGRAL: IN-OUT FORMALISM

[Galley,Tiglio] [Jordan]

- Hamiltonian formalism: Time evolution operator $\hat{H} = \hat{H}_0 + \hat{H}_{\text{int}}$

Background

$$U_J(T,T') = \mathcal{T} \exp\left[\frac{i}{\hbar}\int_{T'}^{T} dt \int d^3x \left\{ \hat{H}_{\text{int}}[\phi_0(\mathbf{x},t), Q(\mathbf{x},t)] + J(x)\, \phi_0(\mathbf{x},t) \right\}\right]$$

Free fields

- Heisenberg picture: $\phi_H(\mathbf{x},t) = U_0(-\infty,t)\, \phi_0(\mathbf{x},t) U_0(t,-\infty)$

- Path integral representation:

$$\langle 0 | U_J(\infty,-\infty) | 0 \rangle = \int [D\phi] \exp\left[\frac{i}{\hbar}\left(S[\phi,Q] + \int d^4x\, J(x)\, \phi(x)\right)\right] = \exp\left[\frac{i}{\hbar} W[J]\right]$$

- One-point function:

$= U_0(t,-\infty)\, \phi_H(\mathbf{x},t) U_0(-\infty,t)$

$$\langle \phi_H(\mathbf{x},t) \rangle_{\text{in-out}} = \frac{\delta W[J]}{\delta J(\mathbf{x},t)}\Big|_{J=0} = \langle 0 |\, U_0(\infty,t)\, \phi_0(\mathbf{x},t)\, U_0(t,-\infty)\, | 0 \rangle$$

$$= \langle 0 |\, U_0(\infty,-\infty)\, \phi_H(\mathbf{x},t)\, | 0 \rangle_{\text{in-out}} = {}_{\text{out}}\langle 0 | \phi_H(\mathbf{x},t) | 0 \rangle_{\text{in}}$$

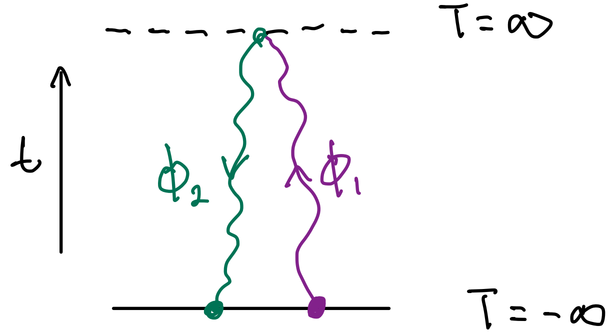
IN-IN (SCHWINGER-KELDYSH) FORMALISM

[Galley,Tiglio] [Jordan]

- Standard path integral yields $\langle \phi_H(x) \rangle_{\mathrm{in-out}} = {}_{\mathrm{out}}\langle 0 | \phi_H(x) | 0 \rangle_{\mathrm{in}}$ but want

$$\langle \phi_H(x) \rangle_{\mathrm{in-in}} = {}_{\mathrm{in}}\langle 0 | \phi_H(x) | 0 \rangle_{\mathrm{in}} = \langle 0 | U(-\infty, t)\, \phi_0(t, \mathbf{x})\, U(t, -\infty)\, | 0 \rangle$$

$\Rightarrow$ need two evolution operators: Double the fields!



$$Z[J_1, J_2] = \langle 0 |\, U_{J_1}(-\infty, \infty)\, U_{J_1}(\infty, -\infty)\, | 0 \rangle$$

$$= \int D[\phi_1, \phi_2] \exp\left[\frac{i}{\hbar} \left(S[\phi_1] - S[\phi_2] + \int_x J_1(x)\, \phi_1(x) - J_2(x)\, \phi_2(x) \right) \right]$$

Boundary conditions:

$$\phi_1(t = +\infty, \mathbf{x}) = \phi_2(t = +\infty, \mathbf{x})$$
$$\phi_1(t = -\infty, \mathbf{x}) = \phi_2(t = -\infty, \mathbf{x}) = 0$$

$$\frac{1}{Z[0,0]} \frac{\delta Z[J_1, J_2]}{\delta J_1(x)} \bigg|_{J_i = 0} = \langle \Phi_H(x) \rangle_{\mathrm{in-in}}$$

KELDYSH BASIS:

$$\phi_+ = \frac{1}{2}(\phi_1 + \phi_2) \qquad\qquad \phi_- = \phi_1 - \phi_2$$

- This yields

$$Z[J_\pm] = \int D[\phi_+, \phi_-] \, \exp\left[\frac{i}{\hbar}\left(S[\phi_+ + \frac{1}{2}\phi_-] - S[\phi_+ - \frac{1}{2}\phi_-] + \int d^4x (J_+\phi_- + J_-\phi_+)\right)\right]$$

- Propagator matrix from free part:

$$\langle \phi_a(x)\, \phi_b(y) \rangle = \begin{array}{cc} \begin{array}{cc} \quad + \quad & \quad - \quad \end{array} & \\ \begin{pmatrix} \frac{1}{2} D_H(x,y) & D_{\mathrm{ret}}(x,y) \\ -D_{\mathrm{adv}}(x,y) & 0 \end{pmatrix} & \begin{array}{c} + \\ - \end{array} \end{array}$$

irrelevant @ tree-level

$$\widetilde{D}_{\mathrm{ret}}(k) = \underset{-}{\bullet}\!\!\longrightarrow\!\!\underset{+}{\bullet} = \frac{-i}{(k^0 + i0)^2 - \mathbf{k}^2},$$

$$\widetilde{D}_{\mathrm{adv}}(k) = \underset{+}{\bullet}\!\!\longleftarrow\!\!\underset{-}{\bullet} = \frac{-i}{(k^0 - i0)^2 - \mathbf{k}^2},$$

- Vertices from:

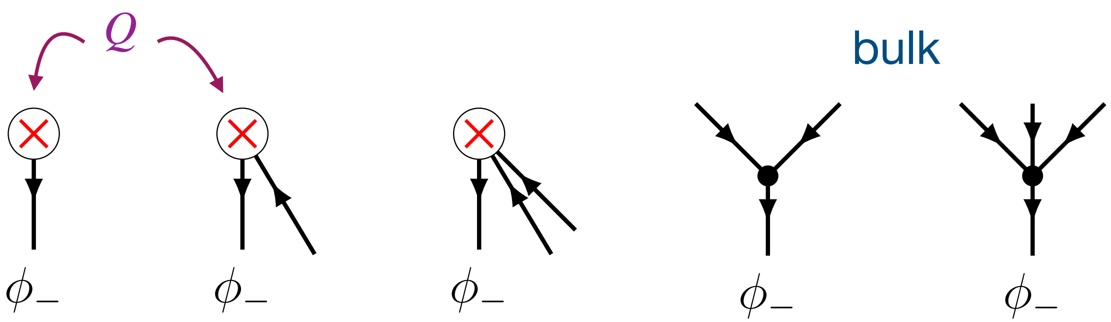
$$S_{\mathrm{int}}[\phi_+ + \frac{1}{2}\phi_-] - S_{\mathrm{int}}[\phi_+ - \frac{1}{2}\phi_-] = \phi_- \left(\frac{\delta S_{\mathrm{int}}[\phi]}{\delta\phi}\right)\bigg|_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$

$\Rightarrow$ only **odd** number of ϕ_- legs!

ONE POINT FUNCTIONS @ TREE-LEVEL:

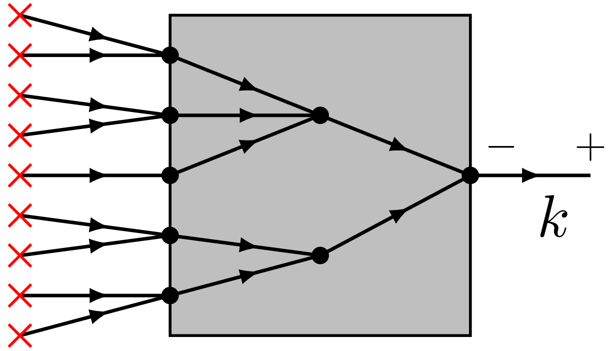
- Vertices including background field Q

$$S_{\rm int}[\phi; Q] \to \phi_- \left(\frac{\delta S_{\rm int}[\phi, Q]}{\delta \phi} \right)\Big|_{\phi \to \phi_+} + \mathcal{O}(\phi_-^3)$$



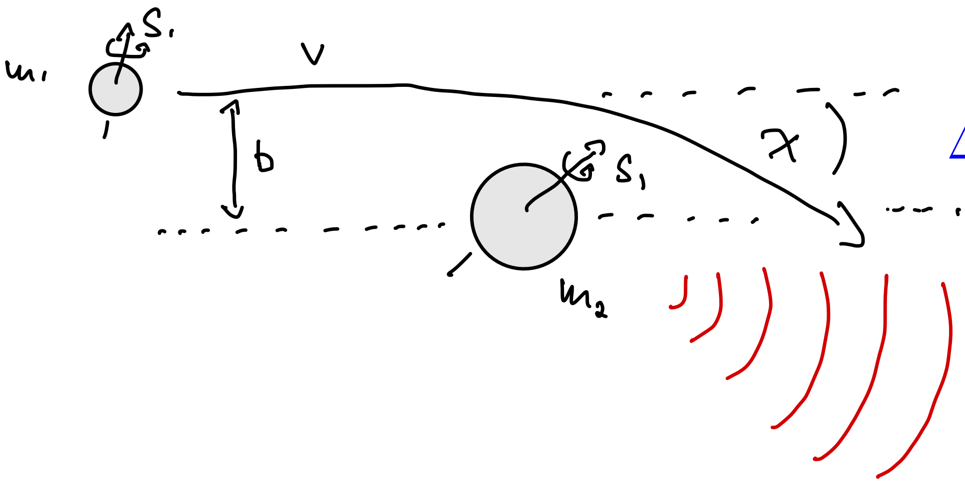
- One point function:

$$\langle \phi(k) \rangle_{\text{in-in}} =$$



Only retarded propagators contribute!

WQFT OBSERVABLES

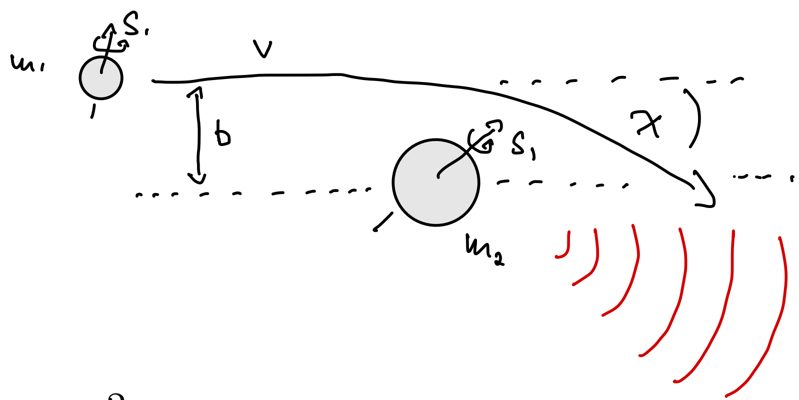


$$\Delta p_1^{\mu} = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

Spin-less BH/NS scattering:



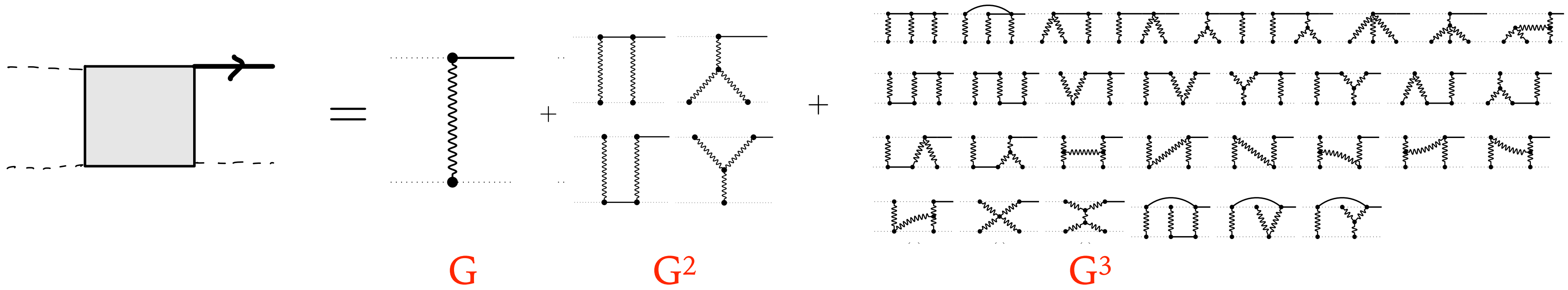
1) Impulse (change of momentum)

$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega\to 0}$$

Fourier trans.

Needs sum of all graphs with outgoing z-line: Jakobsen, Mogull, ***PRL*** 128 *(2022) 14;* Jakobsen, Mogull, **JP**, Sauer, *JHEP* 10 (2022) 128

$$\langle \Delta p^\mu \rangle =$$

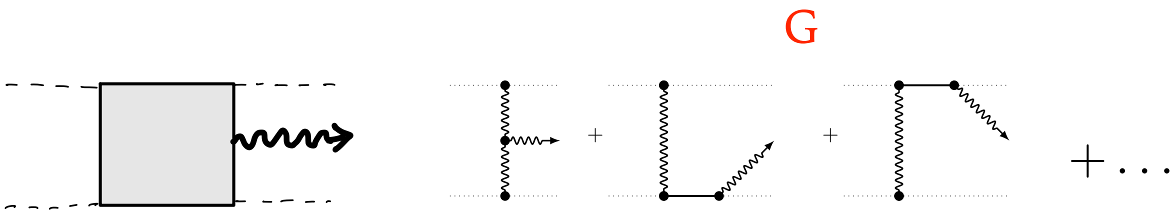


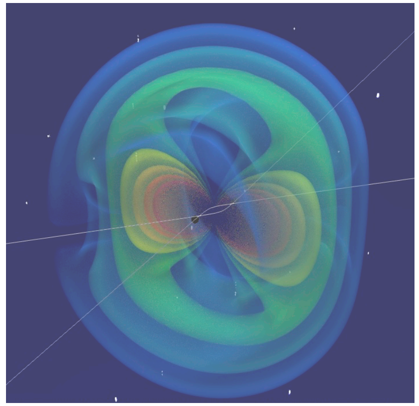
OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

2) Emitted Waveform (Gravitational Bremsstrahlung)

Jakobsen, Mogull, **JP**, Steinhoff, ***PRL*** 126 (2021) 20, ***PRL*** 128 (2022) 1

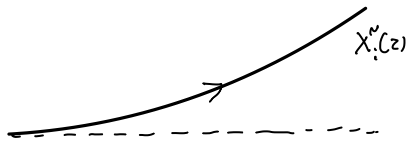
$\langle h_{\mu\nu} \rangle$





3) Trajectory!

$$X^{\mu}_{i}(\tau) = b^{\mu}_{i} + v^{\mu}_{i}\,\tau + \int d\omega\, e^{i\omega\cdot\tau} \left\langle z^{\mu}_{i}(\omega) \right\rangle_{WQFT}$$

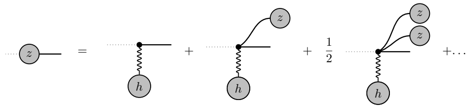


WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion
(tree-level!)



Insert Feynman rules

Form script
(4PM: 5g vertex)

Tensor reduction to
scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \ldots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master
Integral

Fire/Kira
Reduced symmetries
due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff.
Eq. technique

Single scale
integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions
in static limit (PN) $\gamma \to 1$

Method of regions
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ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

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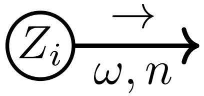
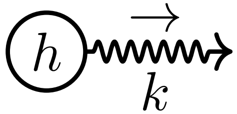
BERENDS-GIELE TYPE RECURSION

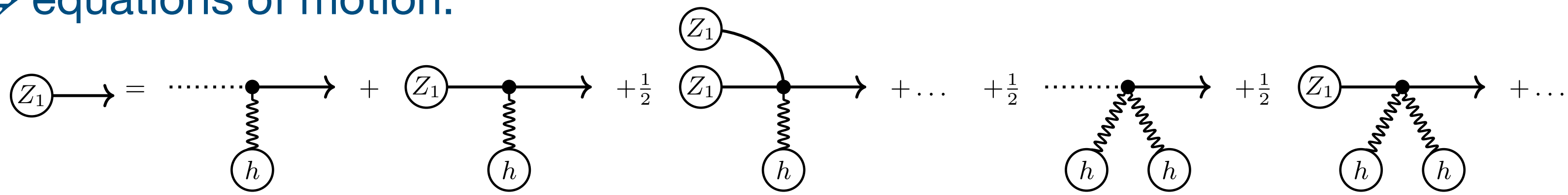
[Jakobsen,Mogull,JP,Sauer]

- One-point functions: $\langle Z_i(\omega)\rangle =$ 

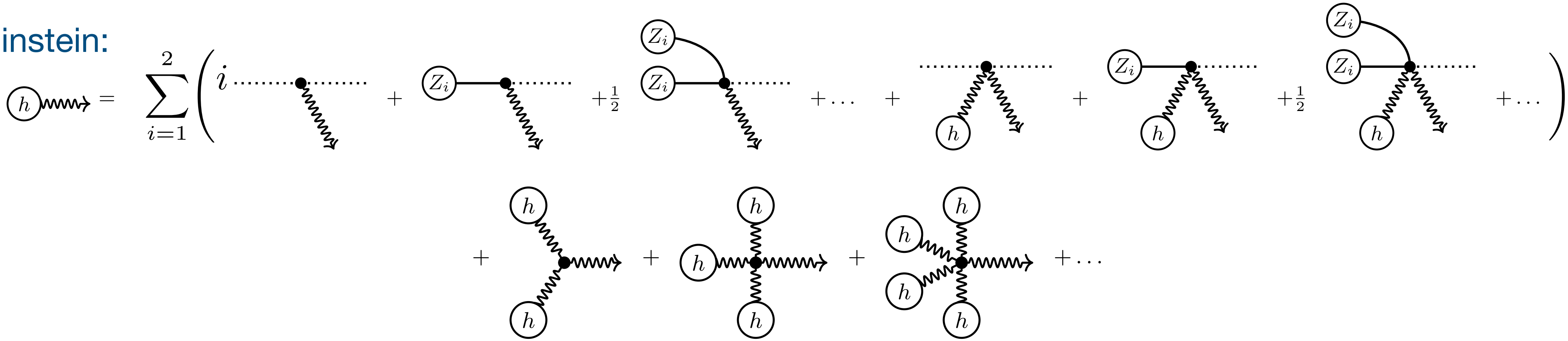
$\langle h_{\mu\nu}(k)\rangle =$ 

- Recursion ⇔ equations of motion:

Geodesic:



Einstein:



Causality flow implented

WORKFLOW WITH RETARDED INTEGRALS

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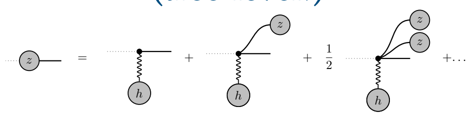

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Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

$i\epsilon$

$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

Tensor reduction to scalar integrals

STRUCTURE OF WQFT INTEGRALS: IMPULSE & SPIN KICK

[Jakobsen,Mogull,JP,Sauer]

- Order **n-PM** : Single scale **(n-1)-loop** integral

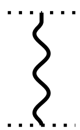
$$I_{\mathrm{nPM}} = \int_q e^{-q\cdot b}\delta(q\cdot v_1)\delta(q\cdot v_2)\int_{l_1,l_2\ldots l_{n-1}} \frac{\mathrm{num}[l_i]}{D_1\ldots D_j}\delta(l_1\cdot v_*)\delta(l_2\cdot v_*)\ldots\delta(l_{n-1}\cdot v_*)$$

$$v_* \in \{v_1, v_2\}$$

Retarded propagators $D_i(l_i, q, v_*)$ are linear $(l_i \cdot v_*) \pm i0$ or quadratic $(l_i + q)^2$

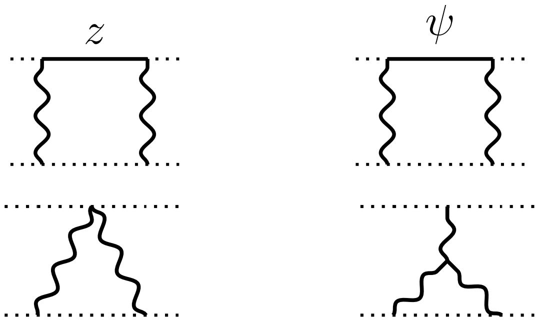
As $v_*^2 = 1$ scale q factors out, left with single parameter integral $\gamma = v_1 \cdot v_2$

- **1PM: Trivial - pure Fourier transform**



$$\sim Gm_1m_2$$

- **2PM: 1-loop**



$$\sim G^2m_1m_2^2$$

$$D_1 = l\cdot v_1 \pm i\epsilon\ ,$$
$$D_2 = l^2\ ,$$
$$D_3 = (l+q)^2\ .$$

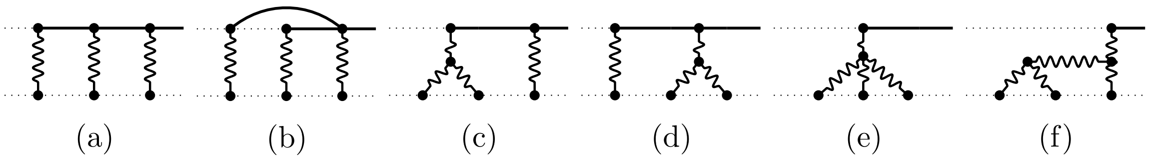
$$\int_l \frac{\mathrm{num}[l]}{D_1D_2D_3}\delta(l\cdot v_*)$$

Tensor reduction to scalar integrals

3PM

[Jakobsen,Mogull,JP,Sauer]

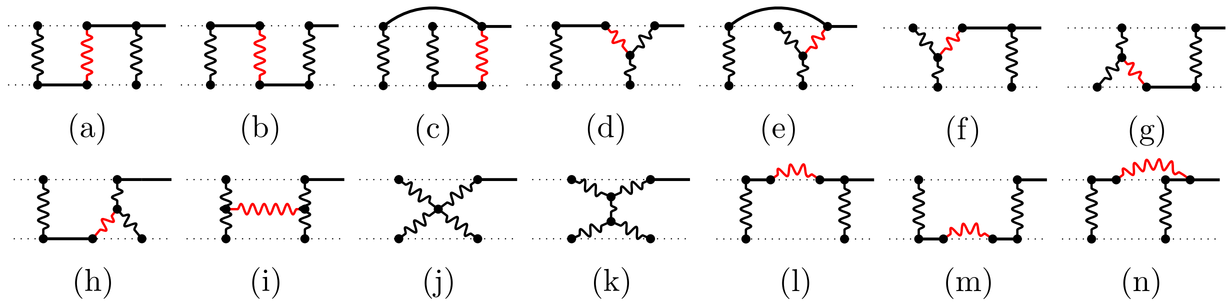
1) Test body diagrams (geodesic motion in Schwarzschild background):



$\sim G^3 m_1\, m_2^3$

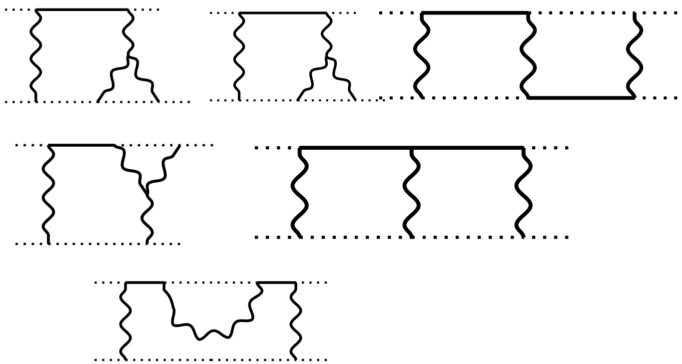
A 2-loop computation!

2) Comparable mass diagrams (i0 prescription relevant for red propagators):



$\sim G^3 m_1^2\, m_2^2$

6 top sectors integrals:



Integral family

$$\boxed{\int_{l_1 l_2} \frac{\delta(l_1\cdot v_{\bar{\imath}_1})\delta(l_2\cdot v_{\bar{\imath}_2})}{\prod_{j=1}^{7} D_j^{n_j}}}$$

$$D_1 = l_1\cdot v_{i_1} + \sigma_1 i0\ , \qquad D_2 = l_2\cdot v_{i_2} + \sigma_2 i0\ ,$$

$$\boxed{D_3 = (k^0 + \sigma_3 i0)^2 - \boldsymbol{k}^2}$$

$$D_4 = l_1^2\ , \qquad D_5 = l_2^2\ ,$$

$$D_6 = (l_1+q)^2\ , \qquad D_7 = (l_2+q)^2\ .$$

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[Jakobsen,Mogull,JP,Sauer]

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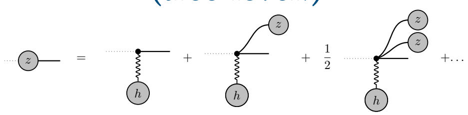
Berends-Giele recursion
(tree-level!)



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$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$

ΔS_i^μ

Find same set of Master integrals for spin and tidal effects@ 3PM

RESULT IMPULSE @ 3PM PRECISION:

[Jakobsen,Mogull,JP,Sauer]

$$\boxed{\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu] - v_2 \cdot P_{\rm rad}\, w_2^\mu}$$

$$w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1}$$

$$\gamma = v_1 \cdot v_2$$

- Scattering angle:

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{\text{1PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{\text{2PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^3\left(2\frac{64\gamma^6-120\gamma^4+60\gamma^2-5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4-12\gamma^2-3)}{(\gamma^2-1)}\frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)}_{\text{3PM conservative}}$$

$$+ \underbrace{\left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3} + \frac{1}{v^2} + \frac{(3v^2-1)}{v^3}\text{arccosh}\gamma\right)}_{\text{3PM radiation-reaction}}$$

$$\Gamma = E/M = \sqrt{1 + 2\nu(\gamma - 1)}$$

$$\nu = \frac{m_1 m_2}{M^2}$$

- Radiated 4-momentum: $P^\mu_{\rm rad} = -\Delta p_1^\mu - \Delta p_2^\mu$ Dissipation! Need for retarded propagators

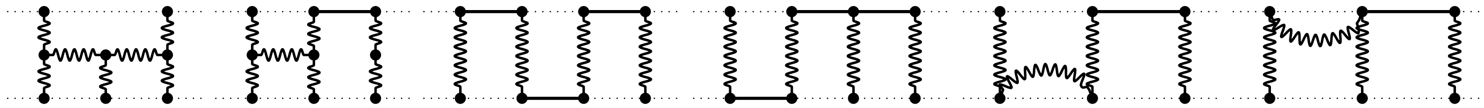
$$P^\mu_{\rm rad} = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma + 1}\left[e_1 + e_2 \log\left(\frac{\gamma+1}{2}\right) + e_3 \frac{\text{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4 - 30\gamma^2 + 11)}{16(\gamma^2-1)^{3/2}}.$$

A GLIMPSE @ 4PM:



[Jakobsen,Mogull,JP,Sauer]

- Two integral families:

$$I^{[i](\sigma_1,\sigma_2,\ldots,\sigma_8)}_{n_1,n_2,\ldots,n_{12}} = \int_{l_1,l_2,l_3} \frac{\delta\!\!\!{}^-(l_1\cdot v_1)\delta\!\!\!{}^-(l_2\cdot v_1)\delta\!\!\!{}^-(l_3\cdot v_i)}{D_1^{n_1}D_2^{n_2}\ldots D_{12}^{n_{12}}}$$

$$\begin{aligned}
D_j &= l_j\cdot v_{i_j} + i0^+\sigma_j\\
D_4 &= -(l_1+l_2+l_3+q)^2 - i0^+\sigma_4 v\cdot(l_1+l_2+l_3)\\
D_5 &= -(l_1+l_2+q)^2 - i0^+\sigma_5 v\cdot(l_1+l_2)\\
D_{5+k} &= -(l_k+l_3)^2 - i0^+\sigma_{6+k} v\cdot(l_k+l_3)\\
D_{7+j} &= -l_j^2\,, \qquad D_{10+k} = -(l_k+q)^2\,.
\end{aligned}$$

$$J^{(\sigma_1,\sigma_2,\ldots,\sigma_5)}_{n_1,n_2,\ldots,n_{12}} := \int_{\ell_1,\ell_2,\ell_3} \frac{\delta\!\!\!{}^-(\ell_1\cdot v_1)\delta\!\!\!{}^-(\ell_2\cdot v_1)\delta\!\!\!{}^-(\ell_3\cdot v_2)}{D_1^{n_1}D_2^{n_2}\ldots D_{12}^{n_{12}}}$$

$$\begin{aligned}
D_j &= \ell_j\cdot v_i + i0^+\sigma_j\\
D_4 &= -(\ell_1-\ell_3)^2 - i0^+\sigma_4 v\cdot(\ell_1-\ell_3)\\
D_5 &= -(\ell_2-\ell_3)^2 - i0^+\sigma_5 v\cdot(\ell_2-\ell_3)\\
D_6 &= -(\ell_1-\ell_2)^2\,, \quad D_{6+j} = -\ell_j^2\,, \quad D_{9+j} = -(\ell_j+q)^2
\end{aligned}$$

- Function space:

$$F^{(-1)} = \left\{\mathrm{arccosh}[\gamma], \log[\gamma], \log[\gamma_\pm], E\left[\frac{\gamma_-}{\gamma_+}\right], K\left[\frac{\gamma_-}{\gamma_+}\right]\right\}$$

$$F^{(-2)} = \left\{\mathrm{Li}_2\left[\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[-\frac{\gamma_-}{\gamma_+}\right], \mathrm{Li}_2\left[\sqrt{\frac{\gamma_-}{\gamma_+}}\right]\right\}$$

Elliptics appear!

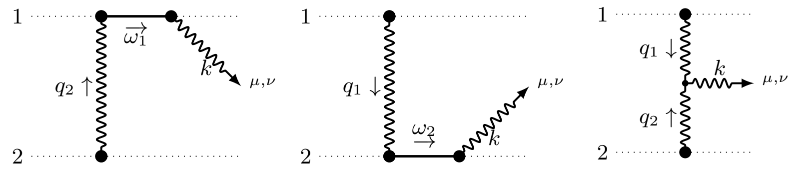
$$\gamma = v_1\cdot v_2 \qquad (\gamma_\pm := \gamma\pm 1)$$

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy : Ω

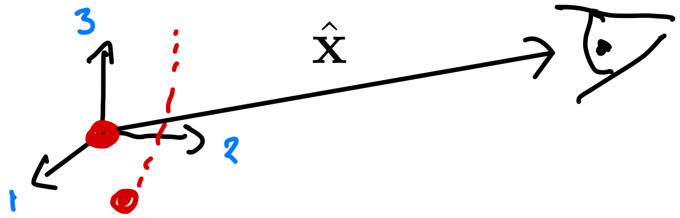
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

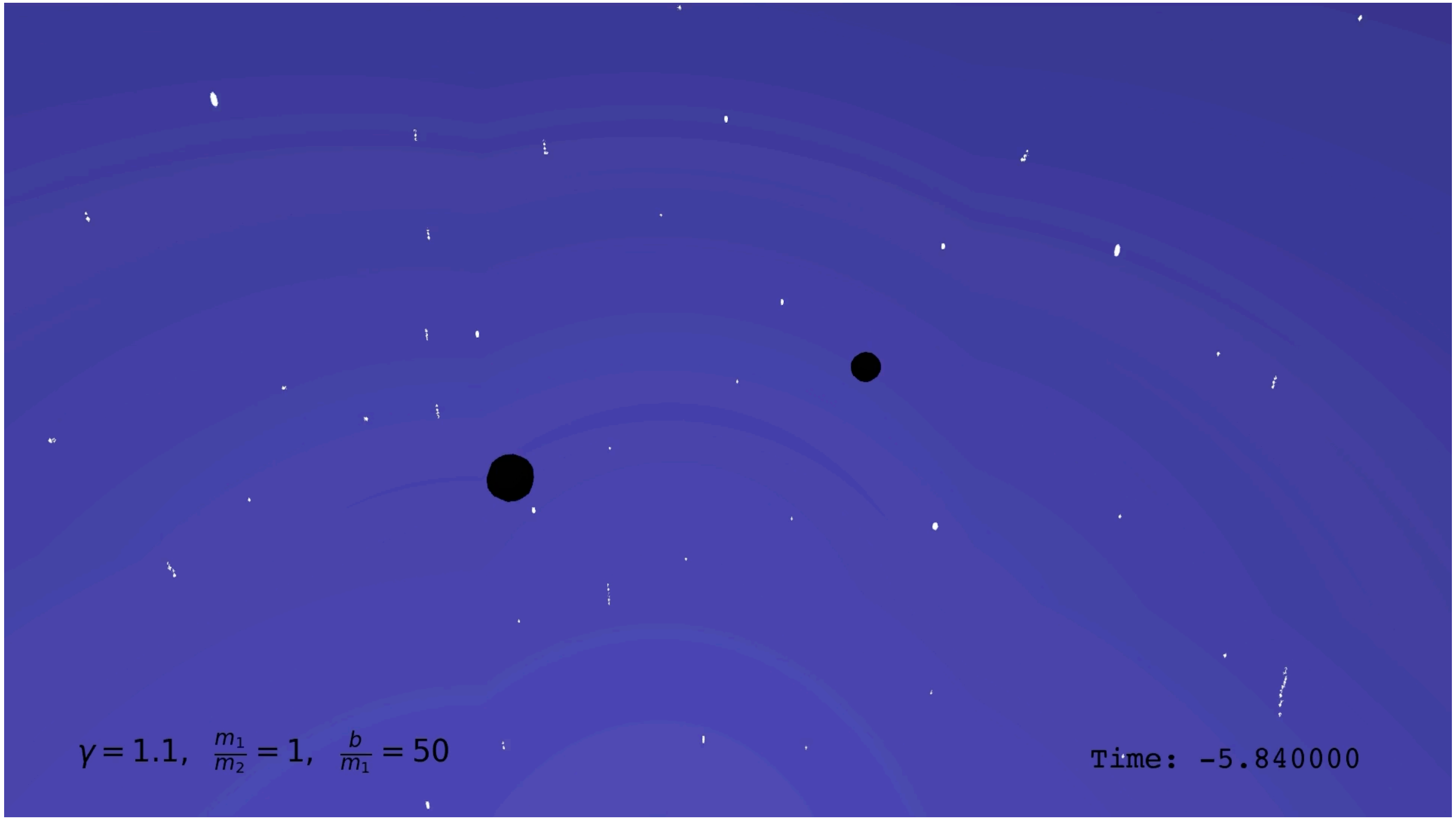
The waveform has two polarizations

$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



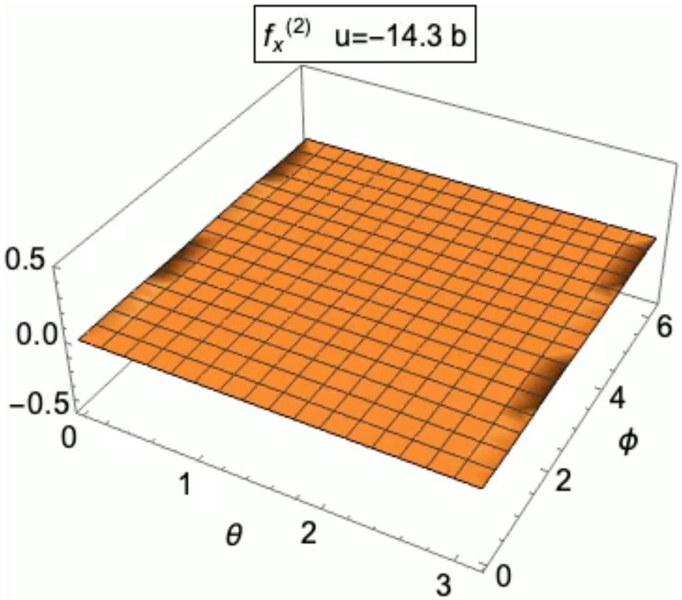
Visualization: Plus-Polarization $f_+^{(2)}$

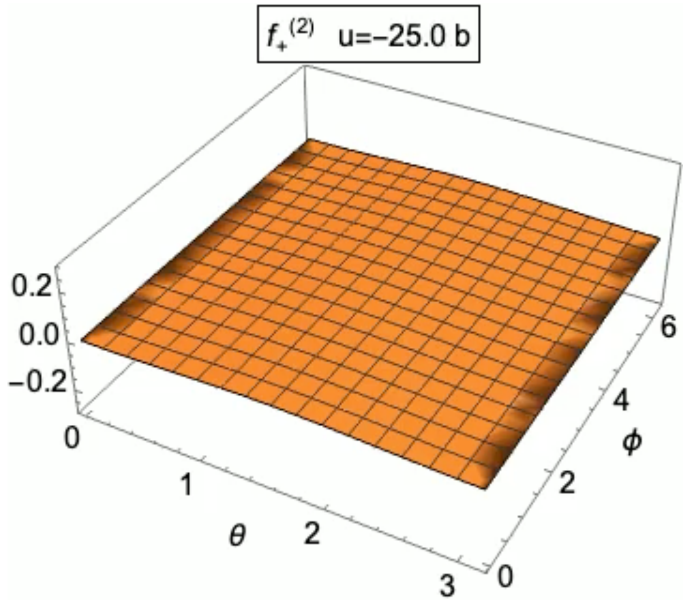


$$v = 0.4$$

[HU/AEI BSc thesis Babayemi]

Memory effect



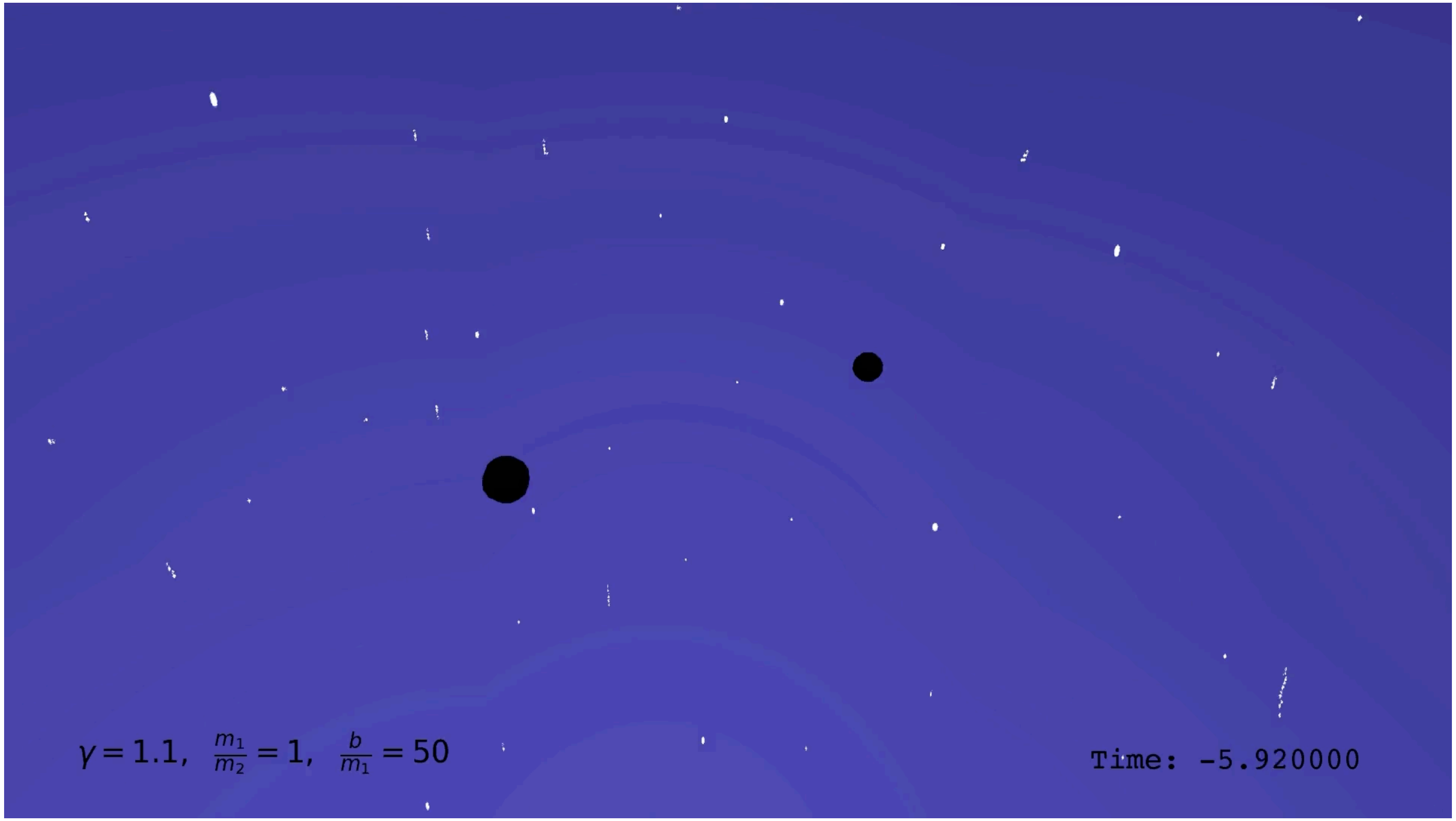


$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$

$$\rho = (1, \hat{\mathbf{x}})$$

$$\gamma = v_1 \cdot v_2$$

Visualization: Plus-Polarization curvature



$$v = 0.4$$

More visualisations at: https://www.youtube.com/channel/UC5UVcydoMoG7ILkJo9bik1w

EXTRACT GRAVITATIONAL POTENTIAL IN PM EXPANSION

[Jakobsen,Mogull]

- Make ansatz for 2-body Hamiltonian

$$H = \sqrt{\vec{p}^{\,2}+m_1^2} + \sqrt{\vec{p}^{\,2}+m_2^2} + V(\vec{r},\vec{p},\vec{S}_i) \qquad \vec{L} = \vec{r}\times\vec{p}$$

$$V = \sum_A \mathcal{O}^A V^A \qquad \mathcal{O}^A = \left\{1, \frac{\vec{L}\cdot\vec{S}_i}{r^2}, \frac{\vec{r}\cdot\vec{S}_i\,\vec{r}\cdot\vec{S}_j}{r^4}, \frac{\vec{S}_i\cdot\vec{S}_j}{r^2}, \frac{\vec{p}\cdot\vec{S}_i\,\vec{p}\cdot\vec{S}_j}{r^2}\right\}$$

$$V^A = \sum_n \left(\frac{G}{r}\right)^n c_n^A(|\vec{p}|)$$

Fix free coefficients c_n^A by matching to scattering data Δp_1^μ and $\Delta s_1^{\mu\nu}$ by solving Hamilton's eqs.

$$\dot{\vec{r}} = \frac{\partial H}{\partial \vec{p}}, \quad \dot{\vec{p}} = -\frac{\partial H}{\partial \vec{r}}, \quad \dot{\vec{S}}_i = -\vec{S}_i \times \frac{\partial H}{\partial \vec{S}_i}$$

Can make contact to bound problem

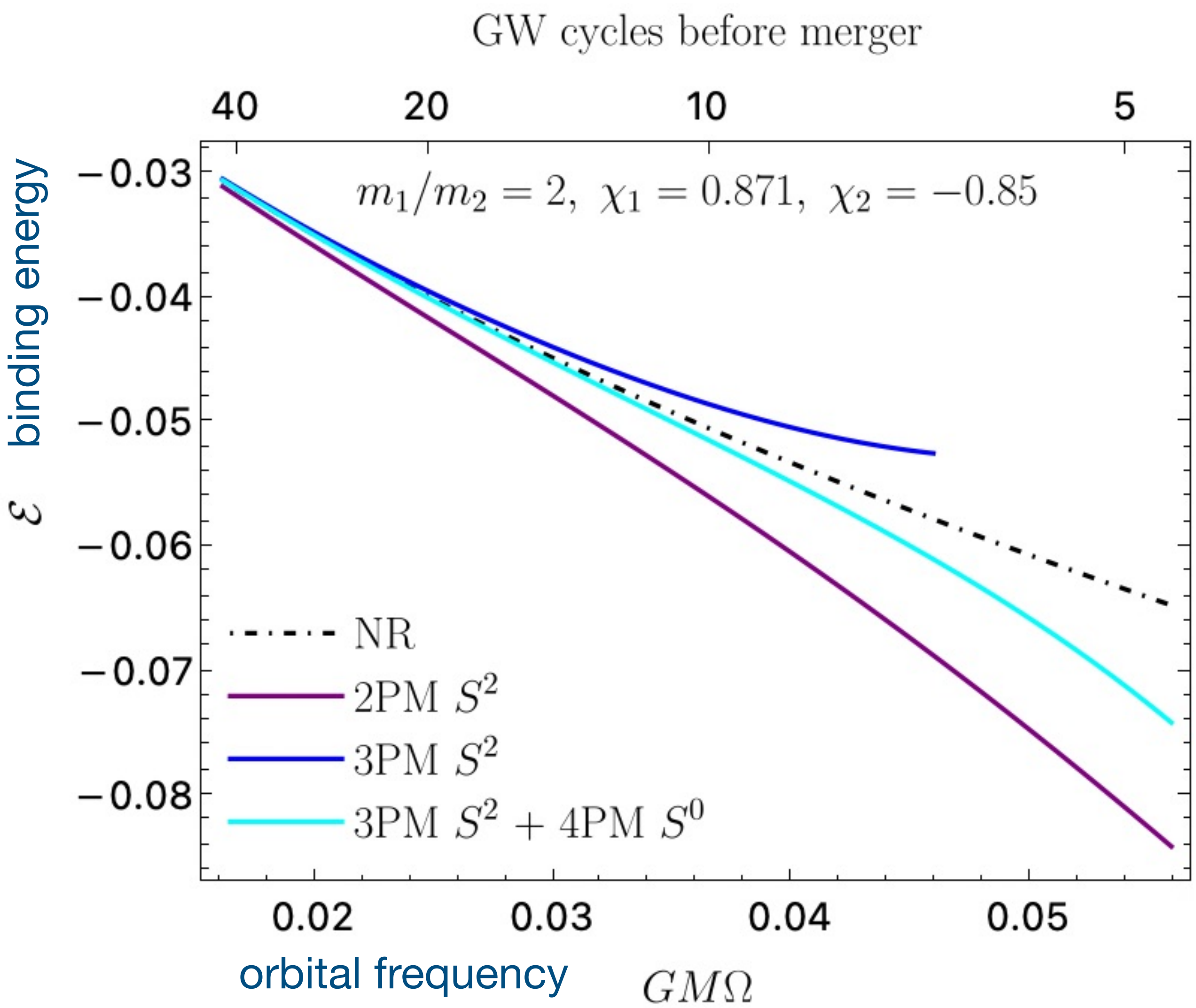
BINDING ENERGY: COMPARISON TO NUMERICAL RELATIVITY

Use 2-body Hamiltonian to compute binding energy $\mathscr{E} = \dfrac{E - M}{\mu}$ for circular orbits and and aligned spins to compare to numerical relativity

$$E = H(p_r, r, J; \chi_1, \chi_2)$$

$$\chi_1 = \frac{a_1}{m_1}, \quad \chi_2 = \frac{a_2}{m_2}$$

$$\Omega(J, r) = \partial H / \partial J$$



PM SCATTERING STATE-OF-THE-ART

WQFT [us]

WEFT **Worldline effective theory**
[Källin,Porto,Dlapa,Cho,Liu,..]
[Riva,Vernizzi,Mougiakakos..]

HEFT **Heavy BH effective theory**
[Aoude,Haddad,Helset,Damgaard]
[Brandhuber,Travaglini,Chen]

Amps **Scattering amplitudes**
[Bern,Roiban,Shen,Parra-Martinez,Ruf,..]
[Bjerrum-Bohr,Damgaard,Vanhove,..]
[Di Vecchia,Veneziano,Heissenberg,Russo][Solon,Cheung,..][Huang,..]
[Guevera,Ochirov,Vines,…]
[Johansson,Pichini[Kosower,O'Connell,Maybee,Cristofoli, Gonzo…]

	deflection & spin kick				waveform			
	plain	spin²	spin>2	tidal	plain	spin²	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps	WQFT WEFT Amps	WQFT WEFT (Amps)	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT (Amps)		WQFT WEFT	HEFT Amps			~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT		WQFT WEFT				
4PM w/o r-r	WEFT Amps							~ 3-loop
4PM w r-r	WEFT							

r-r: Radiation-reaction

(…) : partial results

SUMMARY

WQFT: Highly efficient technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom & focus on observables (=one-point functions)
- Only compute tree-level diagrams (=classical theory). No „super-classical“ contributions
- IN-IN Formalism: Take all propagators retarded.
- Include spin degrees of freedom through Graßmann odd vectors on the world-line (spinning particle)
- Hidden Supersymmetry = Spin Supplementary Condition

OUTLOOK

WQFT still needs to be extended:

- Higher precision (4PM)
- Higher spin (beyond Spin squared)
- Bound orbits? Relation to EOB
- Relation to self force expansion

WE ARE HIRING!

- Fall 23: Long Term Postdoc (5y), 1 PhD
- Fall 24: Postdoc (4y), 1 PhD

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Thank you for your attention!