CLASSICAL BLACK HOLE SCATTERING FROM A WORLDLINE QFT

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Based on joint work with

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2201.07778, ***PRL*** 128 *(2022) 14*

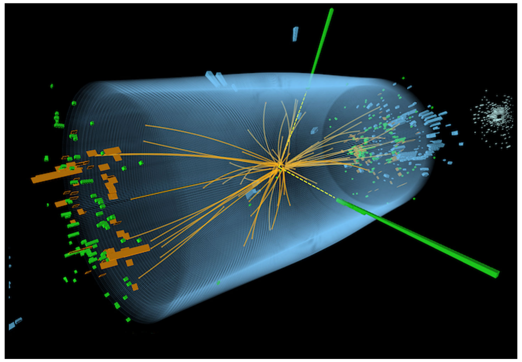
2207.00569,

RTG 2575: Rethinking Quantum Field Theory

Quantum Universe Cluster Day, DESY, 09/22

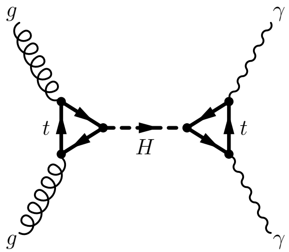
Particle Physics: Paradigmatic experiment is Scattering in Colliders

Theory: Relativistic Quantum Field Theory (QFT)

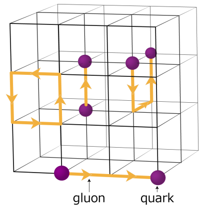


$$\mathcal{L} = -\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} + i\bar{\psi}\not{D}\psi + \psi_i Y_{ij}\psi_j\,\phi + |D_\mu\phi|^2 - \lambda|\phi|^4 - m^2\,|\phi|^2$$

path integral quantization

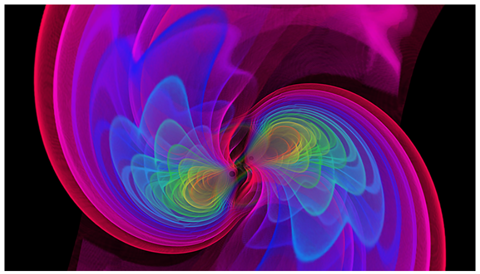


Perturbative QFT: S-matrix



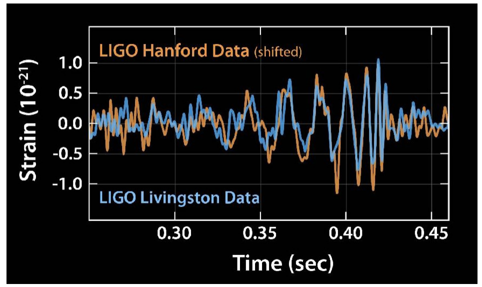
Lattice Field Thy: Bound system

Gravity: Gravitational wave emission in Black Hole and Neutron Star encounters now routinely measured in LIGO-Virgo-Karga GW detectors



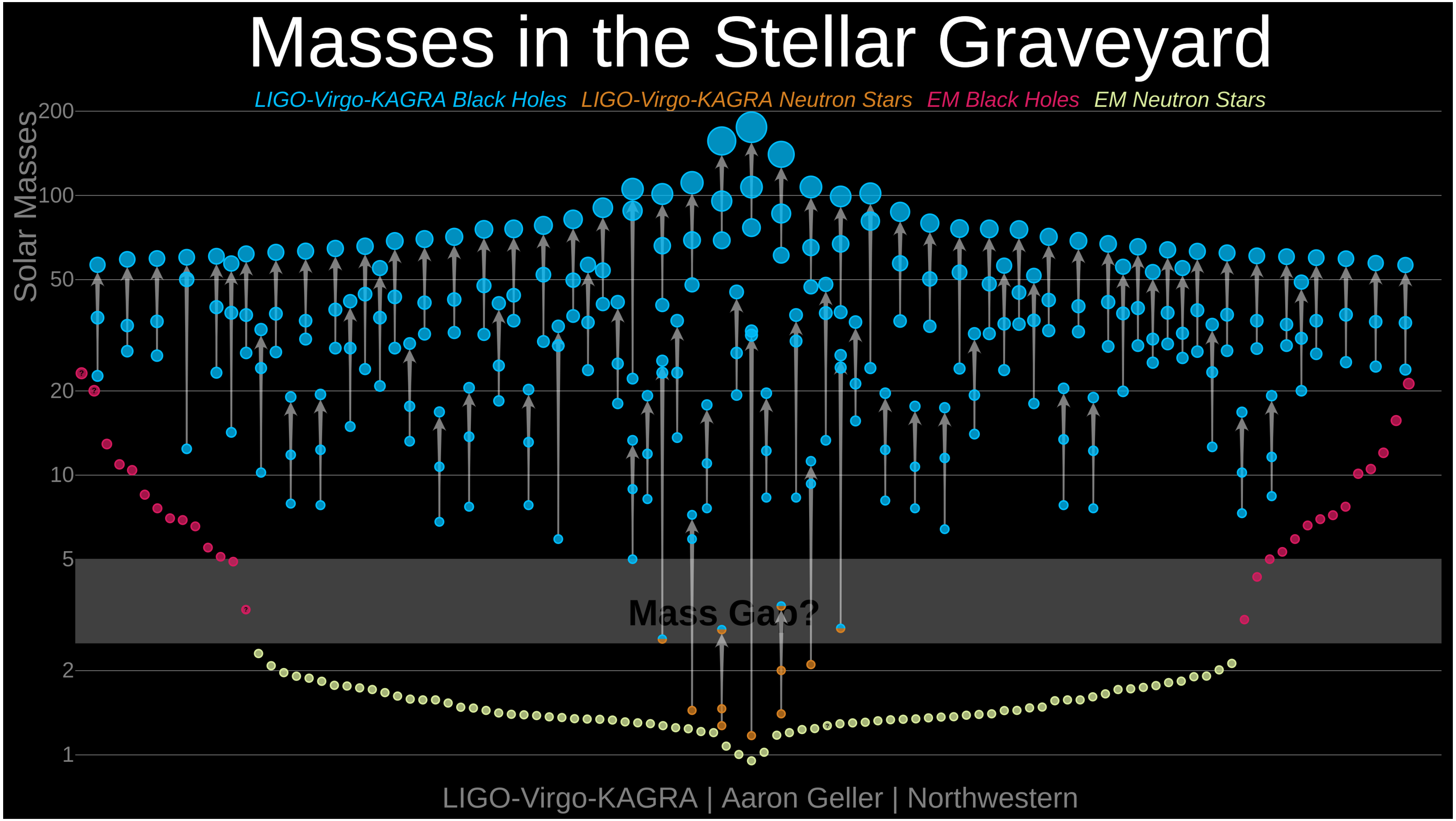
Classical radiative field theory

$$\mathcal{L} = \frac{1}{16\pi G}\sqrt{-g}\,R + \mathcal{L}_{\text{Matter}}$$



Theory: Need for high-precision solution of classical gravitational two-body problem. **Here:** Apply perturbative QFT techniques in classical limit!

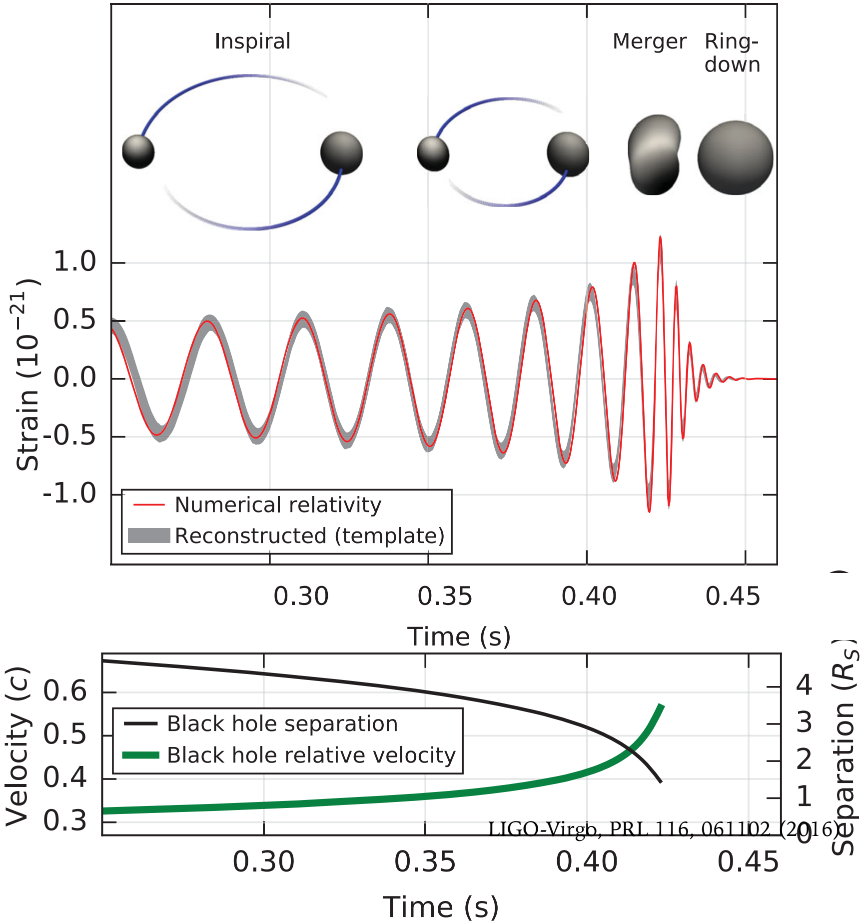
GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA

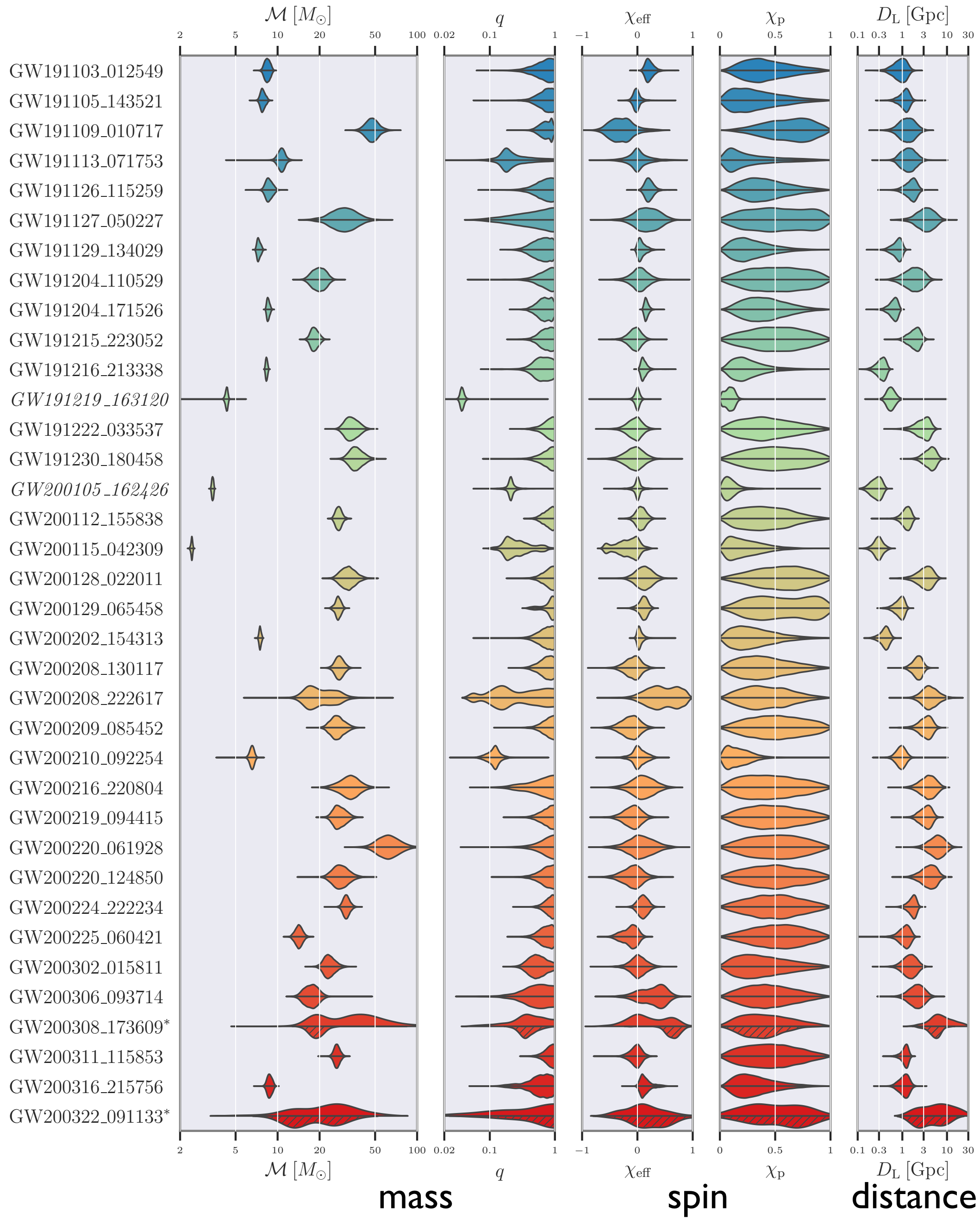


Following GW150914: To date 90 binary mergers detected by LIGO-Virgo-Karga Collaboration

GRAVITATIONAL WAVES: A NEW OBSERVATIONAL ERA

Binary mergers of black holes (BHs) and neutron stars (NS)





LVG collaboration arXiv:2111.03606

Measurement of binary parameters

Masses, Spins, Distance

PHYSICS CASES

AEI

Astrophysics:

- Black hole formation & evolution
- Neutron star properties: Equation of state, strong interacting matter
- Multi-messenger astronomy
- New astrophysical sources of GW

Fundamental physics:

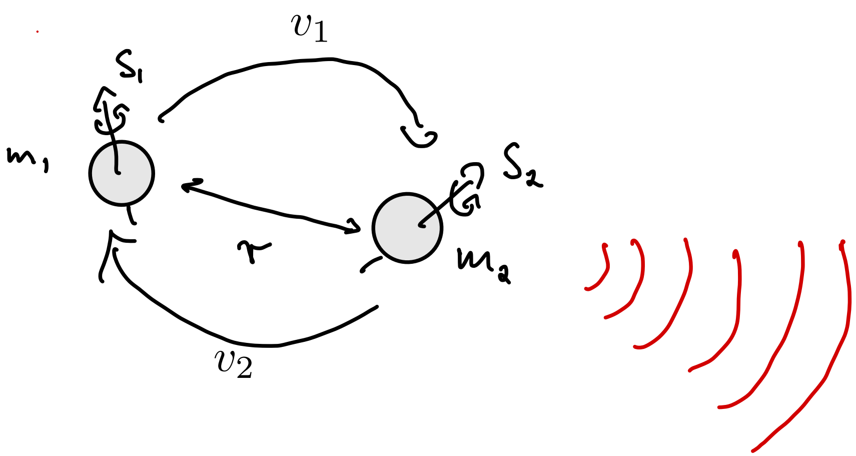
- Precision tests of (strong field) GR
- New physics signals? Modifications of GR, Higher curvature terms, Dark Matter…
- Black hole properties

- 3rd generation of GW observatories (Einstein Telescope; Advanced LIGO, LISA) to start in 2030's. Highly increase of sensitivity.

- **Need for high precision theory predictions**

THE GENERAL RELATIVISTIC 2-BODY PROBLEM

As in Newtonian case has either **bound** or **unbound** orbits.



Inspiral of 2 BHs or NSs:

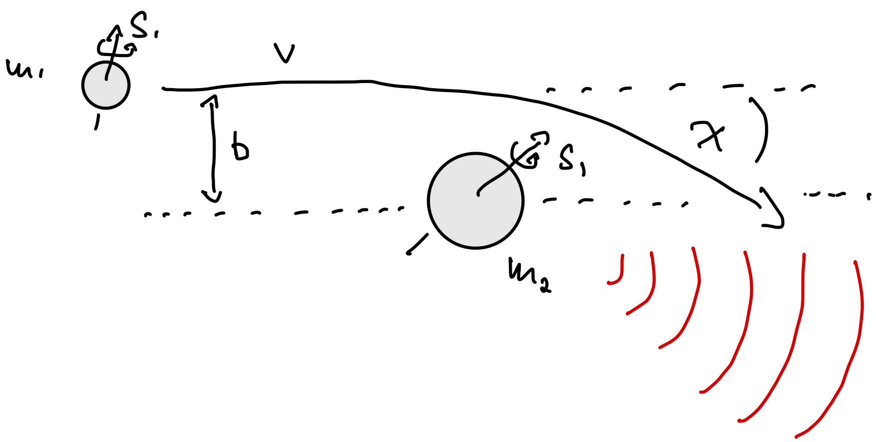
Virial-thm: $\frac{GM}{r} \sim v^2 \qquad (c = 1)$

post-Newtonian (PN) expansion

$$\boxed{\text{Weak field expansion:} \qquad g_{\mu\nu} = \eta_{\mu\nu} + \kappa\, h_{\mu\nu}}$$

$$\kappa = \sqrt{32\pi G}$$

Newton's constant



Scattering of 2 BHs or NSs:

Weak field (G), but exact in v

post-Minkowskian (PM) expansion

THE POST NEWTONIAN EXPANSION

Effective (conservative) action for two massive bodies:

$$S=\sum_i \int dt\Bigg[-m_i+\left(\frac{m_i\mathbf{v}_i^2}{2}+\sum_{j\neq i}\frac{Gm_im_j}{2r_{ij}}\right)$$

1PN: [Newton (1687)]

$$+\left(\frac{m_i\mathbf{v}_i^4}{8}+\sum_{j\neq i}\frac{Gm_im_j}{4r_{ij}}\left(6\,\mathbf{v}_i^2-(\mathbf{n}_{ij}\cdot\mathbf{v}_i)(\mathbf{n}_{ij}\cdot\mathbf{v}_j)-7\,\mathbf{v}_i\cdot\mathbf{v}_j\right)-\sum_{j\neq i}\sum_{k\neq i}\frac{G^2m_im_jm_k}{2r_{ij}r_{ik}}\right)\Bigg]$$

2PN: [Einstein,Infeld, Hofmann(1938)]

$$+\sum_i\int \mathrm{d}t\left\{\frac{m_i\mathbf{v}_i^6}{16}+\sum_{j\neq i}\frac{Gm_im_j}{16r_{ij}}\Big[3(\mathbf{n}_{ij}\cdot\mathbf{v}_i)^2(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2-6\,\mathbf{n}_{ij}\cdot\mathbf{v}_i\,\mathbf{n}_{ij}\cdot\mathbf{v}_j\,\mathbf{v}_{ij}^2-2\,(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2\,\mathbf{v}_i^2\right.$$

$$+3\,\mathbf{v}_i^2\,\mathbf{v}_j^2+2\,(\mathbf{v}_i\cdot\mathbf{v}_j)^2-20\,\mathbf{v}_i^2\,\mathbf{v}_i\cdot\mathbf{v}_j+14\,\mathbf{v}_i^4\Big]+\sum_{j\neq i}\frac{G^2m_im_j^2}{2r_{ij}^2}\left[33\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ij})^2-17\mathbf{v}_{ij}^2\right]$$

$$+\sum_{j\neq i}\sum_{k\neq i}\frac{G^2m_im_jm_k}{8}\Big[\frac{1}{r_{ij}r_{ik}}\left(4(\mathbf{n}_{ij}\cdot\mathbf{v}_j)^2+18\,\mathbf{v}_i^2-16\,\mathbf{v}_j^2-32\,\mathbf{v}_i\cdot\mathbf{v}_j+32\,\mathbf{v}_j\cdot\mathbf{v}_k\right)$$

$$+\frac{1}{r_{ij}^2}\left(14\,\mathbf{n}_{ik}\cdot\mathbf{v}_k\,\mathbf{n}_{ij}\cdot\mathbf{v}_k-12\,\mathbf{n}_{ij}\cdot\mathbf{v}_i\,\mathbf{n}_{ik}\cdot\mathbf{v}_k+\mathbf{n}_{ij}\cdot\mathbf{n}_{ik}\,(\mathbf{n}_{ik}\cdot\mathbf{v}_k)^2-\mathbf{n}_{ij}\cdot\mathbf{n}_{ik}\,\mathbf{v}_k^2\right)\Big]$$

3PN: [Ohta,Okamura,Hiida, Kimura (1974)]

$$+\sum_{j\neq i}\sum_{k\neq i,j}G^2m_im_jm_k\Big[\frac{2(\mathbf{n}_{ij}-\mathbf{n}_{jk})\cdot\mathbf{v}_{ij}}{(r_{ij}+r_{ik}+r_{jk})^2}\left(4\,(\mathbf{n}_{ij}+\mathbf{n}_{ik})\cdot\mathbf{v}_{ij}+(\mathbf{n}_{ik}+\mathbf{n}_{jk})\cdot\mathbf{v}_{ik}\right)$$

$$\left.+\frac{9\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ij})^2-9\,\mathbf{v}_{ij}^2+2\,(\mathbf{n}_{ij}\cdot\mathbf{v}_{ik})^2-2\,\mathbf{v}_{ik}^2}{r_{ij}\,(r_{ij}+r_{ik}+r_{jk})}\Big]\right\}+G^3\times[\text{static term}]\,,\ +\ldots$$

4PN: [Damour, Jaranowski,Schaefer (2016); Blanchet,Bohe,Faye (2015)]

5PN: [Bini,Damour,Geralico (2019); Foffa (2017);Porto, Rothstein, Sturani (2019]

Partial results at 6PN…

POST-NEWTONIAN VS POST-MINKOWSKIAN EXPANSIONS

Conservative non-spinning 2-body dynamics:

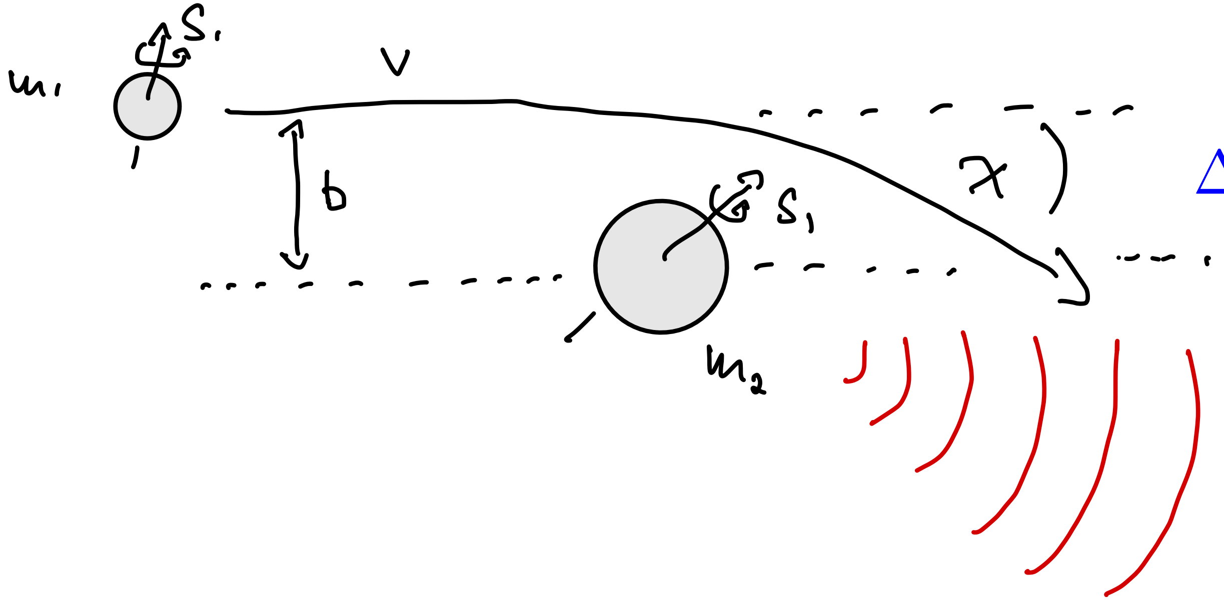
		0PN	1PN	2PN	3PN	4PN	5PN		Integration complexity
0PM [Einstein]	1	v^2	v^4	v^6	v^8	v^{10}	v^{12}	…	
1PM [Westpfahl]		G/r [Newton]	$G\,v^2/r$ [EIH]	$G\,v^4/r$	$G\,v^6/r$	$G\,v^8/r$	$G\,v^{10}/r$	…	~ tree-level
2PM [many]			$G^2\,1/r^2$	$G^2\,v^2/r^2$	$G^2\,v^4/r^2$	$G^2\,v^6/r^2$	$G^2\,v^8/r^2$	…	~ 1-loop
3PM				$G^3\,1/r^3$	$G^3\,v^2/r^3$	$G^3\,v^6/r^3$	$G^3\,v^8/r^3$	…	~ 2-loop
(4PM)	PM state-of-the-art		→		$G^4\,1/r^4$	$G^4\,v^2/r^4$	$G^4\,v^6/r^4$	…	~ 3-loop
….	(w/o radiation)					:	:		

3PM: [Bern,Cheung,Roiban,Shen, Solon,Zeng] [Kälin, Liu, Porto] [Di Vecchia, Heissenberg, Russo,Veneziano] [Bjerrum-Bohr,Vanhove,Damgaard] [Brandhuber,Chen,Travaglini,Wen] [Jakobsen,Mogull,JP,Sauer]

(4PM): [Bern,Parra-Martinez,Roiban,Ruf,Shen,Solon,Zeng] [Dlapa,Källin,Liu,Porto]

PN state-of-the-art ↑

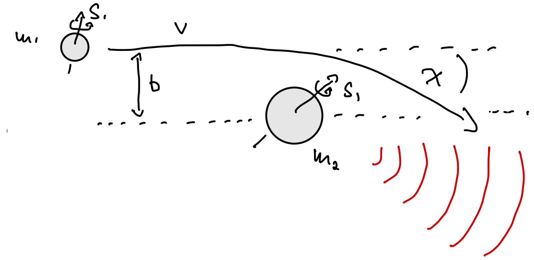
THE POST-MINKOWSKIAN EXPANSION



$$\Delta p_1^\mu = \sum_{n=1}^{\infty} G^n \, \Delta p_1^{(n)\,\mu}$$

$$f_{\mu\nu} = \sum_{n=1}^{\infty} G^n \, f_{\mu\nu}^{(n)}$$

THE GENERAL REALTIVISTIC TWO BODY PROBLEM IN PM: TRADITIONAL APPROACH



Point-particle approximation for BHs (or NSs)

$$S = -\sum_{i=1}^{2} \int d\tau_i \sqrt{g_{\mu\nu}\dot{x}_i^{\mu}(\tau_i)\dot{x}_i^{\nu}(\tau_i)} + \frac{1}{16\pi G}\int d^4x\sqrt{-g}R + S_{\mathrm{g.f.}}$$

Point particle approximation

Bulk gravity & gauge fixing

1) Equations of motion:

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu} = \frac{\kappa^2}{8}T_{\mu\nu} \qquad \ddot{x}_i^{\mu} + \Gamma^{\mu}{}_{\nu\rho}\dot{x}_i^{\nu}\dot{x}_i^{\rho} = 0$$

Einstein's eqs.

Geodesic eqs.

2) Solve iteratively in G

$$g_{\mu\nu} = \eta_{\mu\nu} + \sum_{n=1}^{\infty} G^n\, h_{\mu\nu}^{(n)}(x) \qquad x_i^{\mu}(\tau) = b_i^{\mu} + v_i^{\mu}\,\tau + \sum_{n=1}^{\infty} G^n z_i^{(n)\,\mu}(\tau)$$

emitted radiation

straight line: „in“ state

deflections

3) Construct observables

Far field waveform: $$\lim_{r\to\infty} h_{\mu\nu} = \frac{f_{\mu\nu}(t-r,\theta,\varphi)}{r} + \mathcal{O}\left(\frac{1}{r^2}\right)$$

„Impulse“ (change in momentum): $$\Delta p_i^{\mu} = m_i\dot{x}_i^{\mu}\Big|_{\tau=-\infty}^{\tau=+\infty} = m_i\int d\tau\, \ddot{x}_i^{\mu}(\tau)$$

USE OF QUANTUM FIELD THEORY TECHNIQUES FOR CLASSICAL 2-BODY PROBLEM

1) Effective world-line field theory:

[Källin,Porto,Dlapa] [Mougiakos,Riva,Vernizzi]

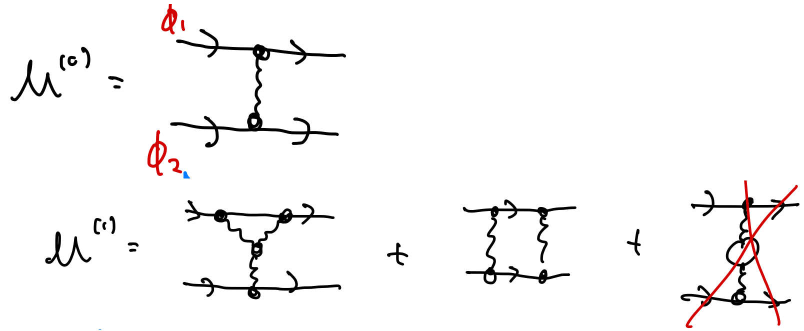
Construct effective action: $$e^{\frac{i}{\hbar}S_{\text{eff}}[x_i]} = \int [Dh_{\mu\nu}] e^{\frac{i}{\hbar}(S_{\text{pp}}[x_i, h_{\mu\nu}] + S_G[h_{\mu\nu}]}$$

Solve e.oms for $x_i(\tau)$: $$\frac{\delta S_{\text{eff}}[x_i]}{\delta x_i} = 0$$

2) Scattering amplitudes:

[Bern,Cheung,Roiban,Solon,Parra-Martinex,Ruf,Zeng,Luna,..] [Bjerrum-Bohr,Damgaard,Vanhove,Cristofoli] [DiVecchia,Heissenberg,Russo,Venneziano] [Kosower,Maybee,O'Connell,Vines]…

Scalar fields as avatars of BHs & NSs:



+ Modern on-shell techniques:

- Non-trivial classical limit
- Opaque relation to observables

$$\mathcal{M} = G\mathcal{M}^{(0)} + G^2\mathcal{M}^{(1)} + \ldots$$

3) World line quantum field theory: Best of 1) & 2)

[Jakobsen,Mogull,JP,Steinhoff]

Philosophy: Focus on observables (here one-point functions @ tree-level

Use 1) but also path integrate over $x_i(\tau)$!

THE BASIC IDEA: USE OF QFT TO SOLVE CLASSICAL EOM

CONSIDER SCALAR FIELD THY AS PROXY :

$$S[\phi;Q] = \frac{1}{2}\int d^4x\,\left[(\partial_\mu\phi)^2 + m^2\phi^2\right] + S_{INT}[\phi;Q]$$

Q: PHYSICAL SOURCE OR BACKGROUND

GOAL: (PERTURBATIVE) SOLUTION OF E.O.M. :

$$\left.\frac{\delta S[\phi,Q]}{\delta\phi}\right|_{\phi=\phi_{class}(x)} \doteq 0$$

QFT: GENERATING FUNCTIONAL

$$e^{\frac{i}{\hbar}W[J]} = \int[D\phi]\,\exp\left\{\frac{i}{\hbar}S[\phi;Q] + \frac{i}{\hbar}\int d^4x\,J(x)\,\phi(x)\right\}$$

ONE-POINT FUNCTION $\quad \langle\hat{\phi}_H(x)\rangle_{IN\text{-}OUT} = \left.\frac{\delta W[J]}{\delta J(x)}\right|_{J=0}$

EFFECTIVE ACTION:

(LEGENDRE-TRANSFORM)

$$S_{eff}[\phi] = \frac{i}{\hbar}\int d^4x\,J(x)\,\phi(x) - W[J]$$

ONE-POINT FUNCTION & E.O.M.

① EFFECTIVE E.O.M. ARE SOLVED BY ONE-POINT FUNCTION '

$$\left.\frac{\delta S_{eff}[\phi]}{\delta \phi(x)}\right|_{\phi(x) = \langle \hat{\phi}_H(x) \rangle} = 0$$

② TREE-LEVEL ⇒ CLASSICAL ACTION : $S_{eff}[\phi] = S[\phi; Q] + \mathcal{O}(\hbar)$

⇒ TREE-LEVEL (FEYNMAN-DIAGRAMMATIC) EVALUATION OF $\langle \hat{\phi}_H \rangle$ YIELDS SOLUTION TO CLASSICAL E.O.M.

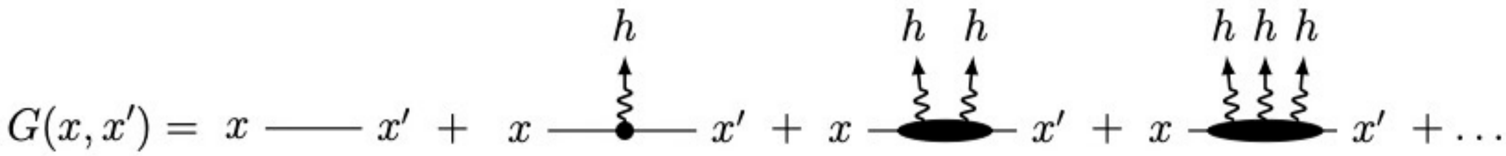
CAUSALITY: EXAMPLE : $S[\phi] = \frac{1}{2}\int d^4x \left[(\partial_\mu \phi)^2 + m^2 \phi(x)^2 + Q(x)\,\phi(x) \right]$

ONE POINT FUNCTION :

$$\langle \hat{\phi}_H(x) \rangle_{\text{IN-OUT}} = \underset{Q}{\otimes}\!\!-\!\!-\!\!-\!\!\bullet\, x = \int d^4y\, G_{\text{FEYN}}(x-y)\, Q(y)$$

SOLVES E.O.M BUT WE WANT RETARDED PROPAGATOR !

WORLDLINE QUANTUM FIELD THEORY



WORLDLINE EFFECTIVE FIELD THEORY

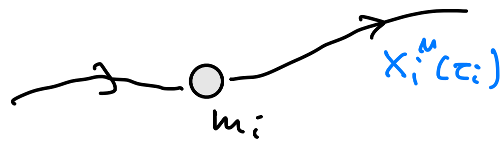
[Goldberger,Rothstein] [Porto,Källin] [Foffa,Sturani]

- MODEL BHs/NSs AS POINT PARTICLES:

$$S_p = -\sum_{i=1}^{2} m_i \int_{-\infty}^{\infty} d\tau_i \sqrt{g_{\mu\nu}\, \dot{x}_i^{\mu}\, \dot{x}_i^{\nu}}$$

BETTER: INTRODUCE EINBEIN $e(\tau)$:

$$S_p = -\frac{m}{2}\int d\tau \left(e^{-1} g_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu} + e\right)$$



ALGEBRAIC E.O.M. YIELD $e^2 = g_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu}$ $\Rightarrow$ PROPER TIME GAUGE $e = 1 \Leftrightarrow \dot{x}^2 = 1$.

- **INCLUSION OF FINITE SIZE/TIDAL EFFECTS** <-> EFT logic

$$S_p = -\frac{m}{2}\int d\tau \Big(g_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu} + c_1 R\, \dot{x}^2 + c_2 R_{\mu\nu}\, \dot{x}^{\mu}\, \dot{x}^{\nu} + c_{E^2}\left(R_{\mu\alpha\nu\beta}\, \dot{x}^{\alpha}\, \dot{x}^{\alpha}\right)^2 + c_{B^2}\left(R^{*}_{\mu\alpha\nu\beta}\, \dot{x}^{\alpha}\, \dot{x}^{\alpha}\right)^2 + \dots$$

- **COUPLE TO GRAVITY**

$$S_G = \frac{2}{\kappa^2}\int d^4x \sqrt{-g}\, R + S_{g.f.}$$

WEAK GRAVITATIONAL FIELD

$$g_{\mu\nu} = \eta_{\mu\nu} + \kappa \cdot h_{\mu\nu}$$

WORLDLINE QFT: FLUCTUATE WORLDLINE & GRAVITON

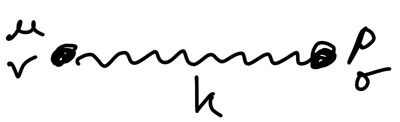
OBJECTIVE: FOCUS ON OBSERVABLES !

[Jakobsen,Mogull,JP,Steinhoff]

$$\boxed{S = -2m_{Pl}^2\int d^4x\sqrt{-g}\,R - \sum_i \frac{m_i}{2}\int d\tau_i\, g_{\mu\nu}\,\dot{x}_i^\mu \dot{x}_i^\nu}$$

$$\left.\begin{aligned} g_{\mu\nu}(x) &= \eta_{\mu\nu} + \kappa\, h_{\mu\nu}(x) \\ x_i^\mu(\tau_i) &= b_i^\mu + \tau_i v_i^\mu + \underbrace{z_i^\mu(\tau_i)}_{\text{QUANTUM FIELDS}} \end{aligned}\right.$$

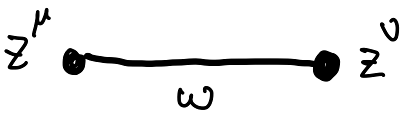
Graviton propagator in de Donder gauge



$$= i\,\frac{P_{\mu\nu;\rho\sigma}}{(k^0+i\varepsilon)^2-\vec{k}^2}$$

$$P_{\mu\nu;\rho\sigma} = \eta_{\mu(\rho}\eta_{\sigma)\nu} - \frac{1}{2}\eta_{\mu\nu}\eta_{\rho\sigma}$$

Worldline fluctuation propagator:



$$= -\frac{i}{m}\,\frac{\eta^{\mu\nu}}{(\omega+i\varepsilon)^2}$$

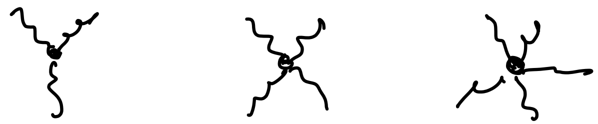
N.B.: $i\varepsilon$ prescription is crucial here!

For classical physics want retarded prop.

$\Rightarrow$ IN-IN FORMALISM

[Schwinger,Keldysh]

Graviton interactions:

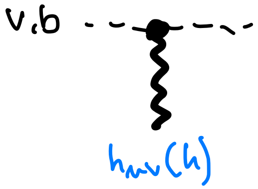


$+ \ldots$

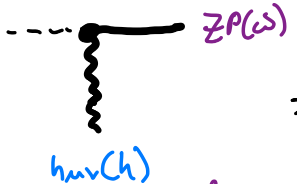
Worldline Interactions

EMERGE FROM $h_{\mu\nu}[X(\tau)]\,\dot{X}^{\mu}(\tau)\,\dot{X}^{\nu}(\tau)$ WITH $X_i^{\mu}(\tau_i) = \underbrace{b_i^{\mu} + \tau_i v_i^{\mu}}_{\text{"}Q\text{"}} + \underbrace{z_i^{\mu}(\tau_i)}_{\text{"}\phi\text{"}}$

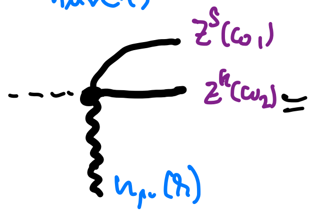
IN MOMENTUM SPACE



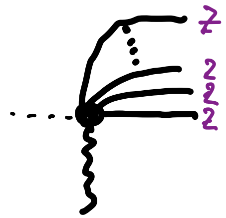
$$= -i m \kappa\, e^{ik\cdot b}\, \delta(k\cdot v)\, v^{\mu} v^{\nu}$$



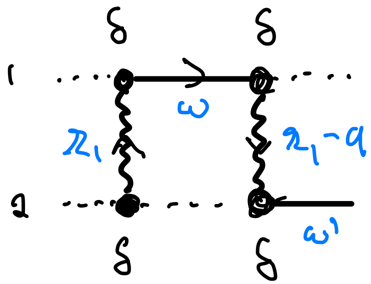
$$= m \kappa\, e^{ik\cdot b}\, \delta(k\cdot v + \omega)\left(2\omega v^{(\mu}\delta^{\nu)}_{\rho} + v^{\mu} v^{\nu} k_{\rho}\right)$$



$= \;\ldots$ and higher!

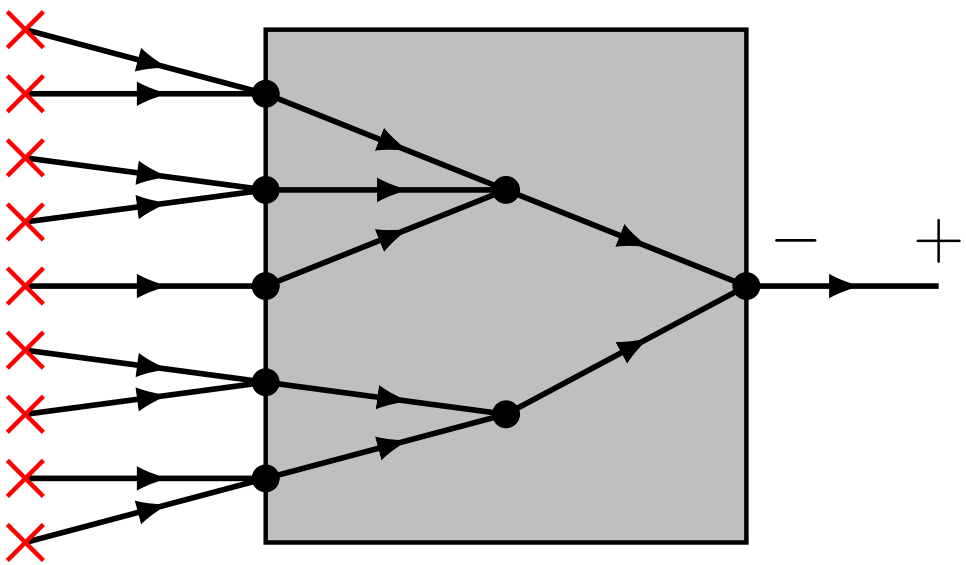


TREE LEVEL WQFT GRAPHS YIELD LOOP-LEVEL FEYNMAN INTEGRALS



$$\hat{=} \int d^4 k_1 \int d\omega\; \delta^4(\ldots)\ldots = \delta(q\cdot v_1)\,\delta(q\cdot v_2)\; \underbrace{\int d^4 k_1\, \delta(k_1\cdot v_1)\ldots}_{\text{1-LOOP}}$$

THE IN-IN (SCHWINGER-KELDYSH) FORMALISM FOR WQFT



[Jakobsen,Mogull,JP,Sauer]

IN-OUT FORMALISM: STANDARD PATH INTEGRAL

[Galley,Tiglio] [Jordan]

TIME EVOLUTION OPERATOR

$$\hat{H} = \hat{H}_0 + \hat{H}_{INT}$$

INTERACTION PICTURE

$$\mathcal{U}_J(T,T') = \tilde{T} \exp\left\{ \frac{i}{\hbar} \int_{T'}^{T} dt \int d^3x \left[\hat{H}_{INT}\left(\phi_I(\vec{x},t), Q(\vec{x},t) \right) + J(x)\, \phi_I(\vec{x},t) \right] \right\}$$

PATH INTEGRAL REPRESENTATION

$|0\rangle$: GROUNDSTATE AT $T = -\infty$

$$\langle 0 | \mathcal{U}_J(\infty,-\infty) | 0 \rangle = \int [D\phi] \exp\left\{ \frac{i}{\hbar} S[\phi; Q] + \frac{i}{\hbar} \int d^4x\, J(x)\, \phi(x) \right\} = e^{\frac{i}{\hbar} W[J]}$$

ONE POINT FUNCTION

NOT A TRUE VACUUM EXPECTATION VALUE !

$$\langle \hat{\phi}_H(t,\vec{x}) \rangle_{IN-OUT} = \left. \frac{\delta W[J]}{\delta J(t,\vec{x})} \right|_{J=0} = \langle 0 | \mathcal{U}(\infty,t)\, \underbrace{\hat{\phi}_I(t,\vec{x})}_{= \mathcal{U}(t,-\infty)\, \hat{\phi}_H(t,\vec{x})\, \mathcal{U}(-\infty,t)}\, \mathcal{U}(t,-\infty) | 0 \rangle$$

$$= \langle 0 | \mathcal{U}(\infty,-\infty)\, \hat{\phi}_H(t,\vec{x}) | 0 \rangle = {}_{OUT}\langle 0 | \hat{\phi}_H(x) | 0 \rangle_{IN}$$

IN-IN (SCHWINGER-KELDYSH) FORMALISM

[Galley,Tiglio] [Jordan]

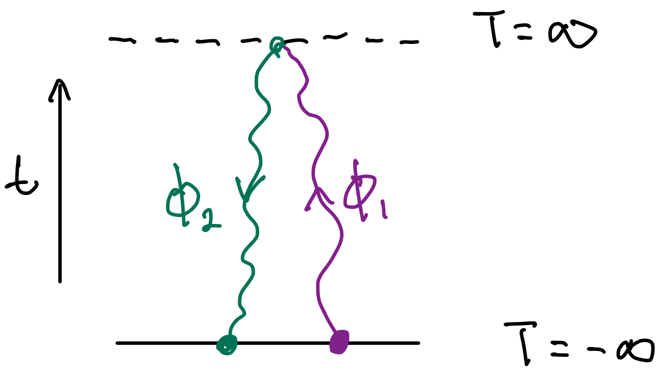
IN-OUT (STANDARD) FORMALISM YIELDS $\langle \hat\phi_H(x) \rangle_{\text{IN-OUT}} = {}_{\text{OUT}}\langle 0 | \hat\phi_H(x) | 0 \rangle_{\text{IN}}$ BUT WANT

$$\langle \hat\phi_H(x) \rangle_{\text{IN-IN}} := {}_{\text{IN}}\langle 0 | \hat\phi_H(x) | 0 \rangle_{\text{IN}} = \langle 0 | \hat{U}(-\infty, t)\, \hat\phi_I(t, \vec{x})\, \hat{U}(t, -\infty) | 0 \rangle$$

NEED TWO TIME EVOLUTION OPERATORS $\Rightarrow$ DOUBLE FIELDS IN PATH-INTEGRAL

$$e^{\frac{i}{\hbar} W[J_1, J_2]} = \langle 0 | \hat{U}_{J_2}(-\infty, \infty)\, \hat{U}_{J_1}(\infty, -\infty) | 0 \rangle$$

$$= \int D\phi_1 D\phi_2 \exp\left\{ \frac{i}{\hbar} \left(S[\phi_1] - S[\phi_2] + \int d^4x\, J_1(x)\phi_1(x) - J_2(x)\phi_2(x) \right) \right\}$$



BOUNDARY CONDITIONS:

$$\phi_1(T=\infty, \vec{x}) = \phi_2(T=\infty, \vec{x})$$

$$\phi_1(T=-\infty, \vec{x}) = \phi_2(T-\infty, \vec{x}) = 0$$

$$\Rightarrow \boxed{\langle \hat\Phi_H(x) \rangle_{\text{IN-IN}} = \left. \frac{\delta W[J_1, J_2]}{\delta J_1(x)} \right|_{J_i=0}}$$

KELDYSH BASIS

$$\phi_+ = \frac{1}{2}(\phi_1 + \phi_2) \qquad \phi_- = \phi_1 - \phi_2$$

(SAME FOR $J_\pm$)

THIS YIELDS

$$e^{\frac{i}{\hbar}W[J_+,J_-]} = \int D\phi_+ D\phi_- \exp\left\{\frac{i}{\hbar}\left(S[\phi_+ + \tfrac{1}{2}\phi_-] - S[\phi_+ - \tfrac{1}{2}\phi_-] + \int d^4x\,(J_+\phi_- + J_-\phi_+)\right)\right\}$$

PROPAGATOR MATRIX FROM FREE PART:

$$\Rightarrow D^{ab}(x,y) = \begin{array}{c} \\ + \\ - \end{array}\!\!\begin{array}{c} \begin{array}{cc} \quad + \quad & \quad - \quad \end{array} \\ \begin{pmatrix} 0 & D_{adv}(x,y) \\ D_{ret}(x,y) & \frac{i}{2}D_H(x,y) \end{pmatrix} \end{array}$$

RETARDED PROPAGATOR

$\langle\{\phi(x),\phi(y)\}\rangle$ IRRELEVANT @ TREE-LEVEL

$$D_{ret}(k) = \bullet_{-}\!\longrightarrow\!\bullet_{+} = \frac{i}{(k^0+i\epsilon)^2 - \vec{k}^2}$$

$$D_{adv}(k) = \bullet_{+}\!\longleftarrow\!\bullet_{-} = \frac{-i}{(k^0-i\epsilon)^2 - \vec{k}^2}$$

VERTICES FROM

$$S_{INT}[\phi_+ + \tfrac{1}{2}\phi_-] - S_{INT}[\phi_+ - \tfrac{1}{2}\phi_-] = \phi_-\left(\frac{\delta S_{INT}(\phi)}{\delta\phi}\right)_{\phi\to\phi_+} + O(\phi_-^3)$$

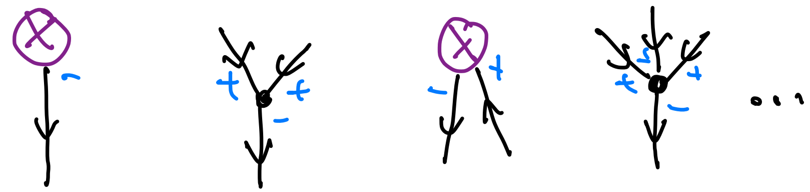
$\Rightarrow$ $\boxed{\text{ONLY **ODD** NUMBER OF } \phi_- \text{ LEGS}}$

ONE-POINT FUNCTIONS @ TREE-LEVEL

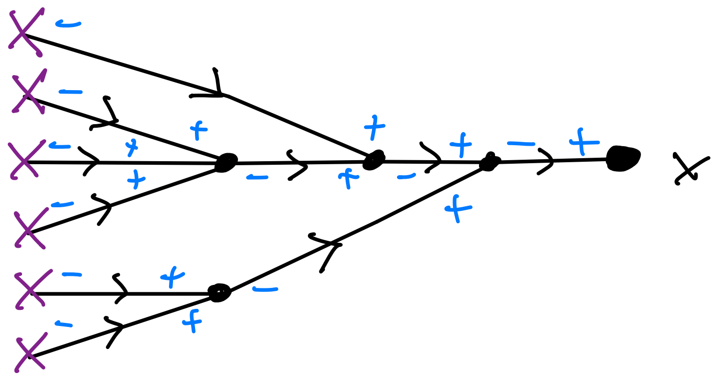
[Jakobsen,Mogull,JP,Sauer]

$$S_{\text{INT}}[\phi;Q] \xrightarrow[\text{IN-IN}]{} \phi_- \left(\frac{\delta S_{\text{INT}}[\phi,Q]}{\delta \phi}\right)_{\phi\to\phi_+} + \mathcal{O}(\phi_-^3)$$

VERTICES:



ONE-POINT FCT. $\Rightarrow$ $\langle \hat{\phi}_H(x) \rangle_{\text{IN-IN}} =$

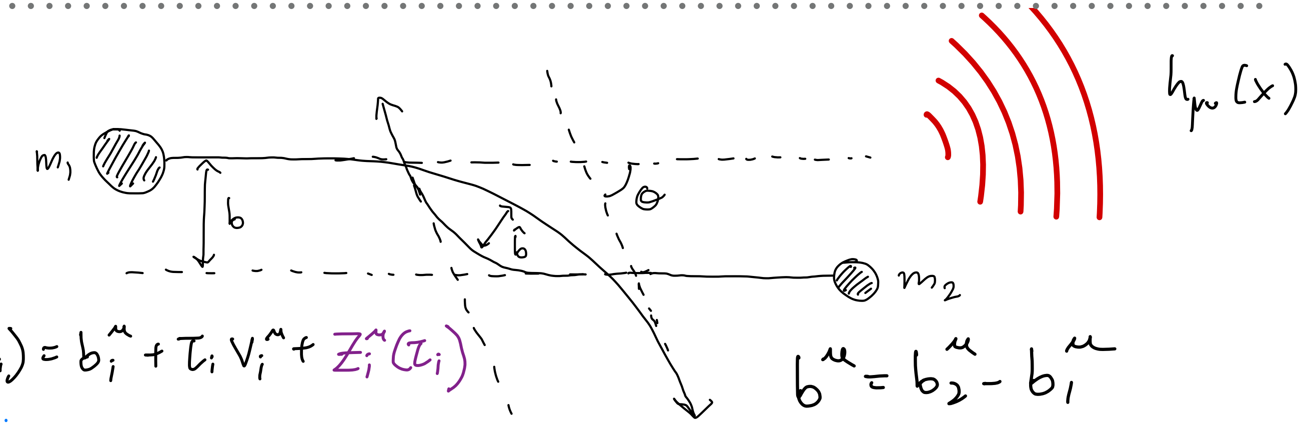


ONLY RETARDED PROPAGATORS CONTRIBUTE

OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

[Jakobsen,Mogull,JP,Steinhoff]

Spin-less BH/NS scattering:

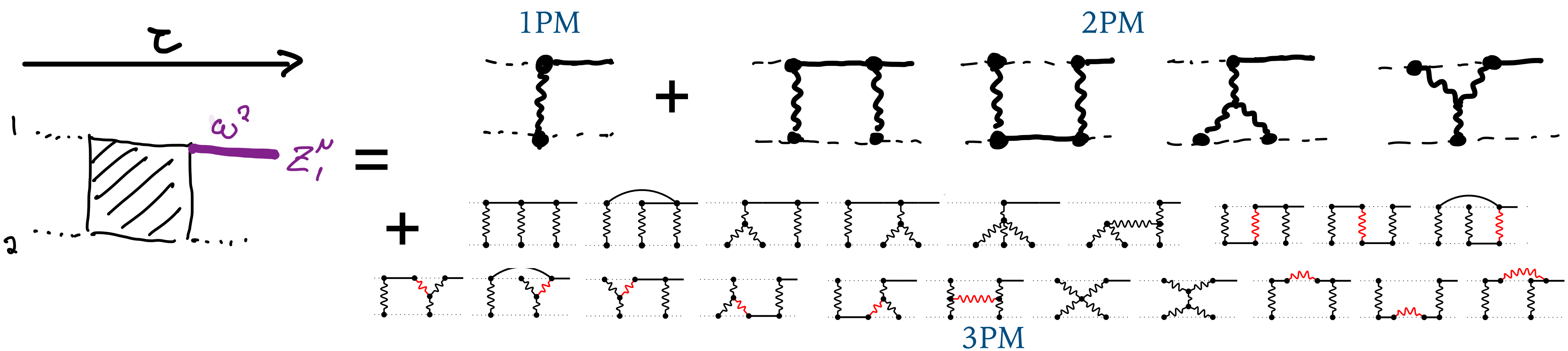


1) **Impulse** (change of momentum)

$$\Delta p_i^\mu = m_i \langle \dot{x}_i^\mu \rangle \Big|_{\tau=-\infty}^{\tau=+\infty} = m_i \int d\tau \langle \ddot{x}_i^\mu(\tau) \rangle = m_i \int d\tau \frac{d^2}{d\tau^2} \langle z_i^\mu(\tau) \rangle = -m_i \omega^2 \langle z_i^\mu(\omega) \rangle \Big|_{\omega \to 0}$$

Fourier trans.

Needs sum of all graphs with outgoing z-line:

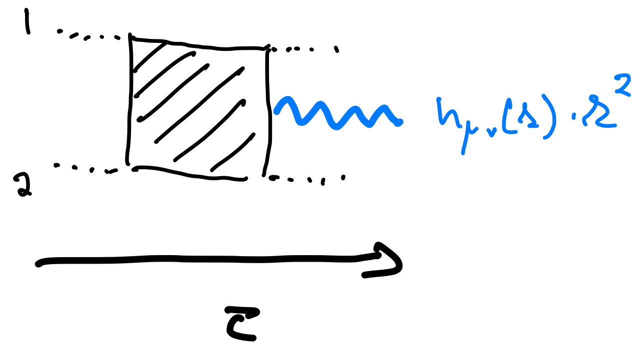


OBSERVABLES OF WQFT: ONE POINT FUNCTIONS

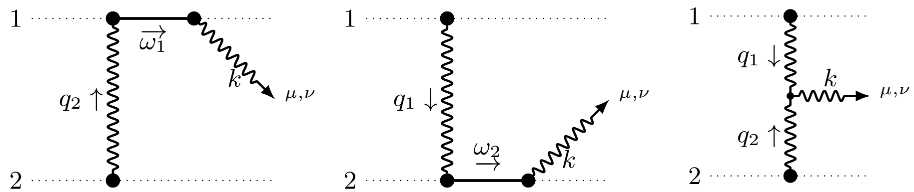
[Jakobsen,Mogull,JP,Steinhoff]

2) Emitted Waveform (Gravitational Bremsstrahlung)

$$\tau_{\mu\nu}^{T.T.} = r^2 \left\langle h_{\mu\nu}(r) \right\rangle_{WQFT} =$$

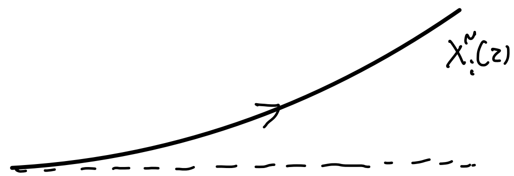


$=$



3) Trajectory!

$$X_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + \int d\omega \, e^{i\omega\cdot\tau} \left\langle z_i^\mu(\omega) \right\rangle_{WQFT}$$



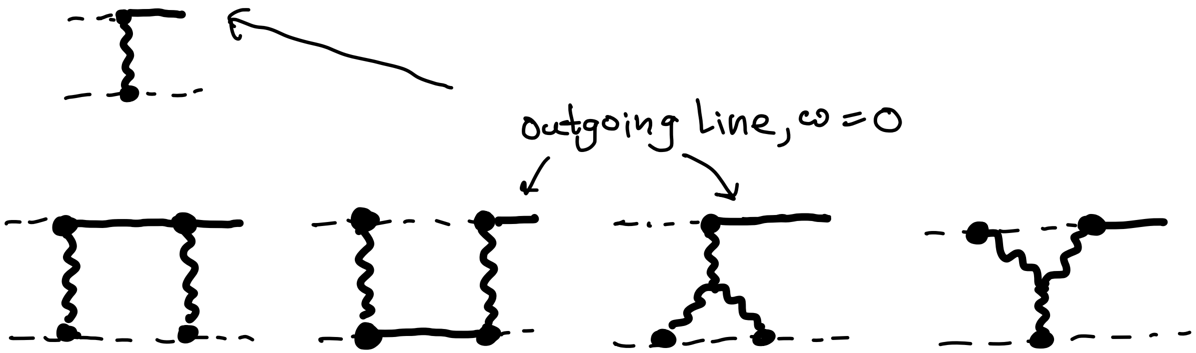
Deflections

$$\Delta p_i^\mu = -m_i \omega^2 \langle Z_i^\mu(\omega) \rangle_{\text{WQFT}} \Big|_{\omega=0}$$

Graphs with single outgoing worldline excitation Z_i^μ

1PM:

2PM:



Δp_1^μ

WE ONLY COMPUTE TREE-LEVEL GRAPHS

$(\hbar = 0)$

Integration gives (w/o spin)

$$\Delta p_1^\mu = \frac{G m_1 m_2 b^\mu}{b^2}\left(\frac{2(2\gamma^2-1)}{\sqrt{\gamma^2-1}} + \frac{3\pi}{4}\frac{(5\gamma^2-1)}{\sqrt{\gamma^2-1}}\frac{G(m_1+m_2)}{b} \right) + O(G^3)$$

$\gamma = v_1 \cdot v_2$

1PM [Mogull,JP,Steinhoff]

2PM [Jakobsen,Mogull,JP,Steinhoff]

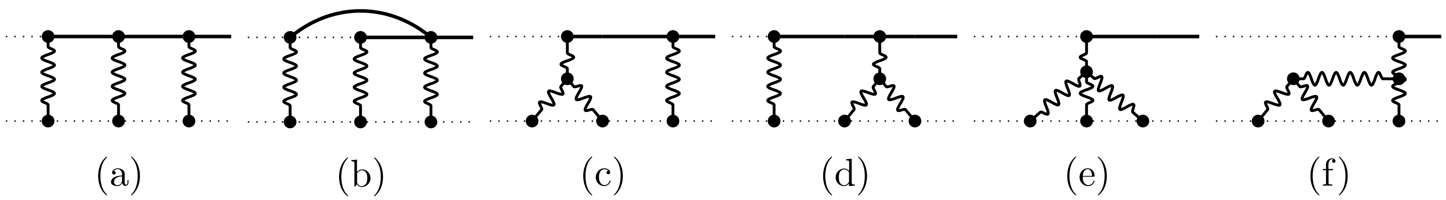
3PM [Jakobsen,Mogull]

AGREES WITH WEFT & AMPLITUDE APPROACHES [Källin,Porto][Bern et al][Brandhuber et al][Bjerrum-Bohr er al]

MOMENTUM DEFLECTION (IMPULSE) @ 3PM ORDER:

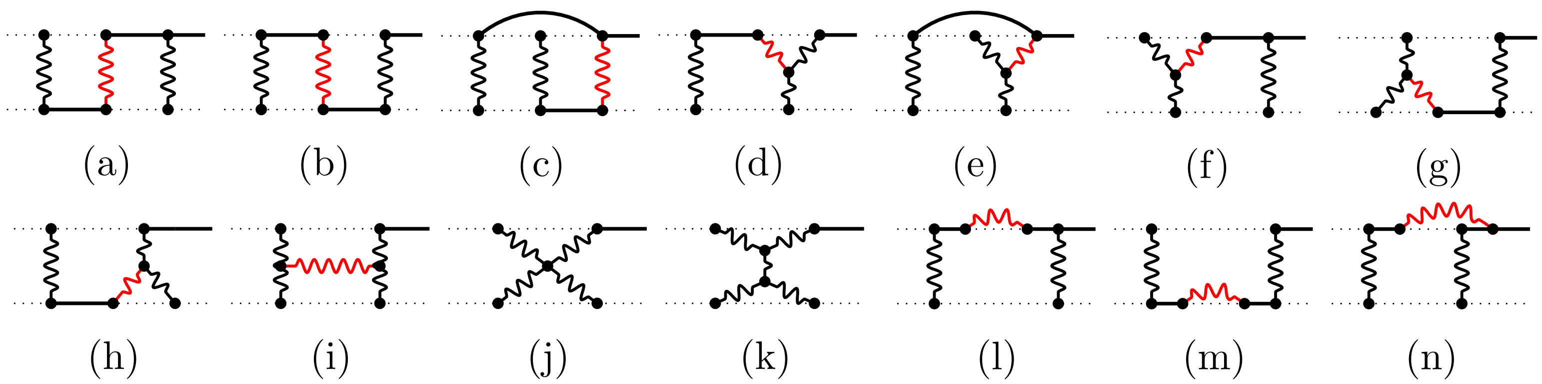
[Jakobsen,Mogull,JP,Sauer]

1) Test body diagrams (geodesic motion in Schwarzschild background):



$\sim G^3 m_1\, m_2^3$

2) Comparable mass diagrams (i0 prescription relevant for red propagators):



$\sim G^3 m_1^2\, m_2^2$

A 2-loop computation!

Integral family (with retarded propagators!)

$$I_{n_1 n_2 n_3 n_4 n_5 n_6 n_7} :=$$

$$\int d^d\ell_1\, d^d\ell_2 \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{\underbrace{(\ell_1\cdot v_1 \pm i\varepsilon)^{n_1}(\ell_2\cdot v_2 \pm i\varepsilon)^{n_2}}_{\text{active worldline prop.}}\underbrace{\left((\ell_1+\ell_2-q)^2 + i\varepsilon\,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}}_{\text{active graviton propagators}}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

RESULT IMPULSE @ 3PM ORDER:

[Jakobsen,Mogull,JP,Sauer]

$$\boxed{\Delta p_1^\mu = p_\infty \sin\theta \frac{b^\mu}{|b|} + (\cos\theta - 1)\frac{m_1 m_2}{E^2}\left[(\gamma m_1 + m_2)v_1^\mu - (\gamma m_2 + m_1)v_2^\mu\right] - v_2 \cdot P_{\rm rad}\, w_2^\mu}$$

Scattering angle:

$$\gamma = v_1 \cdot v_2 \qquad w_1^\mu = \frac{\gamma v_2^\mu - v_1^\mu}{\gamma^2 - 1}$$

$$\frac{\theta}{\Gamma} = \underbrace{\frac{GM}{|b|}\frac{2(2\gamma^2-1)}{\gamma^2-1}}_{\text{1PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^2 \frac{3\pi(5\gamma^2-1)}{4(\gamma^2-1)}}_{\text{2PM}} + \underbrace{\left(\frac{GM}{|b|}\right)^3\left(2\frac{64\gamma^6 - 120\gamma^4 + 60\gamma^2 - 5}{3(\gamma^2-1)^3}\Gamma^2 - \frac{8\nu\gamma(14\gamma^2+25)}{3(\gamma^2-1)} - 8\nu\frac{(4\gamma^4 - 12\gamma^2 - 3)}{(\gamma^2-1)}\frac{\operatorname{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right)}_{\text{3PM conservative}}$$

$$+ \underbrace{\left(\frac{GM}{|b|}\right)^3 \frac{4\nu(2\gamma^2-1)^2}{(\gamma^2-1)^{3/2}}\left(-\frac{8}{3} + \frac{1}{v^2} + \frac{(3v^2-1)}{v^3}\operatorname{arccosh}\gamma\right)}_{\text{3PM radiation-reaction}}$$

$$\Gamma = E/M = \sqrt{1 + 2\nu(\gamma - 1)}$$

$$\nu = \frac{m_1 m_2}{M^2}$$

Radiated 4-momentum:

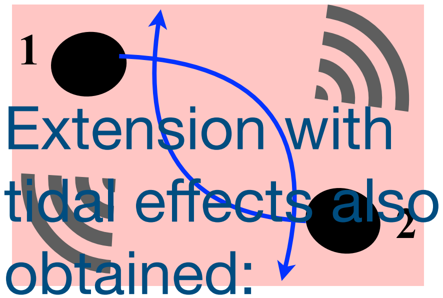
$$P_{\rm rad}^\mu = -\Delta p_1^\mu - \Delta p_2^\mu$$

$$P_{\rm rad}^\mu = \frac{G^3 m_1^2 m_2^2 \pi}{|b|^3}\frac{v_1^\mu + v_2^\mu}{\gamma + 1}\left[e_1 + e_2 \log\left(\frac{\gamma+1}{2}\right) + e_3 \frac{\operatorname{arccosh}\gamma}{\sqrt{\gamma^2-1}}\right]$$

$$e_1 = \frac{210\gamma^6 - 552\gamma^5 + 339\gamma^4 - 912\gamma^3 + 3148\gamma^2 - 3336\gamma + 1151}{48(\gamma^2-1)^{3/2}}$$

$$e_2 = -\frac{35\gamma^4 + 60\gamma^3 - 150\gamma^2 + 76\gamma - 5}{8\sqrt{\gamma^2-1}},$$

$$e_3 = \frac{\gamma(2\gamma^2-3)(35\gamma^4 - 30\gamma^2 + 11)}{16(\gamma^2-1)^{3/2}}.$$



Extension with tidal effects also obtained:

$$C_{E^2}\left(R_{\mu\alpha\nu\beta}\,\dot{x}^\alpha\,\dot{x}^\alpha\right)^2$$

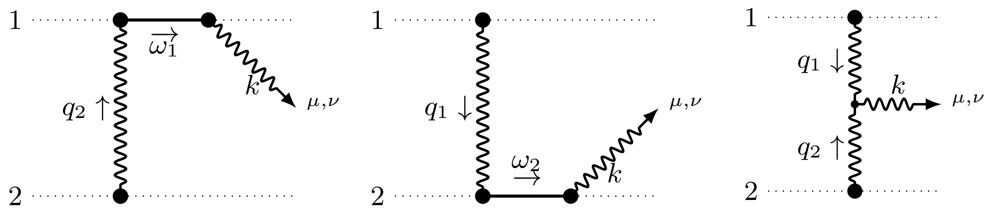
$$+ \; C_{B^2}\left(R^*_{\mu\alpha\nu\beta}\,\dot{x}^\alpha\,\dot{x}^\alpha\right)^2$$

FAR FIELD WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

Sum of diagrams with outgoing graviton:

$$\langle h_{\mu\nu}(k)\rangle =$$



For time-domain waveform needs to integrate over outgoing energy Ω:

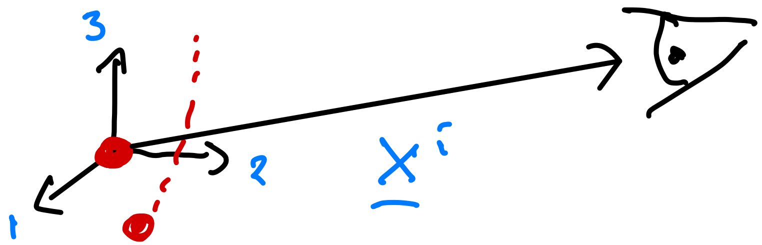
$$\frac{f_{+,\times}(t-r,\hat{\mathbf{x}})}{r} = \frac{4G}{r}\int d\Omega\, e^{-i\Omega(t-r)}\varepsilon^{\mu\nu}_{+,\times}\langle h_{\mu\nu}(k=\Omega(1,\hat{\mathbf{x}}))\rangle$$

where unit vector $\hat{\mathbf{x}}$ points towards the observer

The waveform has two polarizations

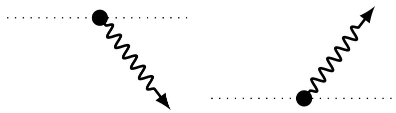
$$f_{+,\times}(\underbrace{t-r}_{u},\underbrace{\theta,\phi}_{\hat{\mathbf{x}}};v,|b|,m_1,m_2)$$

retarded time



INTEGRATED WAVEFORM @ NLO

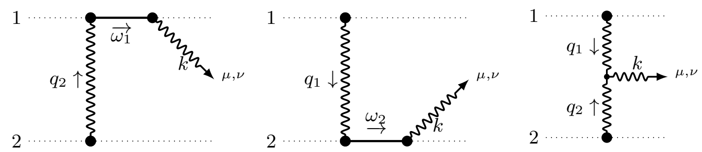
[Jakobsen,Mogull,JP,Steinhoff]

LO non-radiating: 

$$f^{(1)}(\hat{\mathbf{x}}) = \frac{2m_1}{\rho\cdot v_1}(\epsilon\cdot v_1)^2 + \frac{2m_2}{\rho\cdot v_2}(\epsilon\cdot v_2)^2$$

Our NLO result reproduces [Kovacs,Thorne '75] obtained with traditional GR techniques in 4 long papers

$$f^{(2)}(u,\hat{\mathbf{x}}) = \frac{1}{|\tilde{\mathbf{b}}|_1}\left[\alpha_1(u_1,\rho) + \frac{\beta_1(u_1,\rho)}{\tilde{b}^2}\right] + (1\leftrightarrow 2)$$



$$|\tilde{\mathbf{b}}|_{1,2} := \sqrt{|b|^2 + (\gamma^2-1)u_{2,1}^2}$$

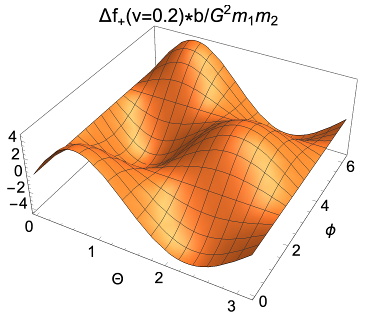
$$u_i = \frac{\rho\cdot(x-b_i)}{\rho\cdot v_i}$$

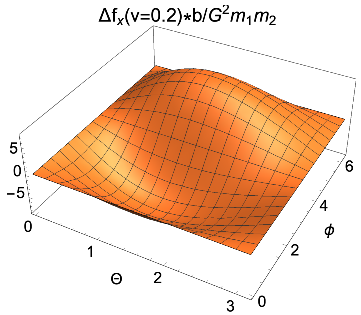
retarded time in ith rest frame

$$\rho = (1,\hat{\mathbf{x}})$$

The wave memory effect: $\Delta f^{2)} = f^{(2)}(u=+\infty) - f^{(2)}(u=-\infty)$

$$\frac{\Delta f^{(2)}_{\mathcal{S}=0}}{m_1 m_2} = \frac{4(2\gamma^2-1)\epsilon\cdot v_1(2b\cdot\epsilon\,\rho\cdot v_1 - b\cdot\rho\,\epsilon\cdot v_1)}{|b|^2\sqrt{\gamma^2-1}(\rho\cdot v_1)^2}$$





$$\gamma = v_1\cdot v_2$$

SUSY IN THE SKY WITH GRAVITONS

PUTTING SPIN ON THE WORLD-LINE

- Generalization of $(\Box + m^2)\, G(x,x') = \delta(x-x')$ to general spin $N/2$ in flat space-time: N-extended supersymmetric particle [Howe,Penati,Pernici,Townsend]

- Worldline fields: $x^\mu(\tau)\ ;\quad \psi^a_\alpha(\tau) \qquad \alpha = 1,\dots,N$

- Supercharge: $Q_\alpha = p\cdot\psi_\alpha \quad \xrightarrow{\{\psi^a_\alpha,\psi^b_\beta\}_{P.B.} = i\,\delta_{\alpha\beta}\,\eta^{ab}} \quad \{Q_\alpha, Q_\beta\}_{P.B.} = -2i\,\delta_{\alpha\beta}\, H$

  Hamiltonian: $H = \frac{1}{2}p^2$ R-Charge: $R_{\alpha\beta} = \psi_\alpha\cdot\psi_\beta$

- In curved space-time background: SUSY only possible for $N \le 2^{*}$

[Bastianelli, Benincasa,Giombi] [Bonezzi, Meyer, Sachs]

$$Q = \psi^a e_a^\mu(x)\left(P_\mu - i\,\omega_{\mu ab}\,\bar\psi^a\psi^b\right)$$

N=2 SUSY $\quad \psi^a = \psi^a_1 + i\,\psi^a_2$

SUSY IN THE SKY WITH GRAVITONS

- "Gauged" 1st order form of action:

$$\Pi_\mu = P_\mu - i\,\omega_{\mu ab}\,\bar{\psi}^a \psi^b$$

$$S = \int d\tau \left[P_\mu \dot{x}^\mu + i \bar{\psi}^a \dot{\psi}^b \eta_{ab} + \frac{e}{2}\left(g^{\mu\nu} \Pi_\mu \Pi_\nu - m^2 + R_{abcd}\, \bar{\psi}^a \psi^b \bar{\psi}^c \psi^d \right) \right.$$

Identify spin-field

$$S^{\mu\nu} = -2i\, \bar{\psi}^{[\mu} \psi^{\nu]}$$

- Is equivalent to "Traditional form" of massive spinning body:

$$S_{ps} = \int dz \left[\pi_\mu \dot{x}^\mu + \frac{1}{2} S_{\mu\nu} \Lambda_A{}^\mu \frac{D\Lambda^{A\nu}}{Dz} - e\left(\pi^2 - \mathcal{M}^2\right) \right]$$

SPIN FIELD BODY FIXED FRAME

[Vines,Kunst,Steinhoff,Hinderer][Steinhoff]
[Porto][Levi]

$$\mathcal{M}^2 = m^2 - \frac{1}{4} R_{\mu\nu\rho\sigma} S^{\mu\nu} S^{\rho\sigma} + C_E\, E_{\mu\nu} S^{\mu\rho} P_{\rho\sigma} S^{\sigma\nu} + \mathcal{O}(S^3)$$

✓ ✓ ✗ ✗

KERR BH AND FINITE SIZE TERMS

- In "traditional form" of [Vines,Kunst,Steinhoff,Hinderer][Steinhoff][Porto][Levi]

$$\boxed{\mathcal{M}^2 = m^2 - \tfrac{1}{4} R_{\mu\nu\rho\sigma} S^{\mu\nu} S^{\rho\sigma} + C_E E_{\mu\nu} S^{\mu\rho} P_{\rho\sigma} S^{\sigma\nu} + \mathcal{O}(S^3)}$$

✓ ✓ ✗ ✗

- Spin induced quadrupole moment: $E_{\mu\nu} = R_{\mu\alpha\nu\beta}\, \pi^\alpha \pi^\beta / m^2$ $\quad P_{\rho\sigma} = g_{\rho\sigma} - \pi_\rho \pi_\sigma / \pi^2$

$$\boxed{\text{KERR-BH has } C_E = 0 \,!}$$

But all order S^n terms....

$\Rightarrow$ Up to S^2-interactions

KERR BH $\hat{=}$ $N=2$ superparticle

- Can include C_E term in our spinning WQFT: Add

$$\boxed{S_{ES^2} = \frac{C_E}{2m} \int dz \; R_{a\mu b\nu}\, \dot{x}^\mu \dot{x}^\nu\, \bar{\psi}^a \psi^b\, \bar{\psi} \cdot P \cdot \psi}$$

PRESERVES SUSY APPROXIMATELY (up to S^3 terms) !

SPINNING WORLDLINE QUANTUM FIELD THEORY

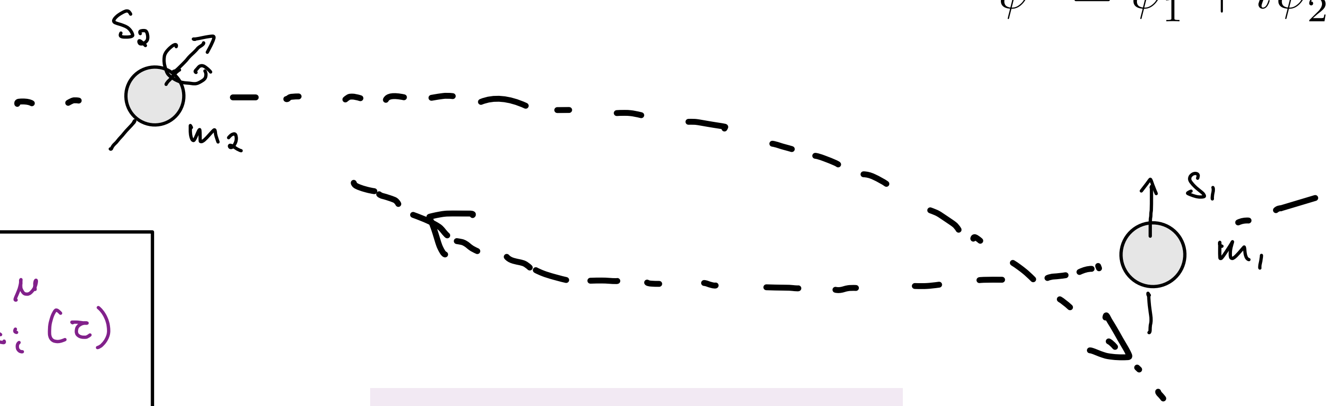
The spinning WQFT action (with exact N=2 SUSY for Kerr-BH)

$$S_{\mathrm{sWQFT}} = \sum_{i=1}^{2} \int d\tau \Big[-\frac{m_i}{2} g_{\mu\nu} \dot{x}_i^\mu \dot{x}_i^\nu + i\bar{\psi}_{ia} D_\tau \psi_i^a + \frac{1}{2m_i} R_{abcd} \bar{\psi}_i^a \psi_i^b \bar{\psi}_i^c \psi_i^d$$

$$+ \frac{C_{E,i}}{2m_i} R_{a\mu b\nu} \dot{x}_i^\mu \dot{x}_i^\nu \bar{\psi}_i^a \psi_i^b \, \bar{\psi}_i^c P_{cd} \psi_i^d \Big] \qquad D_\tau \psi^a = \dot{\psi}^a + \omega_\mu{}^{ab} \dot{x}^\mu \psi_b$$

$$\psi^a = \psi_1^a + i\psi_2^a$$

Scattering scenario:



$$x_i^\mu(\tau) = b_i^\mu + v_i^\mu \tau + z_i^\mu(\tau)$$

$$\psi_i^a(\tau) = \Psi_i^a + \psi_i'^a(\tau)$$

$$\Rightarrow \quad S_i^{ab} = -2i\, \bar{\Psi}_i^{[a} \Psi_i^{b]}$$

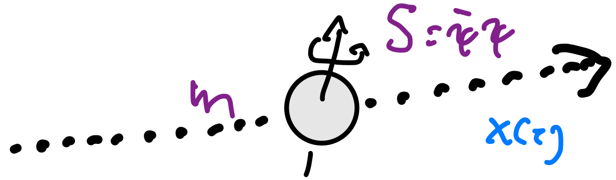
Initial spins of BHs/NSs

Integrate out $z_i^\mu, \psi_i'^a, \bar{\psi}_i'^a$ perturbatively!

[Jakobsen,Mogull,JP,Steinhoff]

Captures spin-orbit and spin-spin interactions up to order $S_1^2, S_2^2, S_1 S_2$

PHYSICAL INTERPRETATION OF SUSY



- **Traditional approach:**

  Spin tensor $S_i^{\mu\nu}(\tau)$ & co-moving frame $\Lambda_i^{A\,\mu}(\tau)$

  Eoms: $\frac{Dp^\nu}{D\tau} + \frac{1}{2} S^{\mu\rho} R_{\mu\rho\nu\kappa} \dot{x}^\kappa = 0 \qquad \frac{DS^{\mu\nu}}{D\tau} + 2\dot{x}^{[\mu} p^{\nu]} = 0$ [Matthisson-Papapetrou-Dixon]

  Freedom of imposing a Spin-Supplementary Condition (SSC): $p_\mu S^{\mu\nu} = 0$

- **Our approach: Spinning super-particle** $\boxed{S_i^{\mu\nu} = -2i\bar{\psi}_i^{[\mu}\psi_i^{\nu]}}$

  Asymptotic SUSY transformations:
  $$\delta b_i^\mu = i\bar{\epsilon}\Psi_i^\mu + i\epsilon\bar{\Psi}_i^\mu\,, \quad \delta v_i^\mu = 0\,, \quad \delta\Psi_i^\mu = -\epsilon v_i^\mu$$
  $$\Rightarrow \quad \delta\mathcal{S}_i^{\mu\nu} = v_i^\mu\,\delta b_i^\nu - v_i^\nu\,\delta b_i^\mu$$

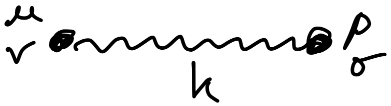
  Are a symmetry of all observables.

- **Interpretation of SUSY:** $\boxed{\text{SUSY = Freedom of picking a SSC.}}$

  Covariant SSC: $v_i \cdot \Psi_i = 0 \Rightarrow v_{i,\mu} S_i^{\mu\nu} = 0$

Spinning WQFT Feynman rules

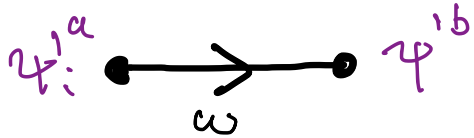
Graviton propagator



$$= i\,\frac{P_{\mu\nu;\rho\sigma}}{(k^0+i\epsilon)^2-\vec{k}^2}$$

Worldline fluctuation propagator:

$$= -\frac{i}{m}\,\frac{\eta^{\mu\nu}}{(\omega+i\epsilon)^2}$$



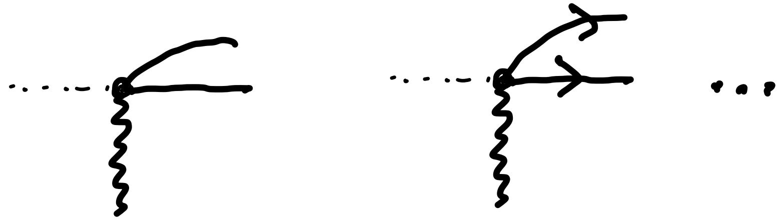
$$= i\,\frac{\eta^{ab}}{\omega+i\epsilon}$$

Worldline interactions

$\sim v^2 + Sv + S^2$ $\qquad \sim S^2$

$\sim v^2 + Sv + v^2$ $\qquad \sim S^2$

$\sim v^2 + Sv + S^2$ $\qquad \sim v + S$



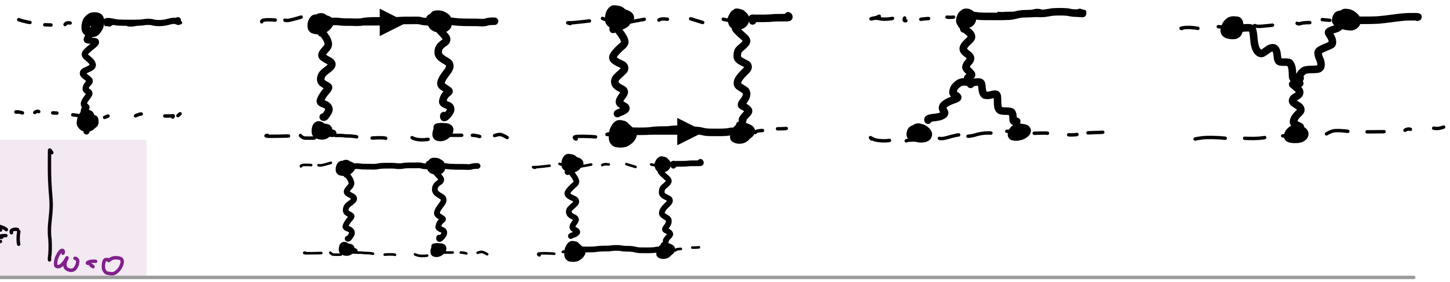
$\sim v^2 + Sv + v^2$ $\qquad \sim v + S$

OBSERVABLES @ NLO

1PM 2PM

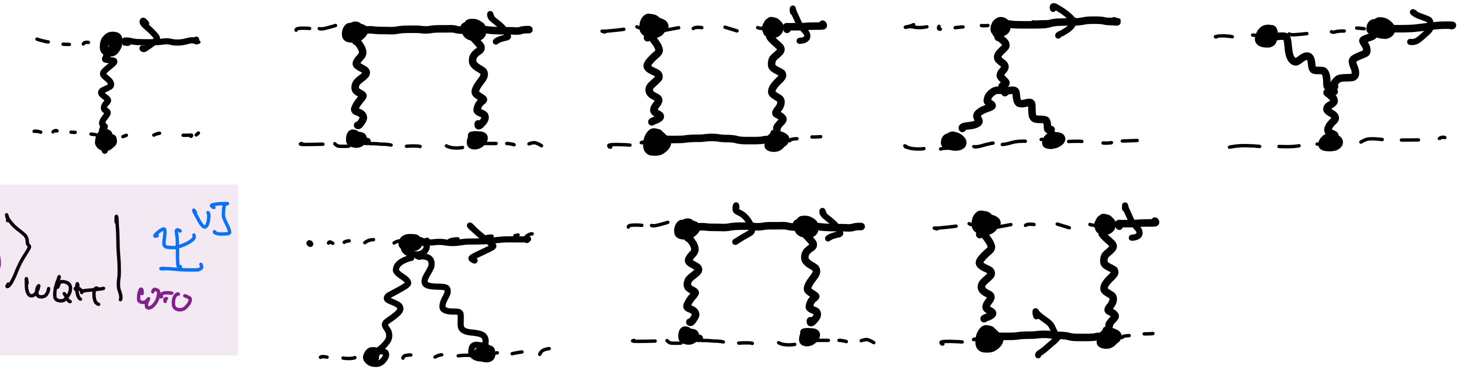
- **DEFLECTION**

$$\Delta p_1^{\mu} = -m_1 \omega^2 \langle z_1^{\mu}(\omega) \rangle_{WQFT} \Big|_{\omega=0}$$



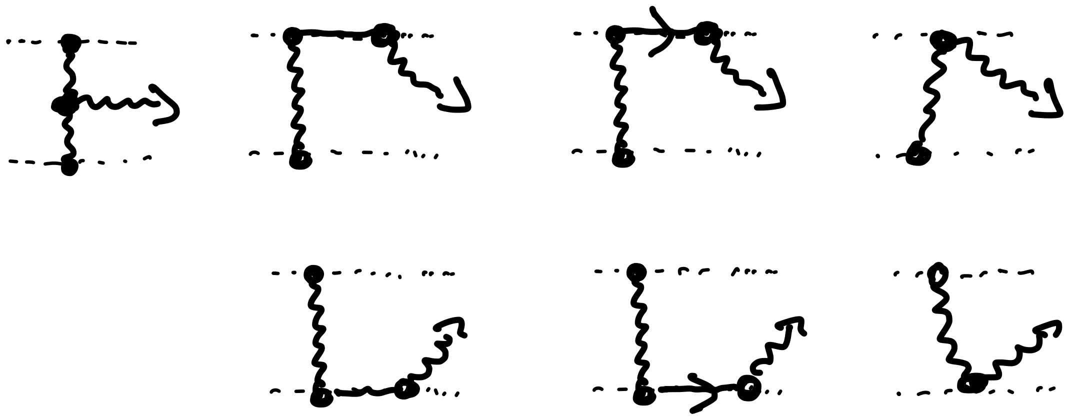
- **SPIN KICK**

$$\Delta S_1^{\mu\nu} = -2i\omega \langle \bar{\psi}^{[\mu}(\omega) \rangle_{WQFT} \Big|_{\omega=0} \psi^{\nu]}$$



- **BREMSSTRAHLUNG**

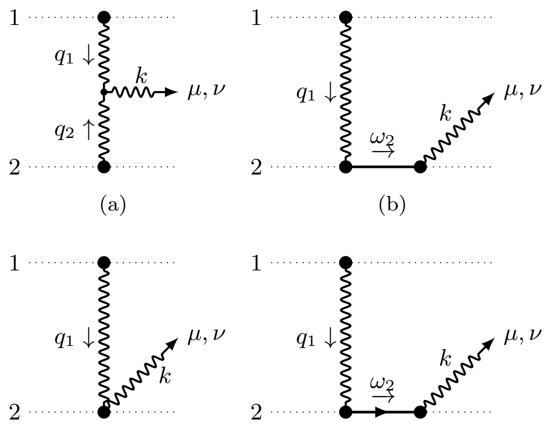
$$-i\kappa^2 \langle h^{\mu\nu}(k) \rangle_{WQFT}$$



SPINNING WAVEFORM @ NLO

[Jakobsen,Mogull,JP,Steinhoff]

$$\frac{f^{(2)}}{m_1 m_2} = \sum_{s=0}^{2} \frac{1}{|\tilde{\mathbf{b}}|_1^{2s+1}} \left[\alpha_1^{(s)} + \frac{\beta_1^{(s)}}{|\tilde{b}|^{2s+2}} \right] + (1 \leftrightarrow 2)$$



(a) (b) (c) (d)

$$|\tilde{\mathbf{b}}|_{1,2} := \sqrt{|b|^2 + (\gamma^2 - 1) u_{2,1}^2} \qquad u_i = \frac{\rho \cdot (x - b_i)}{\rho \cdot v_i} \qquad \rho = (1, \hat{\mathbf{x}})$$

retarded time in ith rest frame

Updates Kovacs-Thorne with spin.

The spinning wave memory: $\Delta f^{2)} = f^{(2)}(u = +\infty) - f^{(2)}(u = -\infty)$

$$\Delta f^{(2)} = \left(1 + \frac{2v|a_3|}{b(1+v^2)} + \frac{|a_3|^2}{|b|^2} - \sum_{i=1}^{2} \frac{C_{E,i}|a_i|^2}{|b|^2} \right) \Delta f^{(2)}_{\mathcal{S}=0}$$

(Aligned spin case)

Using Pauli-Lubanski vector: $\mathcal{S}_i^{\mu\nu} = \epsilon^{\mu\nu}{}_{\rho\sigma} v_i^\rho a_i^\sigma \qquad a_3^\mu = a_1^\mu + a_2^\mu$

Radiated angular momentum in COM:

$$\frac{J_{xy}^{\text{rad}} + i J_{zx}^{\text{rad}}}{J_{xy}^{\text{init}}\big|_{\mathcal{S}=0}} = \frac{4G^2 m_1 m_2}{|b|^2} \frac{(2\gamma^2 - 1)}{\sqrt{\gamma^2 - 1}} \mathcal{I}(v)$$
$$\times \left(1 - \frac{2iv\, \mathbf{a}_3 \cdot \mathbf{l}}{|b|(1+v^2)} - \frac{(\mathbf{a}_3 \cdot \mathbf{l})^2}{|b|^2} + \sum_{i=1}^{2} \frac{C_{E,i}}{|b|^2} (\mathbf{a}_i \cdot \mathbf{l})^2 \right)$$

INTEGRATION TECHNOLOGY

$$\mathcal{I}^{(2;\pm)}_{0,0,0,1,1,0,1} = 0\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,1,1,0,1,1} = (4\pi)^{-3+2\epsilon}\frac{\Gamma^4(\frac{1}{2}-\epsilon)\Gamma^2(\frac{1}{2}+\epsilon)}{\Gamma^2(1-2\epsilon)}\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,1,1,1,0,0} = -(4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{\mathrm{arccosh}\gamma}{4\epsilon\sqrt{\gamma^2-1}} + \mathcal{O}(\epsilon^0)\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,2,1,1,0,0} = -(4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{(1-2\epsilon)\gamma\sqrt{\gamma^2-1}+2\epsilon(\gamma^2-1)\mathrm{arccosh}\gamma}{2\sqrt{\gamma^2-1}} + \mathcal{O}(\epsilon^2)\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,1,1,2,0,0} = -(4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{\mathrm{arccosh}\gamma}{2\sqrt{\gamma^2-1}} + \mathcal{O}(\epsilon)\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,1,1,1,1,1} = (4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{\mathrm{arccosh}\gamma+\epsilon(\mathrm{arccosh}^2\gamma+\mathrm{Li}_2)}{2\epsilon\sqrt{\gamma^2-1}} + \mathcal{O}(\epsilon)\,,$$

$$\mathcal{I}^{(2;\pm)}_{0,0,1,1,2,1,1} = (4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{(1+5\epsilon)\gamma\sqrt{\gamma^2-1}-(1+\epsilon+2\gamma^2\epsilon)\mathrm{arccosh}\gamma-\epsilon(\mathrm{arccosh}^2\gamma+\mathrm{Li}_2)}{2\sqrt{\gamma^2-1}} + \mathcal{O}(\epsilon^2)\,,$$

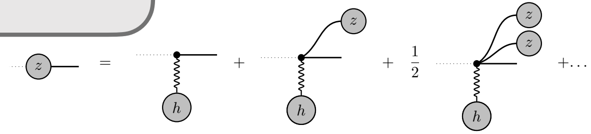
$$\mathcal{I}^{(2;+)}_{1,1,1,1,1,0,0} = \frac{1}{2}\mathcal{I}^{(2;-)}_{1,1,1,1,1,0,0} = (4\pi)^{-2+2\epsilon}e^{-2\epsilon\gamma_{\mathrm{E}}}\frac{1}{2\epsilon^2(\gamma^2-1)} + \mathcal{O}(\epsilon^{-1})\,,$$

WORKFLOW WITH RETARDED INTEGRALS

[Jakobsen,Mogull,JP,Sauer]

Generate graphs

Berends-Giele recursion (tree-level!)



Insert Feynman rules

Form script (4PM: 5g vertex)

Tensor reduction to scalar integrals

$$\int \prod_i d^D l_i \frac{l_{i_1}^{\mu_1} \dots l_{i_n}^{\mu_n}}{\prod_i D(l_i)}$$

$i\epsilon$

IBPs: Basis of Master Integral

Fire/Litered/Kira
Reduced symmetries due to retarded prop.

$l \to -l \quad \epsilon \to -\epsilon$

Solve masters by Diff. Eq. technique

Single scale integrals

$x = \gamma - \sqrt{\gamma^2 - 1}$

Fix boundary conditions in static limit (PN) $\gamma \to 1$

Method of regions (potential & radiation gravitons)

$i\epsilon$

$$\Delta p_i^\mu \quad f_{\mu\nu}(u, \hat{x})$$

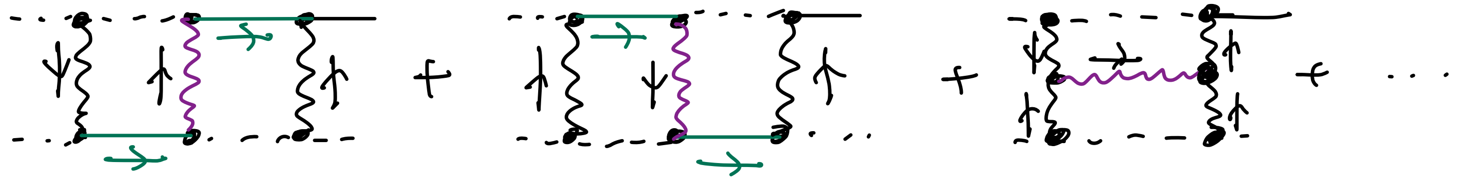
$$\Delta S_i^\mu$$

Find same set of Master integrals for spin and tidal effects@ 3PM

INTEGRATION TECHNOLOGY @ 3PM

3PM DEFLECTION, ONLY RETARDED PROPAGATORS ARISE:

$$\Delta p_1^{\mu} = $$



INTEGRAL FAMILY:

$$I_{n_1 n_2 n_3 n_4 n_5 n_6 n_7} := \int d^d\ell_1\, d^d\ell_2 \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{\underbrace{(\ell_i\cdot v_1 \pm i\varepsilon)^{n_1}(\ell_j\cdot v_2 \pm i\varepsilon)^{n_2}}_{\text{active worldline prop.}}\underbrace{\left((\ell_1+\ell_2-q)^2 \pm i\varepsilon\,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}}_{\text{active graviton propagators}}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

INTEGRALS ARE (PSEUDO)-REAL IN PHYSICAL $\gamma = v_1\cdot v_2$ REGION

$$I^*_{n_1,n_2,\dots,n_7} = (-1)^{n_1+n_2} I_{n_1,n_2,\dots,n_7} \quad \Big\}\ \text{when } 1<\gamma<1$$

Contrast with Feynman integrals, real for $\underline{\underline{-1<\gamma<1}}$.

Performing Retarded Integrals

USE STATE-OF-THE-ART INTEGRATION TECHNOLOGY:

IBP, DIFF. EQUATIONS & METHOD OF REGIONS ADAPTED TO RETARDED PROPAGATORS!

$$I^{(\sigma_1;\sigma_2;\sigma_3)}_{n_1 n_2 \cdots n_7} := \int_{\ell_1 \ell_2} \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_1)}{(\ell_1\cdot v_1+\sigma_1 i\varepsilon)^{n_1}(\ell_2\cdot v_2+\sigma_2 i\varepsilon)^{n_2}\left((\ell_1+\ell_2-q)^2+i\varepsilon\sigma_3\,\mathrm{sgn}(\ell_1^0+\ell_2^0-q^0)\right)^{n_3}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$

$$\frac{\delta^{(n)}(\omega)}{(-1)^n\, n!} = \frac{i}{(\omega+i\varepsilon)^{n+1}} - \frac{i}{(\omega-i\varepsilon)^{n+1}}$$

$\Big\}$ treat $\delta(\omega)$ as a propagator from perspective of IBPs

$i\varepsilon$ RELEVANT FOR SYMMETRIES IN IBP REDUCTION. HERE: 3 FAMILIES

$$I^{(++)}_{n_1 n_2 \cdots n_7} \qquad I^{(--+)}_{n_1 n_2 \cdots n_7} \qquad I^{(+-+)}_{n_1 n_2 \cdots n_7}$$

System of DEs in $x = \gamma - \sqrt{\gamma^2-1}$ takes canonical form:

$$\frac{d\vec{I}}{dx} = \varepsilon\left(\frac{A}{x} + \frac{B_+}{1+x} - \frac{B_-}{1-x}\right)\vec{I}$$

$\Big\}$ $\vec{I} = \vec{I}^{(0)} + \varepsilon \vec{I}^{(1)} + O(\varepsilon^2)$

Method of Regions

Fix boundary conditions to leading order in the static limit $v \to 0$.
Behavior characterized by one graviton:

$$k^{pot} = (k^0, \vec{k}) \sim (v, 1)$$
$$k^{rad} = (h^0, \vec{h}) \sim (v, v)$$

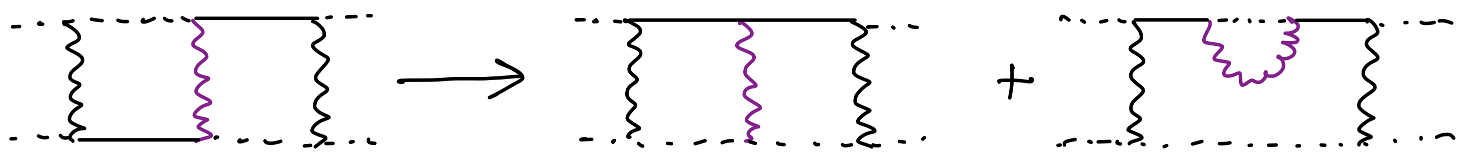
$$I^{(\sigma_1\sigma_2\sigma_3)}_{n_1 n_2 \cdots n_7} = I^{(\sigma_1\sigma_2\sigma_3)pot}_{n_1 n_2 \cdots n_7} + I^{(\sigma_1\sigma_2\sigma_3)rad}_{n_1 n_2 \cdots n_7}$$

Expand integrand in v, assuming all other loop momenta are potential.
Reduce to simpler integrals with manifest dependence on $\gamma = (1-v^2)^{-1/2}$.

$$I^{(\sigma_1\sigma_2\sigma_3)pot}_{n_1 n_2 \cdots n_7} = \int_{\ell_1 \ell_2} \frac{\delta(\ell_1\cdot v_2)\,\delta(\ell_2\cdot v_2)}{(\ell_1\cdot v_1+\sigma_1 i\varepsilon)^{n_1}(\ell_2\cdot v_1+\sigma_2 i\varepsilon)^{n_2}((\ell_1+\ell_2-q)^2)^{n_3}(\ell_1^2)^{n_4}(\ell_2^2)^{n_5}((\ell_1-q)^2)^{n_6}((\ell_2-q)^2)^{n_7}}$$
$$+\, \mathcal{O}(v^{2-n_1-n_2})$$

$$I^{(\sigma_1\sigma_2\sigma_3)rad}_{n_1 n_2 \cdots n_7} = \int_{\ell, k} \frac{\delta((k-\ell)\cdot v_1)\,\delta(\ell\cdot v_2)}{(\ell\cdot v_1+\sigma_1 i\varepsilon)^{n_1}(\ell\cdot v_1+\sigma_2 i\varepsilon)^{n_2}(k^2+\sigma_3\,\mathrm{sgn}(k^0)\,i\varepsilon)^{n_3}(\ell^2)^{n_4+n_7}((\ell-q)^2)^{n_5+n_6}}$$
$$+\, \mathcal{O}(v^{D+1-n_1-n_2-2n_3})$$

Intuitively...



POST-MINKOWSKIAN SCATTERING PRECISSION RACE

WQFT [us]

WEFT Worldline effective theory [Källin,Porto,Dlapa,Cho,Liu,..] [Riva,Vernizzi,Mougiakakos..]

Amps Scattering amplitudes [Bern,Roiban,Shen,Parra-Martinez,Ruf,..] [Bjerrum-Bohr,Damgaard,Vanhove,..] [Di Vecchia,Veneziano,Heissenberg,Russo] [Solon,Cheung,..][Huang,..][Guevera,Ochirov,Vines,…] [Johansson,Pichini[Kosower,O'Connell,Maybee,Cristofoli, Gonzo…]

HEFT Heavy BH effective theory [Aoude,Haddad,Helset] [Brandhuber,Travaglini,Chen]

	deflection & spin kick				waveform			
	plain	spin²	spin>2	tidal	plain	spin²	tidal	Integration complexity
1PM	WQFT WEFT Amps HEFT	WQFT WEFT Amps HEFT	Amps HEFT	X	trivial	trivial	trivial	~ tree-level
2PM	WQFT WEFT Amps HEFT	WQFT WEFT HEFT	Amps	WQFT WEFT Amps	WQFT WEFT (Amps)	WQFT WEFT	WQFT WEFT	~ 1-loop
3PM w/o r-r	WQFT WEFT Amps HEFT	WQFT (Amps)		WQFT WEFT				~ 2-loop
3PM r-r	WQFT WEFT Amps HEFT	WQFT (WEFT)		WQFT WEFT				~ 2-loop
4PM w/o r-r	WEFT Amps							~ 3-loop

r-r: Radiation-reaction

(…) : partial results

SUMMARY

WQFT: Highly efficient technology for classical scattering in GR

- „Quantize“ world-line degrees of freedom & focus on observables (=one-point functions)
- Only compute tree-level diagrams (=classical theory). No „super-classical“ contributions
- IN-IN Formalism: Take all propagators retarded.
- Include spin degrees of freedom through Graßmann odd vectors on the world-line (spinning particle)
- Hidden Supersymmetry = Spin Supplementary Condition

OUTLOOK

WQFT still needs to be extended:

- Higher precision (4PM)
- Higher spin (beyond Spin squared)
- Bound orbits? Relation to EOB
- Relation to self force expansion

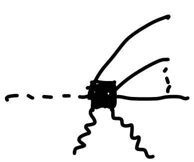
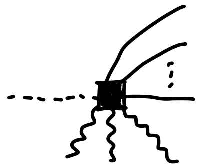
Thank you for your attention!

Tidal effects – work with Benjamin Sauer

Consider a simple extension to the non-spinning theory:

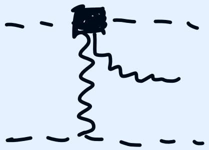
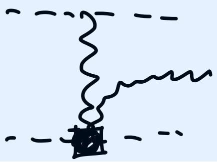
$$S_{pp}+S_{tidal} = m\int d\tau \left[-\frac{1}{2} g_{\mu\nu}\dot{x}^\mu \dot{x}^\nu + C_{E^2} E_{\mu\nu}E^{\mu\nu} + C_{B^2} B_{\mu\nu}B^{\mu\nu}\right]$$

$$E_{\mu\nu} = R_{\mu\alpha\nu\beta}\dot{x}^\alpha\dot{x}^\beta\,, \quad B_{\mu\nu} = R^{*}_{\mu\alpha\nu\beta}\dot{x}^\alpha \dot{x}^\beta\,, \quad R^{*}_{\mu\alpha\nu\beta} = \frac{1}{2} \epsilon_{\nu\beta\rho\sigma} R_{\mu\alpha}{}^{\rho\sigma}$$

Gives rise to new kinds of vertices:  

$\left.\begin{matrix} C_{E^2} \\ C_{B^2} \end{matrix}\right\}$ quadrupole Love no's

We begin with the waveform, consists of 2 diagrams:

$\langle h_{\mu\nu}\rangle =$  $+$ 

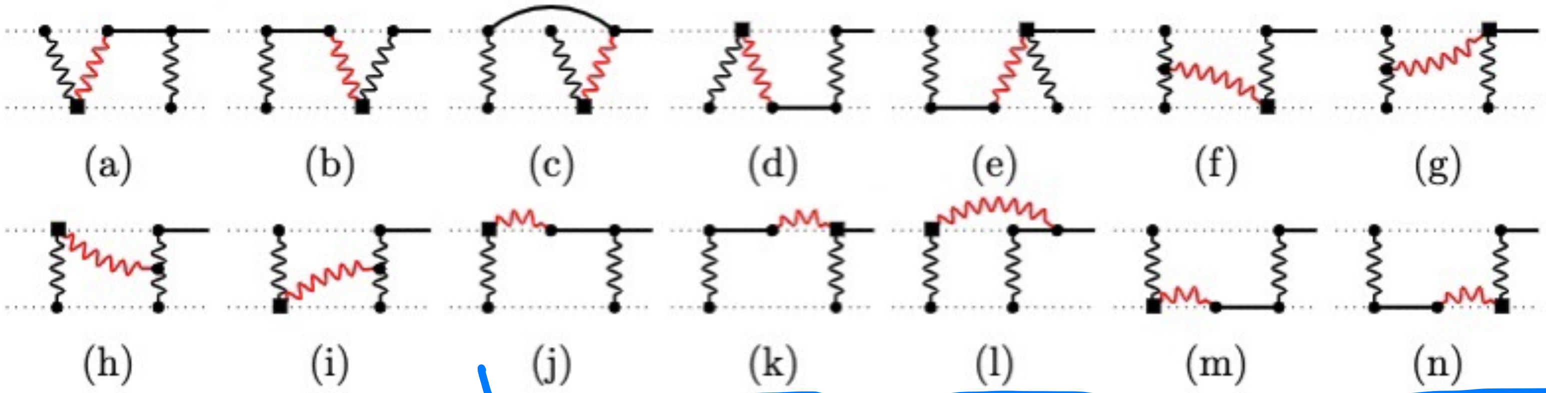
Lack of diagrams with a propagating worldline mode $\Rightarrow$ vanishing wave memory

$$\Delta f_{tidal}(\hat{x}) := f_{tidal}(t=+\infty,\hat{x}) - f_{tidal}(t=-\infty,\hat{x}) = O(G^3)$$

$\Rightarrow J^{rad} \sim P^{rad} \sim O(G^3) \quad \Rightarrow \quad \Theta^{rad,tidal} \sim O(G^4)$

Tidal effects (2)

To compute Δp_1, similar diagrams to spinning calculation:



“mushrooms”

Final result takes a convenient schematic form:

$$\Delta p^{\mu}_{1,\mathrm{cons}} = p_\infty \sin\theta_{\mathrm{cons}} \frac{b^\mu}{|b|} + (\cos\theta_{\mathrm{cons}} - 1)\frac{m_1 m_2}{E^2}\left[(\gamma m_1 + m_2) v_1^\mu - (\gamma m_2 + m_1) v_2^\mu\right]$$

$$\Delta p^{\mu}_{1,\mathrm{rad}} = \underbrace{p_\infty \sin\theta_{\mathrm{rad}} \frac{b^\mu}{|b|}}_{\text{real integrals}} + \underbrace{\frac{P_{\mathrm{rad}}\cdot v_2}{\gamma^2 - 1}\left(v_2^\mu - \gamma v_1^\mu\right)}_{\text{imaginary integrals}}$$

Confirmed: $\theta_{\mathrm{rad}} = 0$, θ_{cons} has finite high-energy $\gamma \to \infty$ limit
P^μ_{rad} agrees with result from squaring waveform (PN expansion)