Massive Gravity and Supersymmetry

Eric Bergshoeff

Groningen University

based on work in progress with

Marija Kovacevic, Jan Rosseel, Ergin Sezgin and Yihao Yin

Ahrenshoop, August 30 2012

rijksuniversiteit groningen

Outline

Introduction

Outline

Introduction

3D New Massive Gravity

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Massive Spin-2 by Higher Derivatives

Einstein Gravity is the unique field theory of interacting massless spin-2 particles around a given spacetime background that mediates the gravitational force

Problem: Gravity is perturbative non-renormalizable

$$\mathcal{L} \sim R + a \left(R_{\mu\nu}{}^{ab}\right)^2 + b\,(R_{\mu\nu})^2 + c\,R^2 \;\; :$$

renormalizable but not unitary

Stelle (1977)

massless spin 2 and massive spin 2 have opposite sign !

Special Case

- In three dimensions there is no (bulk) massless spin 2 !

$\Rightarrow$ “New Massive Gravity”

Hohm, Townsend + E.B. (2009)

Massive Spin-2 by Explicit Mass Term

- Massive Gravity is an IR modification of Einstein gravity that describes a massive spin-2 particle via an explicit mass term

- modified gravitational force

$$V(r) \sim \frac{1}{r} \quad \rightarrow \quad V(r) \sim \frac{e^{-mr}}{r}$$

  - characteristic length scale $r = \frac{1}{m}$

- Cosmological Constant Problem

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Free Fierz-Pauli

- $\left(\Box - m^2\right)\tilde{h}_{\mu\nu} = 0\,, \qquad \eta^{\mu\nu}\tilde{h}_{\mu\nu} = 0\,, \quad \partial^\mu \tilde{h}_{\mu\nu} = 0$

- $\mathcal{L}_{\rm FP} = \frac{1}{2}\tilde{h}^{\mu\nu}G^{\rm lin}_{\mu\nu}(\tilde{h}) + \frac{1}{2}m^2\left(\tilde{h}^{\mu\nu}\tilde{h}_{\mu\nu} - \tilde{h}^2\right)\,, \qquad \tilde{h} \equiv \eta^{\mu\nu}\tilde{h}_{\mu\nu}$

no obvious non-linear extension !

number of propagating modes is $\frac{1}{2}D(D+1) - 1 - D = \begin{cases} 5 & \text{for } 4D \\ 2 & \text{for } 3D \end{cases}$

Higher-Derivative Extension in 3D

$$\partial^\mu \tilde{h}_{\mu\nu} = 0 \quad \Rightarrow \quad \tilde{h}_{\mu\nu} = \epsilon_\mu{}^{\alpha\beta}\epsilon_\nu{}^{\gamma\delta}\partial_\alpha\partial_\gamma h_{\beta\delta} \equiv G^{\text{lin}}_{\mu\nu}(h)$$

$$\left(\Box - m^2\right)\, G^{\text{lin}}_{\mu\nu}(h) = 0\,, \qquad R^{\text{lin}}(h) = 0$$

Non-linear generalization : $\quad g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad \Rightarrow$

$$\mathcal{L} = \sqrt{-g}\left[-R - \frac{1}{2m^2}\left(R^{\mu\nu}R_{\mu\nu} - \frac{3}{8}R^2\right)\right]$$

"New Massive Gravity" : unitary !

What We Now Know

- NMG is (most likely) non-renormalizable

- NMG plus c.c. Λ: massive gravitons $\Leftrightarrow$ black holes

- special value of Λ leads to critical gravity

see talk by Nutma

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Massive Gravity

counting d.o.f. massless gravity

$$6+6\ (g_{ij},\pi^{ij};\, i=1,2,3)-4-4\ (N,N^{i})=2+2\ \text{: massless spin-2}$$

counting d.o.f. massive gravity

$$6+6\ (g_{ij},\pi^{ij})=5+5\ (\text{massive spin-2})\ +1+1\,(\text{BD ghost})\ -1-1$$

4D : Gabadadze, de Rham, Tolley (GdRT) (2010); Chamseddine, Mukhanov (2010)

3D Bi-metric Gravity

Gabadadze, de Rham, Tolley (2010), Hassan, Rosen (2012), Hinterbichler, Rosen (2012), Hassan, Schmidt-May, von Strauss (2012)

3D : Banados, Theisen (2009), Afshar, Alishahiha, Naseh (2009), Zinoviev (2012)

$$I\,[e,f] \;=\; \int d^3x \,\{\, \sigma M_e\, eR(e) + M_f\, fR(f) \;-$$
$$- \frac{1}{16} M m^2\, \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\, (e_\mu^a + f_\mu^a)(e_\nu^b - f_\nu^b)(e_\rho^c - f_\rho^c) \;+$$
$$+ \alpha\, M\, m^2 \varepsilon^{\mu\nu\rho}\, \varepsilon_{abc}\, (e_\mu^a - f_\mu^a)(e_\nu^b - f_\nu^b)(e_\rho^c - f_\rho^c)\}$$

- ${e_\mu}^a$ and ${f_\mu}^a$ are Dreibeins
- $M_e, M_f, M = \frac{M_e M_f}{M_e + M_f}$ and m are (positive) mass parameters
- $\sigma = \pm 1$ and α are dimensionless parameters

The dRGT limit $(\sigma = +1)$

$$f_\mu{}^a = \delta_\mu{}^a + M_f^{-1/2} \delta f_\mu^a \,, \qquad M_f \to \infty \,, M_e = M = M_P$$

$$I_{\text{GdRT}}[e] = M_P \int d^3x \, \{ eR(e) - \frac{1}{16} m^2 \, \varepsilon^{\mu\nu\rho} \, \varepsilon_{abc} \, (e_\mu^a + \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) \; + $$
$$+ \alpha \, m^2 \varepsilon^{\mu\nu\rho} \, \varepsilon_{abc} \, (e_\mu^a - \delta_\mu^a)(e_\nu^b - \delta_\nu^b)(e_\rho^c - \delta_\rho^c) \}$$

The NMG limit $(\sigma = -1)$

$$f_\mu{}^a = e_\mu{}^a + \lambda\, q_\mu^a\,, \qquad \lambda \to 0\,, M_f \to \infty\,, M_e - M_f = \lambda M_f = M_P$$

$$I_{\text{NMG}}\,[e, q] = M_P \int d^3x\,\{-eR(e) + G^{\mu\nu}(e) q_{\mu\nu} - m^2(q^{\mu\nu} q_{\nu\mu} - q^2)\}\,,$$

- $q_{\mu\nu}$ is an auxiliary field

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Massless Spin-2

$$I_{m=0} = \int d^3x \left\{ h^{\mu\nu} G^{\text{lin}}_{\mu\nu}(h) - 4\bar{\psi}_\mu \gamma^{\mu\nu\rho} \partial_\nu \psi_\rho \right\}$$

$$\begin{aligned} \delta h_{\mu\nu} &= \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} \,, \\ \delta\psi_\mu &= -\frac{1}{4}\gamma^{\rho\sigma}\partial_\rho h_{\mu\sigma}\epsilon \end{aligned}$$

$$\delta h_{\mu\nu} = 2\partial_{(\mu} v_{\nu)} \,, \qquad \delta\psi_\mu = \partial_\mu \eta$$

Supersymmetric Fierz-Pauli

$$
\begin{aligned}
I_{m\neq 0} \quad = \quad & \int d^3x \left\{ h^{\mu\nu} G^{\text{lin}}_{\mu\nu}(h) - m^2 \left(h^{\mu\nu} h_{\mu\nu} - h^2 \right) \right. \\
& \left. -4\bar{\psi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \psi_\rho - 4\bar{\chi}_\mu \gamma^{\mu\nu\rho}\partial_\nu \chi_\rho + 8m\bar{\psi}_\mu \gamma^{\mu\nu}\chi_\nu \right\}
\end{aligned}
$$

$$
\begin{aligned}
\delta h_{\mu\nu} &= \bar{\epsilon}\gamma_{(\mu}\psi_{\nu)} + \frac{1}{m}\bar{\epsilon}\partial_{(\mu}\chi_{\nu)}\,, \\
\delta \psi_\mu &= -\frac{1}{4}\gamma^{\rho\sigma}\partial_\rho h_{\mu\sigma}\epsilon\,, \\
\delta \chi_\mu &= \frac{m}{4}\gamma^\nu h_{\mu\nu}\epsilon
\end{aligned}
$$

Bi-metric Supergravity

$$e_\mu^a = \delta_\mu^a + h_\mu^a \,, \qquad f_\mu^a = \delta_\mu^a + k_\mu^a$$

$$\begin{aligned} I^{\text{lin}}_{\text{bi-grav}}[h,k] \;=\; & \int d^3x \,\Big\{ \frac{1}{2}\sigma M_e h^{\mu\nu} G^{\text{lin}}_{\mu\nu}(h) + \frac{1}{2} M_f k^{\mu\nu} G^{\text{lin}}_{\mu\nu}(k) \\ & + \frac{1}{4} M m^2 \delta^{\mu\nu}_{\alpha\beta} (h_\mu{}^\alpha - k_\mu{}^\alpha)(h_\nu{}^\beta - k_\nu{}^\beta) \Big\} \end{aligned}$$

diagonalize : $\quad h_{\mu\nu} = A_{\mu\nu} + B_{\mu\nu} \;,\; k_{\mu\nu} = A_{\mu\nu} - B_{\mu\nu}$

- 2 spin-connections $\rightarrow$ spin-connection + torsion
- supersymmetric GdRT limit ?

New Massive Supergravity

$$\text{NMG limit}: \qquad k_\mu{}^a = h_\mu{}^a + \lambda q_\mu{}^a\,, \quad \lambda \to 0$$

$$I^{\text{lin}}_{\text{NMG}}[h,k] \;=\; \int d^3x\,\{-h^{\mu\nu}G^{\text{lin}}_{\mu\nu}(h) + 2q^{\mu\nu}G^{\text{lin}}_{\mu\nu}(h) - m^2(q^{\mu\nu}q_{\mu\nu} - q^2)\}$$

$$\text{diagonalize}: \qquad h_{\mu\nu} = A_{\mu\nu} + B_{\mu\nu}\ ,\ q_{\mu\nu} = B_{\mu\nu}$$

- elimination of all tensor and vector-spinor auxiliary fields leads to new massive supergravity

Outline

Introduction

3D New Massive Gravity

3D Bi-metric Gravity

Supersymmetry

Conclusions

Summary

- NMG and the 3D GdRT model are different limits of the same bi-metric model

- Bi-metric Supergravity exists with important role for torsion

- New Massive Supergravity Supergravity exists but the GdRT model does not allow (local) supersymmetry

Open Issues

- 3D Chern-Simons formulation ?
- what about 4 dimensions ?
- generalization to higher spins ?