

# SCALAR GEOMETRY AND MASSES IN CALABI-YAU STRING MODELS

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- Scalar masses in supergravity and geometry
- Effective theory of Calabi-Yau string models
- Geometry in the dilaton and no-scale sectors
- Average sgoldstino mass and metastability
- Soft sfermion masses and universality

# SCALAR MASSES IN SUPERGRAVITY

## General structure of soft scalar masses

In a supergravity theory with Kähler potential  $K$  and superpotential  $W$ , the scalar fields have a kinetic function and a potential given by:

$$Z_{I\bar{J}} = K_{I\bar{J}} \quad V = e^K \left[ K^{I\bar{J}} (W_I + K_I W) (\bar{W}_{\bar{J}} + K_{\bar{J}} \bar{W}) - 3|W|^2 \right]$$

At a given vacuum where  $\nabla_I V = 0$ , susy breaking is controlled by  $F_I = -e^{K/2} (W_I + K_I W)$  and  $m_{3/2} = e^{K/2} |W|$ . To get a vanishing cosmological constant  $V = 0$  one then needs to adjust  $|F| = \sqrt{3} m_{3/2}$ . Finally, the masses are  $m_{I\bar{J}}^2 = \nabla_I \nabla_{\bar{J}} V$  and  $m_{I\bar{J}}^2 = \nabla_I \nabla_{\bar{J}} V$ .

The mass matrix depends on  $W$  and  $K$ , with susy and non-susy parts. But along certain particular directions it simplifies and its value is mostly controlled by the geometry defined by  $K$  and less by the form of  $W$ .

## Average sGoldstino mass in the hidden sector

Gomez-Reino, Scrucra 2006

Denef, Douglas 2005

The average mass for the sGoldstino defined by the normalized direction  $f^I$  of susy breaking in the hidden sector is given by:

$$m_{\text{sGold}}^2 = 3 \left( R(f) + \frac{2}{3} \right) m_{3/2}^2$$

in terms of the sectional curvature along  $f^I$ :

$$R(f) = -R_{I\bar{J}P\bar{Q}} f^I f^{\bar{J}} f^P f^{\bar{Q}}$$

A necessary condition for metastability is that  $m_{\text{sGold}}^2$  should be positive.

This implies:

$$R(f) > -\frac{2}{3}$$

This represents a non-trivial constraint on  $K$ , even if  $W$  is allowed to be arbitrary.

## Soft sfermion masses in the visible sector

Kaplunovsky, Louis, 1993  
Brignole, Ibanez, Munoz 1993

The soft mass induced for the sfermions defined by a normalized direction  $v^I$  in the visible sector when susy is broken along a normalized direction  $f^I$  in the hidden sector is given by

$$m_{\text{sferm}}^2 = 3 \left( R(v, f) + \frac{1}{3} \right) m_{3/2}^2$$

in terms of the bisectional curvature along  $v^I$  and  $f^I$ :

$$R(v, f) = -R_{I\bar{J}P\bar{Q}} v^I v^{\bar{J}} f^P f^{\bar{Q}}$$

For positivity and universality, one then needs:

$$R(v, f) > -\frac{1}{3}$$

This represents once again a non-trivial constraint on  $K$ , even if  $W$  is allowed to be arbitrary.

# EFFECTIVE THEORY OF CALABI-YAU STRING MODELS

## Field content

The minimal chiral multiplets are the dilaton  $S$ , the Kähler moduli  $T^A$  and some matter fields  $\Phi^\alpha$ . They naturally split in visible and hidden sectors.

## Effective Kähler potential

The Kähler potential controlling the kinetic energy is always dominated by a classical contributions of the form:

$$K = -\log(S + \bar{S}) - \log[Y(T^A + \bar{T}^A, \Phi^\alpha \bar{\Phi}^\beta)]$$

## Effective superpotential

The superpotential controlling the potential energy can be dominated by non-classical contributions, and can thus a priori be quite arbitrary:

$$W = W(S, T^A, \Phi^\alpha)$$

## Dilaton sector domination

The dilaton belongs to a fixed and factorized manifold  $SU(1, 1)/U(1)$  with constant curvature  $-2$ . One then finds:

$$R(f) = -2 \quad R(v, f) = 0$$

This unavoidably leads to a negative  $m_{\text{sgold}}^2$ , but automatically yields a positive universal  $m_{\text{sferm}}^2$ :

$$m_{\text{sgold}}^2 = -4m_{3/2}^2 \quad m_{\text{sferm}}^2 = m_{3/2}^2$$

This means that it is impossible to realize this scenario in a controllable weak coupling situation.

Cremmer, Ferrara, Kounnas, Nanopoulos 1983

Ellis, Lahanas, Nanopoulos, Tamvakis 1984

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucra 2008

## No-scale sector domination

The moduli and matter fields span a no-scale manifold. For 1 modulus and  $m$  matter fields, one gets  $SU(1, 1 + m)/(U(1) \times SU(1 + m))$  with constant curvature  $-\frac{2}{3}$ . One then finds

$$R(f) = -\frac{2}{3} \quad R(v, f) = -\frac{1}{3}$$

This implies vanishing  $m_{\text{sgold}}^2$  and vanishing  $m_{\text{sferm}}^2$ , which can be a good starting point:

$$m_{\text{sgold}}^2 = 0 \quad m_{\text{sferm}}^2 = 0$$

For  $1 + n$  moduli and  $m$  matter fields, one gets a more general  $\mathcal{M}_{\text{ns}}$  with a curvature that is a priori not constant but must behave as in the previous case along some special direction.

This shows that it may be possible to realize this scenario in a controllable weak coupling situation, at least in models with several moduli.

# GEOMETRY OF NO-SCALE MANIFOLDS

## General no-scale manifolds

A general no-scale manifold spanned by some moduli and matter fields  $Z^i = T^A, \Phi^\alpha$  is described by a Kähler potential of the form

$$K = -\log Y(J^A) \quad J^A = T^A + \bar{T}^A + N^A(\Phi^\alpha \bar{\Phi}^\beta)$$

The real functions  $N^A$  are arbitrary, while the real function  $Y$  must be homogeneous of degree three in the variables  $J^A$ . This implies that:

$$K^i = -\delta_A^i J^A \quad K^i K_i = 3$$

As a consequence, the geometry of such spaces has a restricted form along the special direction  $k^i = -\frac{1}{\sqrt{3}}K^i$  in the hidden sector and any direction  $v^i$  in the visible sector.



The metric, Christoffel symbol and Riemann tensor are found to be:

$$g_{i\bar{j}} = -Y^{-1}Y_{i\bar{j}} + Y^{-2}Y_i Y_{\bar{j}}$$

$$\Gamma_{ij\bar{k}} = -Y^{-1}Y_{ij\bar{k}} + Y^{-2}Y_{ij}Y_{\bar{k}} - Y^{-1}(g_{i\bar{k}}Y_j + g_{j\bar{k}}Y_i)$$

$$R_{i\bar{j}p\bar{q}} = g_{i\bar{j}}g_{p\bar{q}} + g_{i\bar{q}}g_{p\bar{j}} - Y^{-1}Y_{i\bar{j}p\bar{q}} - Y^{-2}Y_{ip\bar{s}}Y^{\bar{s}}_{\bar{j}\bar{q}}$$

Along the special directions  $k^i$  and  $v^i$  one then finds:

$$g_{i\bar{j}}k^i\bar{k}^{\bar{j}} = 1 \quad g_{i\bar{j}}v^i\bar{k}^{\bar{j}} = 0 \quad g_{i\bar{j}}v^i\bar{v}^{\bar{j}} = 1$$

$$\Gamma_{ij\bar{k}}k^ik^j\bar{k}^{\bar{k}} = -\frac{2}{\sqrt{3}} \quad \Gamma_{ij\bar{k}}v^ik^j\bar{k}^{\bar{k}} = 0 \quad \Gamma_{ij\bar{k}}v^iv^j\bar{k}^{\bar{k}} = 0$$

$$R_{i\bar{j}p\bar{q}}k^i\bar{k}^{\bar{j}}k^p\bar{k}^{\bar{q}} = \frac{2}{3} \quad R_{i\bar{j}p\bar{q}}v^i\bar{k}^{\bar{j}}k^p\bar{k}^{\bar{q}} = 0 \quad R_{i\bar{j}p\bar{q}}v^iv^j\bar{k}^{\bar{q}}k^p\bar{k}^{\bar{q}} = \frac{1}{3}$$

This implies that

$$R(k) = -\frac{2}{3} \quad R(v, k) = -\frac{1}{3}$$

# HETEROTIC MODELS

## Geometry of the no-scale sector

One finds:

$$Y = \frac{1}{6} d_{ABC} t^A t^B t^C$$

where

$$t^A = J^A$$

$$J^A = T^A + \bar{T}^A - c_{\alpha\beta}^A \Phi^\alpha \bar{\Phi}^\beta$$

The function  $Y(J^A)$  is homogeneous of degree **3** and also polynomial.

The quantities  $d_{ABC}$  and  $c_{\alpha\beta}^A$  are defined by integrals of harmonic forms:

$$d_{ABC} = \int_X \omega_A \wedge \omega_B \wedge \omega_C$$
$$c_{\alpha\beta}^A = \int_X \omega^A \wedge \text{tr}(u_\alpha \wedge \bar{u}_\beta)$$

Cecotti, Ferrara, Girardello 1988

Candelas, de la Ossa 1990

Buchbinder, Ovrut 2003

Paccetti Correia, Schmidt 2008

Andrey, Scrucca 2011

# ORIENTIFOLD MODELS

## Geometry of the no-scale sector

Grimm, Louis 2004

Graña, Grimm, Jockers, Louis 2005

Jockers, Louis 2005

One finds:

$$Y = \left( \frac{1}{6} d^{ABC} t_A t_B t_C \right)^2$$

where

$$t_A = t_A(J^B) \text{ such that } d^{ABC} t_B t_C = 2J^A$$

$$J^A = T^A + \bar{T}^A - c_{\alpha\beta}^A \Phi^\alpha \bar{\Phi}^\beta$$

The function  $Y(J^A)$  is homogeneous of degree **3** but not polynomial.

The quantities  $d^{ABC}$  and  $c_{\alpha\beta}^A$  are defined by integrals of harmonic forms:

$$d^{ABC} = \int_X \omega^A \wedge \omega^B \wedge \omega^C$$

$$c_{\alpha\beta}^A = \int_C i^* \omega^A \wedge \text{tr}(u_\alpha \wedge \bar{u}_\beta)$$

# CANONICAL PARAMETRIZATION

Gunaydin, Sierra, Townsend 1984

**Canonical frame** Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985

Farquet, Scrucca 2012

At any reference point corresponding to  $T^A \neq 0$  and  $\Phi^\alpha = 0$ , one may switch to a canonical parametrization where

$$T^0 = \frac{\sqrt{3}}{2} \quad T^a = 0 \quad \Phi^\alpha = 0$$

One may moreover require that  $g_{i\bar{j}} = \delta_{ij}$  and  $Y = 1$ , by a further linear field redefinition and a Kähler transformation.

In this new frame,  $T^0$ ,  $T^a$  and  $\Phi^\alpha$  correspond to the volume modulus, cycle moduli and suitably rotated matter fields, and one finds:

$$d_{000} = \frac{2}{\sqrt{3}} \quad d_{00a} = 0 \quad d_{0ab} = -\frac{1}{\sqrt{3}} \delta_{ab} \quad d_{abc} = \text{generic}$$
$$c_{\alpha\beta}^0 = \frac{1}{\sqrt{3}} \delta_{\alpha\beta}, \quad c_{\alpha\beta}^a = \text{generic}$$

# GEOMETRY IN THE CANONICAL FRAME

## Metric

The metric is by construction trivial:

$$g_{0\bar{0}} = 1 \quad g_{a\bar{b}} = \delta_{ab}$$
$$g_{\alpha\bar{\beta}} = \delta_{\alpha\beta}$$

## Christoffel symbol

The Christoffel symbol is found to be identical in heterotic and orientifold models and reads:

$$\Gamma_{00\bar{0}} = -\frac{2}{\sqrt{3}} \quad \Gamma_{0a\bar{b}} = -\frac{2}{\sqrt{3}}\delta_{ab} \quad \Gamma_{ab\bar{c}} = -d_{abc}$$
$$\Gamma_{0\alpha\bar{\beta}} = -\frac{1}{\sqrt{3}}\delta_{\alpha\beta} \quad \Gamma_{a\alpha\bar{\beta}} = -c_{\alpha\beta}^a$$

## Riemann tensor

Andrey, Scrucca 2011  
Farquet, Scrucca 2012

The Riemann tensor for heterotic and orientifold models is instead:

$$\begin{aligned}
 R_{0\bar{0}0\bar{0}} &= \frac{2}{3} & R_{0\bar{0}a\bar{b}} &= \frac{2}{3}\delta_{ab} & R_{a\bar{b}c\bar{0}} &= \frac{1}{\sqrt{3}}d_{abc} & R_{a\bar{b}c\bar{d}} &= (x \mp a)_{abcd} \\
 R_{\alpha\bar{\beta}0\bar{0}} &= \frac{1}{3}\delta_{\alpha\beta} & R_{\alpha\bar{\beta}a\bar{b}} &= -(y + b)_{\alpha\beta ab} & R_{\alpha\bar{\beta}0\bar{b}} &= \frac{1}{\sqrt{3}}c_{\alpha\beta}^b \\
 R_{\alpha\bar{\beta}\gamma\bar{\delta}} &= \frac{1}{3}(\delta_{\alpha\beta}\delta_{\gamma\delta} + \delta_{\alpha\delta}\delta_{\gamma\beta}) + c_{\alpha\beta}^r c_{\gamma\delta}^r + c_{\alpha\delta}^r c_{\gamma\beta}^r
 \end{aligned}$$

in terms of the following combinations of parameters:

$$\begin{aligned}
 a_{abcd} &= \frac{1}{2}(d_{abr}d_{r cd} + d_{adr}d_{r bc} + d_{acr}d_{r bd}) - \frac{1}{3}(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} + \delta_{ac}\delta_{bd}) \\
 x_{abcd} &= \frac{1}{2}(d_{abr}d_{r cd} + d_{adr}d_{r bc} - d_{acr}d_{r bd}) + \frac{2}{3}(\delta_{ab}\delta_{cd} + \delta_{ad}\delta_{bc} - \delta_{ac}\delta_{bd}) \\
 b_{\alpha\beta ab} &= \frac{1}{2}\{c^a, c^b\}_{\alpha\beta} - \frac{1}{3}\delta_{ab}\delta_{\alpha\beta} - \frac{1}{2}d_{abr}c_{\alpha\beta}^r \\
 y_{\alpha\beta ab} &= \frac{1}{2}[c^a, c^b]_{\alpha\beta} - \frac{1}{3}\delta_{ab}\delta_{\alpha\beta} - \frac{1}{2}d_{abr}c_{\alpha\beta}^r
 \end{aligned}$$

## Coset spaces

Cremmer, Kounnas, Van Proeyen, Derendinger, Ferrara, de Wit, Girardello 1985  
Farquet, Scrucca 2012

The space is **symmetric**, with a covariantly constant Riemann tensor, whenever:

$$a_{abcd} = 0 \quad b_{\alpha\beta ab} = 0$$

There is also another mild algebraic condition on the matrices  $c_{\alpha\beta}^a$ , but it is essentially automatically satisfied whenever these form an algebra.

## Degeneracy of heterotic and orientifold models

D'Auria, Ferrara, Trigiante 2004  
Farquet, Scrucca 2012

The manifolds arising in the **heterotic** and **orientifold** models based on the same Calabi-Yau space **coincide** if and only if:

$$a_{abcd} = 0 \quad b_{\alpha\beta ab} = \text{arbitrary}$$

# AVERAGE SGOLDSTINO MASS

## Sectional curvature

Covi, Gomez-Reino, Gross, Louis, Palma, Scrucra 2008  
Farquet, Scrucra 2012

The sectional curvature controlling  $m_{\text{sgold}}^2$  is, for real  $f^i$ :

$$R(f) = -\frac{2}{3} \pm a(f) + 4b(f) - 2\omega^a(f)\omega^a(f)$$

where

$$a(f) = a_{abcd} f^a f^b f^c f^d \quad b(f) = b_{\alpha\beta ab} f^a f^b f^\alpha f^\beta$$

$$\omega^a(f) = \frac{2}{\sqrt{3}} f^a f^0 + \frac{1}{2} d_{abc} f^b f^c + c_{\alpha\beta}^a f^\alpha f^\beta$$

## Metastability and the lightest scalar

The condition  $R(f) > -\frac{2}{3}$  for metastability implies:

$$\pm a(f) > 0 \quad \text{or} \quad b(f) > 0$$

The lightest scalar then has  $m_{\text{light}}^2 \leq \max \{ 0, (\pm a)_{\text{up}}, 2b_{\text{up}} \} m_{3/2}^2$ .



# SOFT SFERMION MASSES

## Bisectional curvature

Andrey, Scrucra 2011  
Farquet, Scrucra 2012

The bisectional curvature controlling  $m_{\text{sferm}}^2$  is, for real  $f^i$  and  $v^i$ :

$$R(v, f) = -\frac{1}{3} + b(v, f) - c^a(v) \omega^a(f)$$

where

$$b(v, f) = b_{\alpha\beta ab} v^\alpha v^\beta f^a f^b$$

$$c^a(v) = c_{\alpha\beta}^a v^\alpha v^\beta \quad \omega^a(f) = \frac{2}{\sqrt{3}} f^a f^0 + \frac{1}{2} d_{abc} f^b f^c + c_{\alpha\beta}^a f^\alpha f^\beta$$

## Positivity, universality and global symmetries

The condition  $R(v, f) > -\frac{1}{3}$  for positivity and universality calls for:

$$\omega^a(f) = 0 \quad \text{and} \quad b(v, f) > 0$$

A set of global symmetries might explain the first of these conditions.

# CONCLUSIONS

- The scalar **geometry** in Calabi-Yau string models is controlled by two kinds of parameters  $a_{abcd}$  and  $b_{\alpha\beta ab}$ , related to the **deviations** from **coset** situations in the moduli and matter sectors.
- **Heterotic** and **orientifold** models lead to dual geometries, which coincide in symmetric situations with  $a_{abcd} = 0$  and  $b_{\alpha\beta ab} = 0$ , but also in non-symmetric cases with  $a_{abcd} = 0$  but  $b_{\alpha\beta ab} \neq 0$ .
- The properties of the **sgoldstino** and **sfermion** masses are directly linked to the parameters  $a_{abcd}$  but  $b_{\alpha\beta ab}$ , and this allows to study the possibilities of achieving **metastability** and **universality**.