

# Fluxes and M5-instantons in F-theory

- 1109.3454 (NPB) with **S. Krause, C. Mayrhofer**
- 1202.3138 (JHEP) with **S. Krause, C. Mayrhofer**
- 1205.4720 (NPB) with **M. Kerstan**

Timo Weigand

Institute for Theoretical Physics, University of Heidelberg

# Invitation

**F-theory** is an attractive corner to study string compactifications:

- combines **brane world paradigm** with presence of **exceptional gauge symmetry** [Beasley,Heckman,Vafa][Donagi,Wijnholt] '08
- rests on **holomorphic aspects of geometry** and is thus tractible
- amenable to **moduli stabilisation** see R.Valandro, M.Grana, A.Hebecker

Recent years have seen great progress in F-theory compactifications to 4 dimensions

↪ realisation of local GUT model building ideas in global models

↪ more formal aspects of F-theory vacua

Focus of this talk:

I) **Geometry (singularities)**

II) **Gauge fluxes**

III) **M5-instantons**

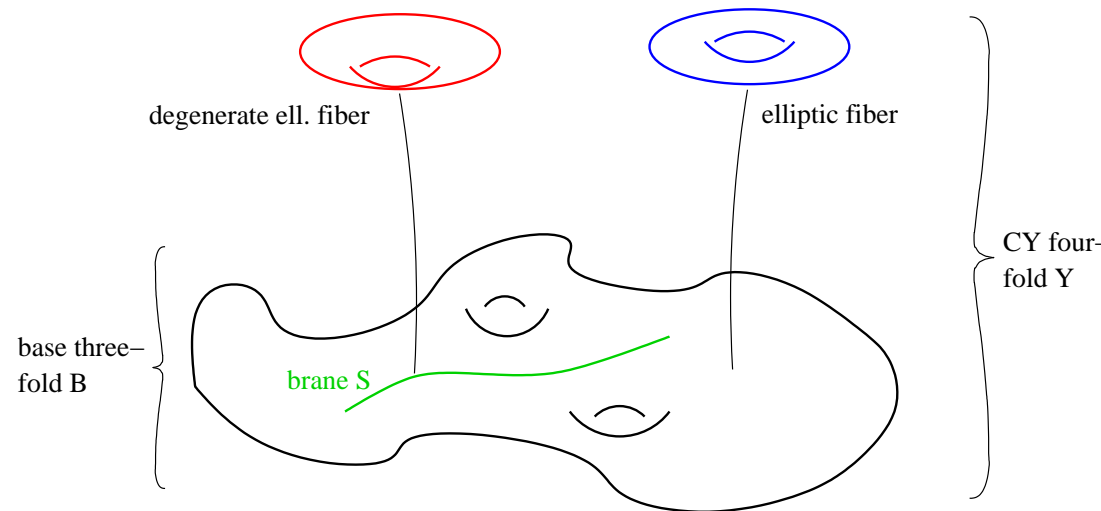
# I) The geometry

# The framework

**Elliptic fibration**  $Y_4 : T^2 \rightarrow B$

$$y^2 = x^3 + f(u_i)xz^4 + g(u_i)z^6$$

- **fibre** coordinates  
 $(x, y, z) \simeq (\lambda^2 x, \lambda^3 y, \lambda z)$
- $B_6$  coordinates:  $u_i$
- singular over  $\Delta \equiv 4f^3 + 27g^2 = 0$



**Matter and interactions** in 4D  $\leftrightarrow$  **singularities of elliptic fiber**

Hierarchy of localisation of structure within base  $B$ :

- **gauge fields**  $G = \text{SU}(5) \leftrightarrow$  GUT **divisor** (codim. 1)
- massless **matter** states  $\mathbf{10}, \bar{\mathbf{5}}_m, \mathbf{5}_H \leftrightarrow$  **curve** (codim. 2)
- **Yukawa** couplings  $\mathbf{10} \mathbf{10} \mathbf{5}_H, \mathbf{10} \bar{\mathbf{5}}_m \bar{\mathbf{5}}_H \leftrightarrow$  **point** (codim. 3)

$\implies$  Understanding **singularities** and their **resolution** is key

# $SU(5) \times U(1)_X$ Tate models

- Example: **SU(5) singularity along  $w = 0$  in Tate form:**

$$P_T : y^2 + a_1xyz + a_3yz^3 = x^3 + a_2x^2z^2 + a_4xz^4 + a_6z^6$$

$$a_1 = a_1, \quad a_2 = a_{2,1} w, \quad a_3 = a_{3,2} w^2, \quad a_4 = a_{4,3} w^3, \quad a_6 = a_{6,5} w^5$$

- **Extra  $U(1)$ s** often desirable for phenomenology

Simplest example:  $a_6 \equiv 0 \leftrightarrow SU(5) \times U(1)_X$  [Grimm, TW '10]

**Complete resolution of all singularities** by sequence of blow-ups:

- Resolution of **SU(5) singularity** on divisor by 4 blow-ups:

[Esole, Yau] [Nameki, Marsano] [Krause, Mayrhofer, TW] [Grimm, Hayashi] '11

$$(x, y, w) \rightarrow (xe_1e_4e_2^2e_3^2, ye_1e_4^2e_2^2e_3^3, we_1e_2e_3e_4)$$

✓ in agreement with earlier toric resolution of [Blumenhagen, Jurke, Grimm, TW]

[Grimm, Krause, TW] '09

- $U(1)_X$  [Grimm, TW '10], [Braun, Collinucci, Valandro] [Krause, Mayrhofer, TW] '11

curve of **SU(2) singularities** at  $C_{34} = \{a_3 = 0\} \cap \{a_4 = 0\}$  resolved by

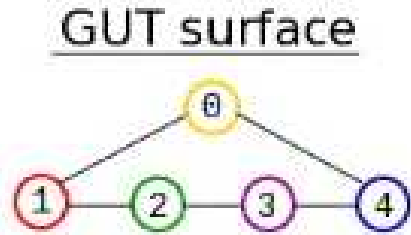
$$y = \tilde{y}s, \quad x = \tilde{x}s$$

# Structure of resolved fiber

**Over divisor:** intersecting  $\mathbb{P}_i^1 - 1$  for each resolution divisor  $E_i : e_i = 0$

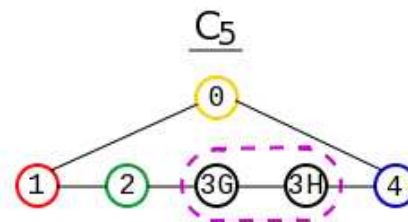
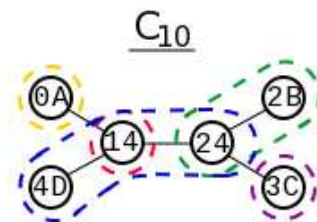
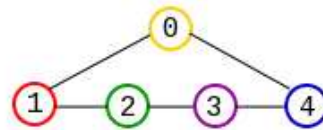
$\Rightarrow$  affine Dynkin diagram of  $SU(5)$

(together with  $\mathbb{P}_0^1$ )



**Over certain curves** some  $\mathbb{P}_i^1$  split  $\rightarrow$  Dynkins of higher groups

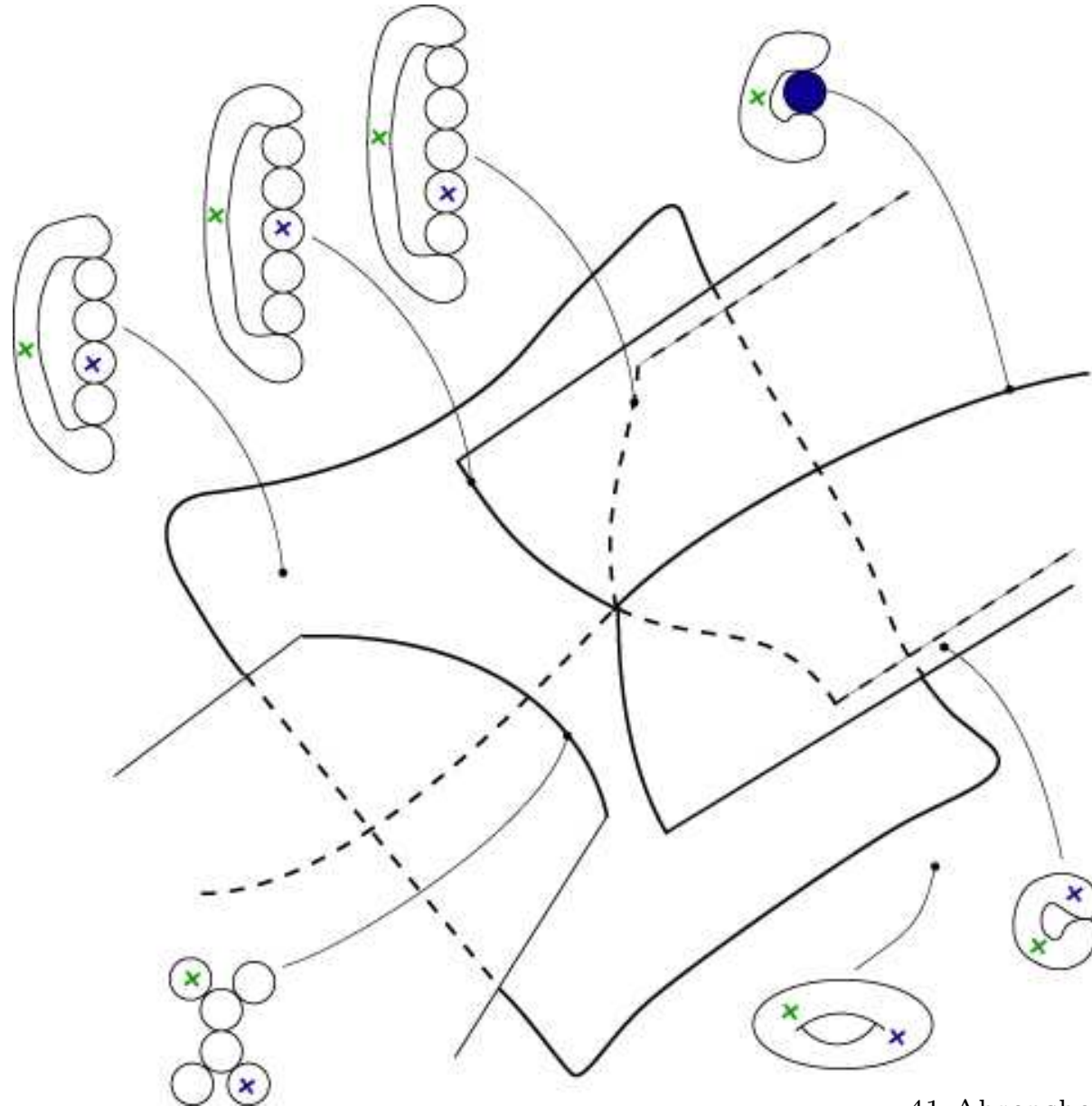
GUT surface



Each weight  $\beta^k$  of repr.  $R \leftrightarrow$  surface  $C_R^k$  fibered over curve  $\mathcal{C}_R \subset B$

# Structure of resolved fiber

[Krause, Mayrhofer, TW'11]



## II) Gauge fluxes



# $G_4$ -Fluxes - Overview

Chiral matter spectrum requires gauge fluxes

Gauge fluxes described by  $\mathbf{G}_4 \in \mathbf{H}^{2,2}(\mathbf{Y}_4)$  with '1 leg along fiber'

$$\text{a) } \int_{\hat{Y}_4} G_4 \wedge D_a \wedge D_b = 0 \quad \text{b) } \int_{\hat{Y}_4} G_4 \wedge D_a \wedge Z = 0 \quad \forall D_i \in H^2(B), Z: \text{ fibre}$$

$$\mathbf{H}^{2,2}(\mathbf{Y}_4) = \mathbf{H}_{\text{vert}}^{2,2}(\mathbf{Y}_4) + \mathbf{H}_{\text{hor}}^{2,2}(\mathbf{Y}_4) \quad [\text{Greene, Plesser, Morrison '94}]$$

- $\mathbf{H}_{\text{vert}}^{2,2}(\mathbf{Y}_4)$  generated by elements of  $H^{1,1} \wedge H^{1,1}$ : factorisable fluxes
  - fluxes associated with massless  $U(1)$ s [Grimm, TW '10],  
[Braun, Collinucci, Valandro '11], [Krause, Mayrhofer, TW'11], [Grimm, Hayashi '11]
  - extra special fluxes, e.g. 'spectral cover' fluxes  
[Marsano, Nameki '11] [Küntzler, Nameki '12] [Tatar, Walters '12]
- $\mathbf{H}_{\text{hor}}^{2,2}(\mathbf{Y}_4)$ : non-factorisable fluxes [Braun, Collinucci, Valandro '11],  
[Krause, Mayrhofer, TW'11]

# Gauge flux from $U(1)_X$

Extra resolution  $S$  gives rise to **extra  $U(1)_X$  not contained in  $SU(5)$**

$$w_X = S - Z - \bar{K} + \sum t_i E_i \in H^{1,1}(\hat{Y}_4) \rightarrow C_3 = A_X \wedge w_X$$

$$\text{repr. } R : \quad 10_1, \quad 5_3, \quad 5_{-2}, \quad 1_5 \quad \mathbf{q}_R = \int_{\mathbb{P}_R} w_X$$

Associated with  $U(1)_X$ : **Gauge flux  $G_4$**  [Grimm, TW' 10]

[Braun, Collinucci, Valandro '11] [Krause, Mayrhofer, TW '11] [Grimm, Hayashi '11]

$$C_3 = A_X \wedge w_X \implies \mathbf{G}_4 = \mathbf{F}_X \wedge w_X, \quad F_X \in H^{1,1}(\hat{Y}_4) \cap H^2(B_3)$$

Importance of  $G_4$  flux: induces **chirality for charged matter**

$$\chi(R) = \int_{C_R} G_4 = \int_{C_R} F_X \wedge w_X = q_R \int_{C_R} F_X \quad \mathbf{q}_R = \int_{\mathbb{P}_R} w_X$$

- ✓ computation of **charged singlets away from  $SU(5)$  GUT brane**
- ✓ result **accounts for corrections to semi-local split spectral cover** due to global features

# Flux consistency conditions

- **Quantisation**:  $G_4 + \frac{1}{2}c_2(\hat{Y}_4) \in H^4(\hat{Y}_4, \mathbb{Z})$  [Collinucci, Savelli '10 & '12]
- **D3/M2 tadpole**:  $N_{M2} + \frac{1}{2} \int_{\hat{Y}_4} G_4 \wedge G_4 = \frac{1}{24} \chi(\hat{Y}_4)$

$$\frac{1}{2} \int_{\hat{Y}_4} G_4 \wedge G_4 = \int_{B_3} F_X \wedge F_X \wedge (15 [W] - 25 c_1(B_3))$$

[Krause, Mayrhofer, TW '11]

- **F-term** condition:  $G_4 \in H^{2,2}(\hat{Y}_4) \checkmark$
- **D-term** condition: from detailed analysis of F/M- theory effective action [Grimm '10] [Grimm, Kerstan, Palti, TW '11]

$$D_X = -\frac{2}{V_B} \int_{\hat{Y}_4} J \wedge G_4 \wedge w_X = \int_{B_3} J \wedge F_X \wedge (15 [W] - 25 c_1(B_3))$$

- ✓ **explicit example of 3-generation model** on concrete 4-fold

[Krause, Mayrhofer, TW '11]

- ✓ **classification of fluxes and explicit match with Type IIB fluxes**

[Krause, Mayrhofer, TW '12]

### III) M5-instantons

# M5-instantons - Setup

Non-perturbative superpotentials from M5-branes are important

- Kähler moduli stabilisation à la KKLT [Witten'96]
- generation of matter couplings  
neutrino masses,  $\mu$ -terms, SUSY breaking F-terms, corrections to Yukawas, ...  
[Blumenhagen,Cvetič,TW] [Ibanez,Uranga] [Florea,Kachru,McGreevy,Saulina]'06
- interesting from formal perspective e.g. relation to U(1)s and  $G_4$ -fluxes

Superpotential requires M5-instanton on vertical divisor [Witten'96]

$$D_M = \pi^{-1} D_M^b \subset Y_4$$

- effect not encoded in singular geometry, extra info
- generic fiber of  $D_M$  smooth except at intersections with 7-branes

## Challenges:

- 1) Structure of M5-partition function
- 2) Interplay with 7-branes/ gauge flux

# Partition function - Overview

**Technical complication:** M5 hosts 2-form  $\mathcal{B}$  with self-dual 3-form field strength  $*\mathcal{H} = i\mathcal{H}$  see [Pasti,Sorokin,Tonin '96&'97] for first explicit action

We work with **Witten's auxiliary action** [Witten'96]

- $S_{M5} = 2\pi(\text{Vol}_{D_M} + i \int C_6) + S_{\mathcal{B}}, \quad S_{\mathcal{B}} = -2\pi \int [\mathcal{H} \wedge *\mathcal{H} - 2i \mathcal{H} \wedge C_3^- + \frac{1}{4} C_3 \wedge *C_3]$
- $*\mathcal{H} = i\mathcal{H}$  only imposed later by extracting holomorphic piece in  $W$

For superpotential need holomorphic piece of the partition function

$$Z_{M5} = e^{-2\pi(\text{Vol}_{D_M} + i \int C_6)} \int \mathcal{D}\mathcal{B} e^{-S_{\mathcal{B}}}$$

- This is merely part of the partition function corresponding in IIB language to fluctuations of gauge potential along instanton
- to be supplemented by fermionic partners and deformation moduli(ni)  
[Kallosh,Kashani-Poor,Tomasiello][Martucci et al.]'05, [Tsimpis]'07
- assume first absence of bulk fluxes (isolated instanton)

# Partition function - Overview

Partition function includes **classical + quantum piece (Pfaffian)**:

$$Z_{M5} = \sum_{\mathcal{H}_0 \in H^3} e^{-S_{M5}[\mathcal{H}_0]} \int \mathcal{D}\delta\mathcal{B} e^{-S'_{M5}[d\delta\mathcal{B}, \mathcal{H}_0]}$$

**Quantum piece (Pfaffian)  $\leftrightarrow$  non-zero modes  $\delta\mathcal{B}$**

$\rightsquigarrow$  non-trivial info on bulk moduli

$\rightsquigarrow$  zeroes expected whenever instanton hits other branes    IIB: [Baumann et al'06], ...

$\rightsquigarrow$  hard to compute explicitly in SUGRA/closed string channel

(cf. Type II: open string channel very helpful!)

To first order (ignoring moduli dependence):

Classical and quantum piece independent

$\implies$  focus on  $\sum_{\mathcal{H}_0 \in H^3} e^{-S_{M5}[\mathcal{H}_0]}$

# Classical partition function

Formal evaluation of classical sum over  $\mathcal{H}_0$  possible

- Expansion of  $\mathcal{H}_0, C_3$  along integer symplectic basis  $(E_M, F^M)$  of  $H^3(D_M)$  and summation yields [Henningson, Nilsson, Salomonson '99]

$$Z_{M5} = \sum_{\alpha, \beta=0, \frac{1}{2}} e^{-2\pi(\text{Vol}_{M5} + i \int C_6)} \mathcal{Z}_- [\alpha]_{\beta} \mathcal{Z}_+ [\alpha]_{\beta}$$

- $\mathcal{Z}_- [\alpha]_{\beta} = e^{\frac{\pi}{2} b^2 C_-^M} Z_{MN} (C_-^N - C_+^N) \times$   

$$\sum_{k_M} e^{i\pi \left( (k+\alpha)_M \bar{Z}^{MN} (k+\alpha)_N + 2(k+\alpha)_M (\beta^M - i b C_-^M) \right)}$$
- $\int_{D_M} E_M \wedge F^N = \delta_M^N, \quad F^N = X^{MN} E_N + Y^{MN} (*E_N),$   

$$Z^{MN} = X^{MN} + iY^{MN}$$

Witten ('96): The final superpotential corresponds to choice of correct spin structure  $\alpha_c, \beta_c$ :

$$W_{\text{cl.}} = e^{-2\pi(\text{Vol}_{M5} + i \int C_6)} \mathcal{Z}_- [\alpha_c]_{\beta_c}$$

Finding  $\alpha_c, \beta_c$  in M-theory is in general hard!



# Comparison with Type IIB

[Kerstan, TW'12]

✓ Quantitative match with fluxed  $O(1)$  E3-instantons in Type IIB  
as described in [Grimm, Kerstan, Palti, TW'11]

✓ sum over  $\mathcal{H}$ -flux  $\leftrightarrow$  sum over  $E_3$ -flux  $\mathcal{F}_E$

$$W_{E3}^{cl.} = \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]},$$

$$W_{E3}^{cl.} = \exp \left[ -\pi \left( \frac{1}{2} C_E^\alpha \mathcal{K}_{\alpha\beta\gamma} v^\beta v^\gamma + i C_E^\alpha (c_\alpha - \frac{1}{2} \mathcal{K}_{\alpha ab} c^a b^b) \right) \right]$$

$$\times \exp \left[ -\frac{i\pi}{\tau - \bar{\tau}} \delta_{MN} G^M (G^N - \bar{G}^N) \right] \sum_{\mathcal{F}^M \in \mathbb{Z}} e^{-i\pi (2\delta_{MN} G^M \mathcal{F}^N + \tau \delta_{MN} \mathcal{F}^M \mathcal{F}^N)}$$

✓ allows us to fix spin structure of M5-instanton:  $\alpha_M = 0 = \beta^M$

$$\mathcal{F}^M \leftrightarrow \delta^{MN} k_N ; \quad G^M \leftrightarrow C_-^M ; \quad \bar{G}^M \leftrightarrow C_+^M ; \quad \bar{Z}^{MN} = -\tau \delta^{MN}.$$

# M5-instantons and gauge flux

Now: intersection of M5 with 7-branes in presence of flux

Flux dependence via  $S_{\mathcal{B}, G_4} = S_{\mathcal{B}} - 2\pi i \int \mathcal{B} \wedge \iota^* G_4$

Superpotential without 'charged matter operators'  $\Rightarrow \iota^* G_4 \stackrel{!}{=} 0$

- $G_4$  breaks large gauge trafos on  $\mathcal{B}$  to  $H^2(D_M, \mathbb{R}) \rightarrow H^2(D_M, \mathbb{Z})$   
extra integration over  $H^2(D_M, \mathbb{R})/H^2(D_M, \mathbb{Z})$  gives zero:

$$\int d\mathcal{B} e^{2\pi i \int \mathcal{B} \wedge \iota^* G_4} \simeq \delta(G_4) \quad [\text{Donagi, Wijnholt '10}]$$

$\iff$  integration over charged zero modes in IIB language

$$\int \mathcal{D}\lambda_a \mathcal{D}\tilde{\lambda}_b \sum_{\mathcal{F}_E} e^{-S_E[\mathcal{F}_E]} = 0$$

- equivalently:  $\iota^* G_4 \neq 0$  implies **Freed-Witten anomaly**  
e.g. [Marsano, Saulina, Schäfer-Nameki '11; Grimm, Savelli '11]

Quantitative criterion: **Compute**  $\int_{C_{(4)}} G_4$  **with**  $C_{(4)} \in H_4(D_M)$

[Kerstan, TW '12]

# M5-instantons and gauge flux

[Kerstan, TW'12]

## Task:

Test for surface  $C_{(4)} \in H_4(D_M)$  with  $C_{(4)}$  neither inside base nor wrapping fiber

**Result:** There are two types of such surfaces

**1st type of  $C_{(4)} \leftrightarrow U(1)_X$**

- $U(1)_X$  from  $C_3 = A_X \wedge w_X \quad w_X \in H^{1,1}(\hat{Y}_4)$
- 4-cycle  $C_4^X \subset D_M$  by intersecting divisor in class  $w_X$  with  $D_M$

$$[C_{(4)}^X] = -D_M \wedge w_X \in H_{\text{vert.}}^{2,2}(\hat{Y}_4)$$

- $q_X = \int_{C_{(4)}^X} \iota^* G_4 = - \int_{\hat{Y}_4} D_M \wedge w_X \wedge G_4 \leftrightarrow U(1)_X$  shift of  $C_6$
- In Type IIB picture: This is **sensitive to linear combination of zero modes** with two brane stacks!

# M5-instantons and gauge flux

## 2) Proposal

- If  $D_M \cap \mathcal{W} \neq 0$  **extra surfaces** exist which are **not in**  $H_{\text{vert}}^{2,2}(\hat{Y}_4)$
- These are algebraic only for special complex structure.  
in different contexts: [Braun,Collinucci,Valandro'11;Collinucci,Savelli'12]

## Evidence:

- Suppose  $D_M \cap \mathcal{W} \subset C_{10}$  (10-matter surface)  $\implies$  take surface  $C_{10}|_{D_M}$ 
  - ✓  $\int_{C_{10}|_{D_M}} \iota^* G_4 \neq 0$  yields correct  $U(1)_a$  charge in IIB limit!
  - ✓ measures **effect of intersection with  $SU(5)$  stack individually**
- If  $D_M \cap \mathcal{W}$  not in  $C_{10}$ , one can deform intersection locus, possibly on auxiliary space defined in [Kerstan,TW'12]

## Conclusion:

Consistent with intuition about zero modes even in absence of IIB dual

# Conclusions

- ✓ F-theory is an attractive corner for model building.
- ✓ Our understanding of the underlying geometry has improved a lot.
- ✓ Explicit resolution of singularities gives a handle on gauge fluxes.
- ✓ Structure of partition function and of selection rules for M5-instantons agrees with Type IIB intuition

## Next steps:

- Better understanding of non-generic fluxes
- Vertex operators/microscopic picture for M5-instantons
- M5-instantons with chiral  $\mathcal{H}$  flux?

# Don't miss....

**MATCH** **MATHEMATICS CENTER HEIDELBERG** **UNIVERSITÄT HEIDELBERG**  
ZUKUNFT SEIT 1386

## Mathematics & Physics of Moduli Spaces

Heidelberg, September 24 – 28, 2012

Lecture series by:  
V. Fock, Strasbourg  
E. Frenkel, Berkeley  
S. Gukov, Caltech & MPI Bonn  
F. Labourie, Paris-Sud  
G. Moore, Rutgers \*  
J. Teschner, DESY Hamburg

Speakers include:  
M. Alim, Harvard  
P. Boalch, ENS Paris  
T. Dimofte, IAS Princeton  
S. Fomin, Ann Arbor  
O. Guichard, Paris-Sud  
A. Neitzke, Austin  
\* to be confirmed

<http://modulispaces2012.uni-hd.de>

Organized by:  
M. Banagl, A. Hebecker, D. Roggenkamp, T. Weigand, A. Wienhard

NSF CEAR EUROPEAN SCIENCE FOUNDATION