Confining dyon gas with finite-volume effects under control

Speaker: Benjamin Maier

Authors:

Falk Bruckmann, Simon Dinter, Benjamin Maier, Ernst-Michael Ilgenfritz,
Michael Müller-Preußker, Marc Wagner

Physics Department
Humboldt-University of Berlin

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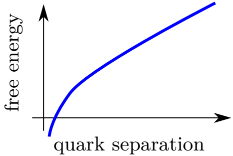
HUMBOLDT-UNIVERSITÄT ZU BERLIN

Motivation / SU(2) YM Theory

- rough approximation of QCD (2 colors instead of 3)
- describes gluons (and infinitely heavy quarks)
- defined via euclidean action $S_{\text{YM}}[A]$
- evaluation of observable O with path integral

$$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A \; O[A] \exp\left(-S_{\text{YM}}[A]\right)$$

$$Z = \int \mathcal{D}A \;\; \exp\left(-S_{\text{YM}}[A]\right)$$



$\Rightarrow$ obtain qualitative understanding of YM theory and confinement

Dyons

$\langle O \rangle = \frac{1}{Z} \int \mathcal{D}A \; O[A] \exp\left(-S_{\mathrm{YM}}[A]\right)$

- approach to solve path integral: semi-classical approximation ⇒ find fields with small action, or their generators

⇒ Transformation in the path integral

$$\int \mathcal{D}A \rightarrow \prod_{j=1}^{n_D} \int d^3\mathbf{r}_j \times \text{Jacobian}$$

- consider calorons, (anti) selfdual classical solutions of YM equations
- consider calorons to be dissolved in their constituents ⇒ dyon gas
- dyons: magnetic monopoles, topological charge $q = \pm 1$

Procedure of Observable Evaluation

what we want

- calculate free energy between a static quark antiquark pair at separation $d = |\mathbf{r} - \mathbf{r}'|$ from

$$F_{Q\bar{Q}}(d) = -T \ \log \left\langle P(\mathbf{r}) P^{\dagger}(\mathbf{r}') \right\rangle$$

- choose dyon density ρ, temperature T
- consider dyons to be located in a volume at positions $\{\mathbf{r}_k\}$
- Polyakov-loop correlator $\left\langle P(\mathbf{r}) P^{\dagger}(\mathbf{r}') \right\rangle$ via

$$P(\mathbf{r}) = -\sin\left(\frac{1}{2T}\Phi(\mathbf{r})\right)$$

- gauge field (superposition of relevant component of the dyon gauge field in abelian limit)

$$\Phi(\mathbf{r}) = \sum_{i=1}^{n_D} \frac{q_i}{|\mathbf{r}_i - \mathbf{r}|}$$

$\Rightarrow$ investigate behavior of free energy for growing quark separation

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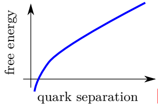

hopefully!

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No Interactions

- in this work: no interactions between dyons
- path integral

$$\left\langle O \right\rangle = \int\limits_V \prod_{i=1}^{n_D} d\mathbf{r}_i \; O\Big(\{\mathbf{r}_k\}, \{q_k\}\Big) \;\Big/\; \int\limits_V \prod_{i=1}^{n_D} d\mathbf{r}_i$$

n_D - number of dyons

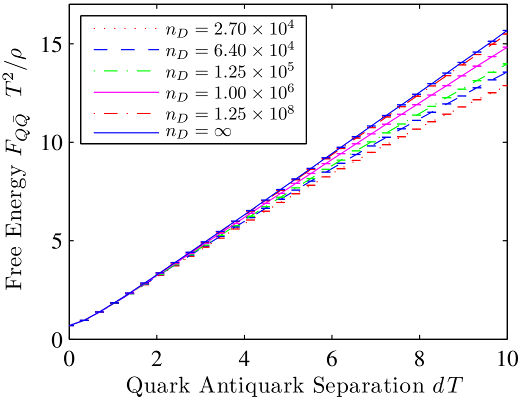
$\{\mathbf{r}_k\}$ - positions of positive (negative) dyons

$\{q_k\}$ - topological charge ± 1

V - volume of gas

- Polyakov loop factorizes to single integrals - possible to solve for arbitrary volume!

Analytical Results for Non-Interacting Dyons



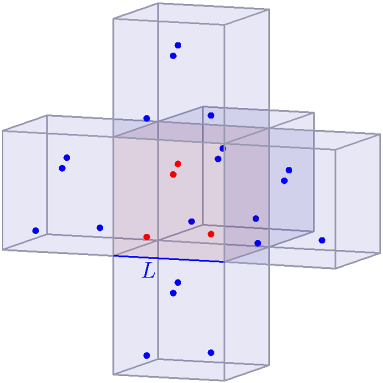
Result

$F_{Q\bar{Q}}$ linear in quark antiquark separation $\Rightarrow$ confinement

Problems with Long-range Dyon Fields

problem long-range potential $\Phi(\mathbf{r}) = \sum_{i=1}^{n_D} \frac{q_i}{|\mathbf{r}_i - \mathbf{r}|} \Rightarrow$ rather large volume is needed

- copy a cubic volume of length L infinitely often in all directions



$\Rightarrow$ Ewald's method

Ewald's Method

- split superposition of dyon potentials into short range part and long range part

$$\Phi(\mathbf{r}) = \sum_{\mathbf{n}\in\mathbb{Z}^3} \sum_{j=1}^{n_D} \frac{q_j}{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|} = \Phi^{\text{Short}}(\mathbf{r}) + \Phi^{\text{Long}}(\mathbf{r})$$

- Φ^{Short} converges exponentially

$$\Phi^{\text{Short}}(\mathbf{r}) = \sum_{\mathbf{n}\in\mathbb{Z}^3} \sum_{j=1}^{n_D} \frac{q_j}{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|} \operatorname{erfc}\left(\frac{|\mathbf{r}-\mathbf{r}_j-\mathbf{n}L|}{\sqrt{2}\lambda}\right)$$

- Φ^{Long} converges exponentially in Fourier space (with momenta $\mathbf{k} = \frac{2\pi}{L}\mathbf{n}$)

$$\Phi^{\text{Long}}(\mathbf{r}) = \frac{4\pi}{L^3} \sum_{\mathbf{k}\neq 0} \sum_{j=1}^{n_D} q_j e^{i\mathbf{k}(\mathbf{r}-\mathbf{r}_j)} \frac{e^{-\lambda^2\mathbf{k}^2/2}}{\mathbf{k}^2}$$

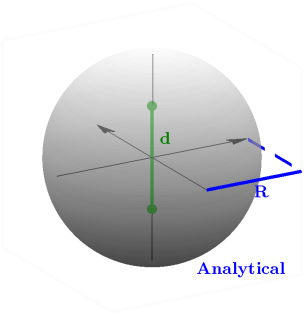
- pedagogical introduction: "Ewald Summation for Coulombic Interactions in a Periodic Supercell" by H. Lee & W. Cai

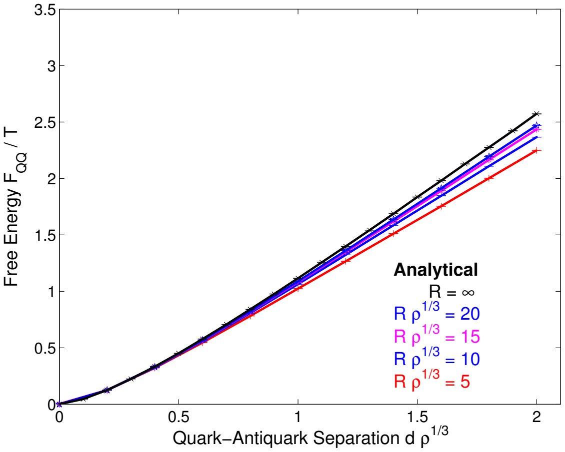
Numerical Results - Ewald's method

Setup and method of computation

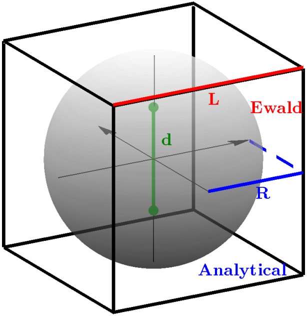
- choose dyon density ρ and temperature T
- put dyons on random positions in a cubic spatial volume for 30 to 800 configurations
- vary dyon number (between 1 000 and 125 000) at fixed density to extrapolate to infinite volume
- evaluate $\Phi(\mathbf{r})$ at various points $\mathbf{r}$ using Ewald's method
- evaluate Polyakov-loop correlator and free energy from Φ

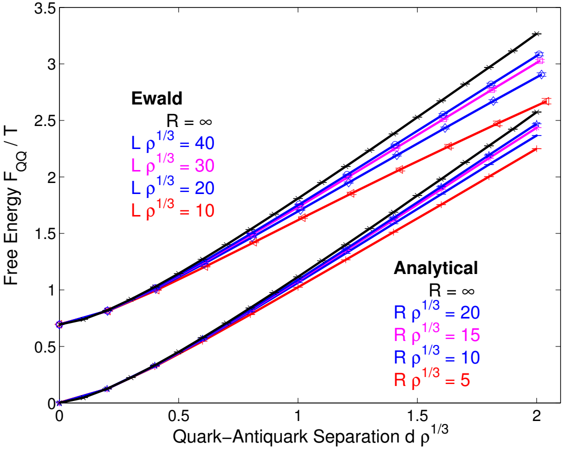
Numerical Results - Ewald's method





Numerical Results





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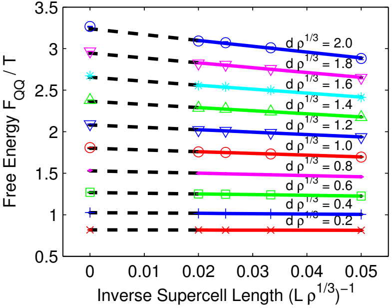

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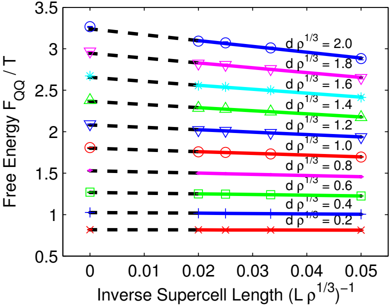

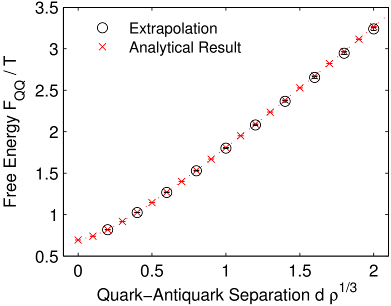

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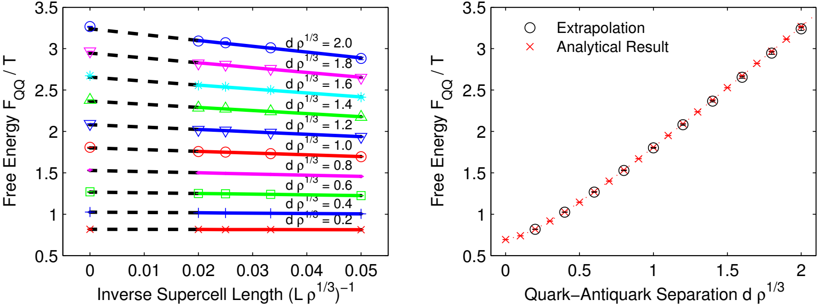

Result

- Ewald's method: controlled extrapolation to infinite volume – equal results to analytical calculations

Summary & Outlook

Summary

- analytically shown: dyon model generates confinement (even without interactions)
- Ewald's method: controlled extrapolation to infinite volume for long-range objects

Ongoing Projects / Future Plans

- simulate an interacting dyon model by expanding an "effective action" $S_{\text{eff}} = \frac{1}{2}\sum_j \sum_i \log\left(1 - \frac{2q_i q_j}{\pi|\mathbf{r}_i - \mathbf{r}_j|}\right)$ (based on the moduli space metric) in inverse powers of r using Ewald's method
- understand effects of interacting/non-interacting dyon model on the free energy

SU(2) YM Action

$$S[A] = \frac{1}{4g^2} \int d^4x \; F^a_{\mu\nu} \; F^a_{\mu\nu},$$
$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + \varepsilon^{abc} A^b_\mu A^c_\nu$$