Quantized Conductance of Point Contacts

Lucas F. Hackl* and Benjamin F. Maier†
Humboldt-Universitaẗ zu Berlin, Department of Physics, Newtonstr. 15, 12489 Berlin
(Dated: August 16, 2011)

We studied quantum effects of point contacts which are realized within a two-dimensional electron gas of GaAs-AlGaAs heterostructure (MODFET). We find quantized steps of $2e^2/h$ the conductance changes in by varying the contact width. This is done by the gate voltage. Varying the gate voltage of the MODFET, we found a change of the conductance in steps of $2e^2/h$. We observed six steps by increasing the gate voltage from $-0.1\,\mathrm{V}$ to $0.6\,\mathrm{V}$. Furthermore, we present an explanation which takes momentum quantization and band structure into account.

I. INTRODUCTION

The effect of quantized conductances is already known from the quantum hall effect, where we can find multiples of e^2/h. The two-dimensional electron gas, which is necessary to investigate those effects, can be realized in GaAs-AlGaAs heterostructures: Doping between the two materials causes a potential minimum under the Fermi edge. This leads to eigenstates of the potential well along this direction while the electrons are free along the other both directions. Therefore, a quantum layer is formed because the effective mass of these electrons is much smaller than the mass of free electrons. Their energy lies slightly beneath the Fermi energy. For our analysis we demand that the mean free path l_e is much greater than the width W ($l_e \gg W$) of the contact and additionally that the Fermi wavelength λ_F is much smaller than W ($\lambda_F \ll W$).

II. EXPERIMENTAL SETUP

For our measurement we use a lock-in amplifier of the type SR830 DSP. The reference signal has the frequency $\nu = 433\,\mathrm{Hz}$ and the amplitude $U = 10\,\mathrm{mV}$. For the integration time we choose $t = 0.1\,\mathrm{s}$ which corresponds to 50 periods.
In the first step we measure the conductance by using several high precision resistors. While in an ideal setting we would expect the relation

$$R_{\text{measured}} = R_{\text{real}}$$

between the measured resistance R_{measured} and the resistance R_{real} of the precision resistor, we find a linear correction:

$$R_{\text{measured}}(R_{\text{real}}) = a\,R_{\text{real}} + R_S$$

Therefore, we have to correct the measured value for the conductance according to

$$G_{\text{real}}(G_{\text{measured}}) = a\,G_{\text{measured}}.$$

Because R_S is the serial resistance of the circuit with precision resistors we do not use it for corrections.

For a we find:

$$a = 1.006 \pm 0.001$$

After this calibration measurement we measure the conductance of our MODFET at low temperature $T = 4.2\,\mathrm{K}$. We use liquid helium as cooling substance to ensure a sharp Fermi distribution caused by the fact that almost all electrons are in states under the Fermi energy. Otherwise the electrons could have enough energy to transcend the potential minimum of the two-deminsional electron gas. The voltage is impressed orthogonal to the contact which shifts the Fermi edge relative to the band structure of the electron gas. This enables us to control the width of the band.

III. MEASUREMENT

Now we can look at the concrete measurement for a MODFET of a certain width l_x. The quantization can be observed

1. if the length of the contact l_z is much greater than the width l_x.
2. if the Fermi wave length is of the same order as l_x.

Under these conditions the contact works as a wave guide. The condition for an electron to pass is then:

$$\lambda_F \approx \frac{2\,l_x}{n}$$

We can rewrite this for the integer n as:

$$n \approx \frac{k_F\,l_x}{\pi}$$

Only those modes m with $m < n$ are allowed and each mode increases the conductance by $2e^2/h$, which leads to:

$$G_n = n\,\frac{2\,e^2}{h}$$

For our measurement we use different structures of the GaAs-AlGaAs MODFET. But in fact we show the results with the best resolution only.
Again, we have to take into account an additional serial resistor R_s which causes the relation $R_{\text{measured}} = aR_n + R_s$. Combining these quantities, we find:

$$G_n(G_{\text{measured}}) = a\,\frac{G_{\text{measured}}}{1 - R_s\,G_{\text{measured}}}$$

*Electronic address: lucas.hackl@physik.hu-berlin.de
†Electronic address: benjamin.maier@physik.hu-berlin.de

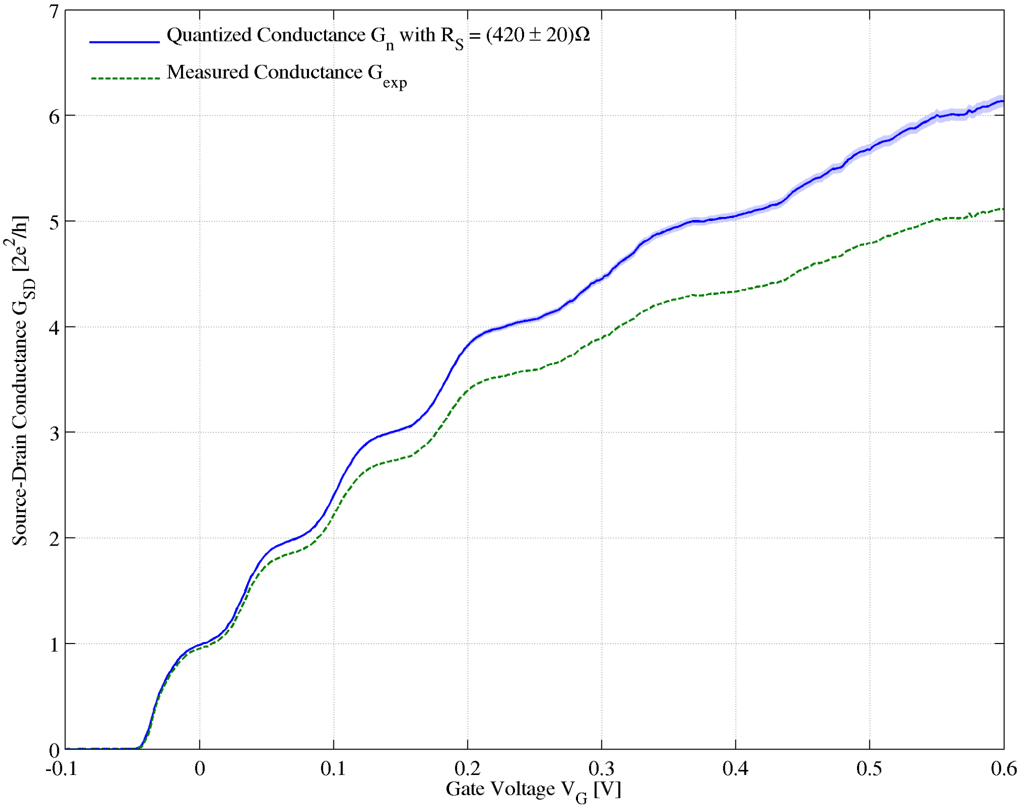

FIG. 1: Quantized conductance dependent on the applied gate voltage V_G. We show the measured values (green, dashed) and the corrected values (blue, solid) by introducing the serial resistor R_s.

We determine R_s by variation to get integer multiples of $2e^2/h$. In fig. 1 we show the uncorrected (green, dashed) and corrected (blue, solid) behavior of the conductance dependent on the applied voltage. One can easily notice the discrete steps which always appear when the potential is high enough to bring the electrons into the next higher band.
For the serial resistor we obtain:

$$R_s = (420 \pm 20)\,\Omega$$

It is important to note that we stress the quantization of the conductance and not of the resistance. Actually, the resistance is quantized as well, which naturally follows from $R = G^{-1}$ but in a more intricate way: While the conductance comes along in integer steps, the resistance, namely the inverse of the conductance, shows a less clear behavior as its steps are inverse to integer steps.

IV. EXPLANATION

We can explain the behavior of the conductance by using the basics of quantum mechanics. The effective one-dimensional area of the quantum point contact can be modelled as a potential well with characteristical lengths l_x and l_y. For the eigenstates we find:

$$\psi(x) = A \sin\left(n\pi \frac{x}{l_x}\right) \sin\left(n\pi \frac{y}{l_y}\right)$$

The corresponding wave numbers follow according to:

$$k_x = n\,\frac{\pi}{l_x}$$

$$k_y = m\,\frac{\pi}{l_y}$$

Therefore, we find stationary electron waves orthogonal to the propagation direction within the quantum point. This leads us to a quantized energy:

$$E_{nm} = \frac{p^2}{2m_e} = \frac{\pi^2\hbar^2}{2m_e}\left(\frac{n^2}{l_x^2} + \frac{m^2}{l_y^2}\right)$$

Because the characteristical length l_z along the propagation direction is much greater than l_x and l_y we can take the corresponding wave numbers as continously distributed with the additional energy:

$$E_z = \frac{\hbar^2 k_z^2}{2m_e}$$

The total energy then is:

$$E_{\text{tot}} = E_{nm} + E_z$$

From the Pauli principle we know that each momentum state can be taken twice because of the spin as additional degree of freedom. This leads to the state density $D(E)$ depending on the energy:

$$D(E)\,\mathrm{d}E = \frac{1}{\pi\hbar}\sqrt{\frac{m}{2E}}\,\mathrm{d}E$$

A small voltage dI causes a small current

$$\mathrm{d}I = e\,v\,\mathrm{d}n$$

with a small linear electron density dn of velocity v. This is exactly the number of electrons per length which are excited by the voltage from the valence band into the conduction band. For this we can find:

$$\mathrm{d}n = D(E)\,\mathrm{d}E = D(E)\,e\,\mathrm{d}V$$

In the last step we have to estimate the velocity by using the relation of the mean values $E = \frac{m}{2}v^2$. This leads to an electric current

$$\mathrm{d}I = \frac{2e^2}{h}\,\mathrm{d}V.$$

Due to the quantization along the two directions we find several sub bands which show the same behavior and which are filled one after the other. Each of those adds a conductance of

$$G = \frac{\mathrm{d}I}{\mathrm{d}V} = \frac{2e^2}{h}$$

which finally leads to the conductance steps

$$G_n = n\,\frac{2e^2}{h}$$

we have measured because the sub bands are filled successively.

V. CONCLUSION

In summary we have presented the measurement of the conductance of single point contacts in a two-dimensional electron gas. Thereby, we found the quantum effect of quantized conductance within these contacts at low temperature, according to steps of $2e^2/h$. Furthermore, we delivered an explanation for the appearance of this effect.

[1] B. J. van Wees, H. van Houten, C.W.J. Beenakker, J.G. Williamson, L.P. Kouwenhovenm, D. van der Marel and C.T. Foxon, Quantized Conductance of Point Contacs in a Two-Dimensional Electron Gas, Phys. Rev. Lett. [72.20.Jv,73.40.Cg, 73.40.Lq] (1988)

[2] T. J. Thornton, M. Pepper, H. Ahmed, D. Andrew, and G. J. Davies, Phys. Rev. Lett. 56, 1198 (1986)

[3] Saskia F. Fischer und Rüdiger Mitdank, Quantisierter Leitwert von Punktkontakten, April 2011

[4] H. Z. Zheng, H. P. Wei, D. C. Tsui, and G. Weimann, Phys. Rev. B. 34, 5635 (1986)

[5] H. van Houten, C. W. J. Beenakker, B. J. van Wees, and J. E. Mooij, in Proceedings of the Seventh International Conference on the Physics of Two-Dimensional Systems, Santa Fe, 1987, Surf. Sci.

[6] G. Timp, A. M. Chang, J. E. Cunningham, T. Y. Chang, P. Mankiewich, R. Behringer, and R. E. Howard, Phys. Rev. Lett. 58, 2814 (1987).

[7] I. B. Levinson, E. V. Sukhorukow, and A. V. Khaetskii, Pis'ma Zh. Eksp. Teor. Fiz. 45, 384 (1987) [JETP Lett. 45, 488 (1987)]

[8] R. Landauer, IBM J. Res. Dev. 1, 223 (1957); R. Landauer, Phys. Lett. 85A, 91 (1981)

[9] P. C. van Son, H. van Kempen, and P. Wyder, Phys. Rev. Lett. 58, 1567 (1987)

[10] G. Timp, A. M. Chang, P. Mangiewich, R. Behringer, J. E. Cunningham, T. Y. Chang, and R. E. Howard, Phys. Rev. Lett. 59, 732 (1987).

[11] Yu. V. Sharvin, Zh. Eksp. Teor. Fiz. 48, 984 (1965) [Sov. Phys. JETP 21, 655 (1987)].