8.6.04 (1)

Complete Saturation

$$E' = \tfrac{1}{2}(g_0 - \alpha)E - i\alpha_H \frac{g_0 E}{2} - \frac{g_0}{2}\varepsilon |E|^2 E$$

$$= \underbrace{\tfrac{1}{2}\left(\bar{g}_0(1 - \varepsilon \underbrace{\overline{|E|^2}}_{S}) - \alpha - i\alpha_H \bar{g}_0\right)}_{G} E - \frac{\bar{g}_0}{2}\overbrace{\tilde{\varepsilon}}^{\tilde{\epsilon}} \delta|E|^2 E$$

including CPP
complex numbers

$$E = \sum_m f_m^{(t,t)} e^{im(\omega t - z/v_g)}$$

$$E' = \sum_m f_m' e^{m} = \sum_m \left(G f_m e^m - \sum_{k,l} \tilde{\epsilon}_{kl} f_k^* f_l f_m e^{m+l-k}\right)$$

mitteln

$$f_0' = G f_0 - \epsilon_{10} f_1^* f_0 f_1 - \cancel{\epsilon_{11}^* f_1^* f_1 f_0}$$

fällt raus weil nicht variabel, schon voll in S!

FWM-Gleichung

$$\boxed{f_0' = G f_0 - \epsilon_{10}|f_1|^2 f_0}$$ bzw. $(\ln f_0)' = G - \epsilon_{10}|f_1|^2$

$$P_0' = (f_0^* f_0)' = f_0^{*\prime} f_0 + f_0^* f_0'$$

$$= (\ln f_0')^* P_0 + P_0 (\ln f_0)'$$

$$(\ln P_0)' = 2\,\mathrm{Re}\,(\ln f_0)' = 2\,\mathrm{Re}\,G - 2\,\mathrm{Re}\,\epsilon_{10}|f_1|^2$$

Also:

$$(\ln P_0)' = 2\mathrm{Re}\, G_{\cancel{1}} - 2\mathrm{Re}(\epsilon_{10})P_1$$

$$\left(\partial_t + \frac{1}{v_g}\partial_z\right)\overset{\ln(p)}{P_k} = G(n, P) - \mathrm{Re}(\epsilon_{\ell k})P_\ell$$

$$\boxed{\left(\partial_z + \frac{1}{v_g}\partial_t\right)\ln\frac{P_1}{P_0} = \mathrm{Re}(\epsilon_{10})P_0 - \mathrm{Re}(\epsilon_{01})P_1}$$

$$\epsilon_{k\ell} = g_0(\bar{N})\left[\frac{(\alpha_H + i)/P_e}{1 + i\,\omega_{k\ell}\tau_e} + \varepsilon\right]$$

$$P_e = \frac{\hbar\omega_0}{g'\tau_e}\ ; \quad \frac{1}{\tau_e} = \frac{1}{\tau_d}(\bar{N}) + \frac{g'}{\hbar\omega_0}P_0$$

$\omega = \omega_1 - \omega_0$

Carrier-Response to $e^{i\omega t} f_1 f_0^*$

$$\dot{N} = I - R(N) - v_g g'(N-N_{tr})|E|^2 \; ; \; I - \frac{J}{e\sigma}$$

$N = \bar{N} + n$; linearisieren

$$\dot{n} = \underbrace{I - R(\bar{N}) - v_g g'(\bar{N}-N_{tr})\overline{|E|^2}}_{0} - R'n - v_g g' n \bar{S} - v_g \bar{g} f_1 f_0^* e^{i\omega t}$$

$$\dot{n} + \frac{n}{\tau_e} = -v_g \bar{g} f_1 f_0^* e^{i\omega t} \; , \; \boxed{\frac{1}{\tau_e} = R'(\bar{N}) + v_g g' \bar{S}}$$

Ansatz: $n = n_0 e^{i\omega t}$

$$\rightarrow (i\omega + 1/\tau_e) n_0 = -v_g \bar{g} f_1 f_0^* e^{i\omega t}$$

$$\rightarrow \boxed{n(t) = \frac{-v_g \bar{g} f_1 f_0^* e^{i\omega_{10} t}}{i\omega_{10} + 1/\tau_e}}$$

$\rightarrow$ Beitrag zu $\underline{E}'$

$$\delta E' = \frac{1}{2} g_0' \cdot n \, (1 - i\alpha_H) \underline{E}$$

$$= \underbrace{\frac{1}{2} \bar{g}_0 \cdot \left[\frac{-v_g g_0' (1+i\alpha_H)}{i\omega_{10} + 1/\tau_e}\right]}_{-\epsilon_{011}} f_1 f_0^* e^{i\omega_{10} t}$$

8.6.04 (4)

$$\boxed{\tilde{\epsilon}_{10} = +\frac{1}{2}\bar{g}_0 \frac{v_g g_0' \tau_e}{1 + i\omega_{10}\tau_e}}$$

dv eine Reibung

$\rightarrow$ daraus kommt $\frac{1}{2}\bar{g}_0 \varepsilon$

$$\cancel{\varepsilon_{ij} = \frac{v_g g_0'}{1/\tau' + i\omega}}$$

$$\boxed{\varepsilon_{ij} = \frac{v_g g' (1+i\alpha_H)}{R'(\bar{N}) + v_g g' \bar{S} + i\omega_{ij}} + \varepsilon}$$

$$= \quad v_g g'$$

Also sind die FWM-Formeln

$$\boxed{\begin{aligned} &\left(\tfrac{1}{v_g}\partial_t + \partial_z\right) f_0 = \tfrac{1}{2} G f_0 - \tfrac{1}{2} g_0 \varepsilon_{10} S_1 f_0 \\ &G = g_0(1 + i\alpha_H - \varepsilon S) - i\alpha_H g_0 - \alpha \\ &\varepsilon_{ij} = \frac{v_g g' (1 + i\alpha_H)}{R' + v_g g' S + i\omega_{ij}} + \varepsilon \end{aligned}}$$

- $S_k = P_k / v_g \hbar\omega_k = |f_k|^2$

u. f. Photondichte

$$\boxed{\left(v_g^{-1}\partial_t + \partial_z\right) S_0 = G S_0 - g_0 \operatorname{Re}(\varepsilon_{10}) S_1 S_0}$$

- $\operatorname{Re}\varepsilon \sim \operatorname{Re}(1 + i\alpha_H)(1 + i\omega\tau)$

$$= \operatorname{Re}(1 - \alpha_H \omega\tau$$

$$\frac{\Delta}{\cancel{S}} = \frac{\Delta_0 / \cancel{S}\, S}{\sqrt{\Delta_0^2 + (S^2 - \Delta_0^2)\, e^{-z/L_\varepsilon}}}$$

$$\Delta S(z) = \frac{\Delta S(0) \cdot S_{sat}}{\sqrt{\Delta S(0)^2 + (S_{sat}^2 - \Delta S(0)^2)\, e^{-z/L_\varepsilon}}}$$

$$r(z) = \frac{r(0)}{\sqrt{r^2(0) + (1 - r^2(0))\, e^{-z/L_\varepsilon}}}$$

$$0 = \frac{(J - J_{tr})}{e\sigma} - v_g g S$$

$$\boxed{S_{sat} = \frac{J - J_{tr}}{e\sigma v_g a}}$$

$$\alpha \varepsilon S_{sat} = \frac{\varepsilon (J - J_{tr})}{e\sigma v_g} = L_\varepsilon^{-1} \quad L_{TWC}$$

$$\boxed{L_{TWC} = \frac{e v_g \sigma}{\varepsilon (J - J_{tr})}} = \boxed{\begin{array}{l} \tau = \frac{10}{7}\,\text{ns} \\ \tau_d = \frac{10}{10}\,\text{ns} = 1\,\text{ns} \end{array}}$$

$$J_{tr} = eR(n_{tr}) = e \cdot (5 + 1 + 1) \cdot 10$$

$$\frac{e n_{tr} \sigma}{\tau} = \frac{1.6 \cdot 10^{-19}\,\text{As}\, 10^{24} \cdot 7\,\text{m}^{-3}}{10^{-8}\,\text{s}} = \cancel{1.6 \cdot 10}$$

$$\cdot\, 5 \cdot 10^{-13}\,\text{m}^2 = 8.7 \cdot \cancel{10}\,\frac{A}{m} = 56\,\frac{A}{m}$$