TWE for SOA with 2 input waves

- optical field $E(z,t) = f(z,t)e^{i(\omega_0 t - k_0 z)}$ only forward.

- dimensionless form of TWE:

$$
\begin{aligned}
2(\partial_t + \partial_z) f &= \left[\frac{g' n(1-i\alpha)}{1+\varepsilon f^* f} - 1\right] f \\
\partial_t n &= j - n\left[\frac{1}{\tau} + \frac{g' f^* f}{1+\varepsilon f^* f}\right] \\
\text{RB: } f(t,0) &= e^{i\omega t}\left(f_{00}(t) + f_{10}(t)e^{i\delta t}\right)
\end{aligned}
$$

- separation into partial waves:

<u>Ansatz:</u> $f(t,z) = e^{i\omega(t-z)} \sum_m f_m(t,z) e^{im\delta(t-z)}$.

with $f_m(t,z)$ slow compared to $1/\delta$

yields for photon density $s = f^* f = \bar{s} + s$

stationary contribution $\bar{s} = \sum_m f_m^* f_m$

beating contribution $s = \sum_{m,n}^{(m \neq n)} f_m^* f_n e^{i(n-m)(t-z)}$.

- restriction to high-δ-limit:
  negligible beating contributions to n

$$\partial_t n = j - n\left[\frac{1}{\tau} + \frac{g'\bar{s}}{1+\varepsilon\bar{s}}\right].$$

• f-equ: linearisation with respect to beating term s

$$2(\partial_t + \partial_z)f_k = -f_k + (1-i\alpha)\frac{g'n\bar{s}}{1+\varepsilon\bar{s}} \times$$
$$\times\left(f_k - \frac{\varepsilon}{1+\varepsilon\bar{s}}\sum_{m,l}^{m\neq k} f_m f_l^* f_{l+k-m}\right).$$

• the nonlinear term mixes the 3 partial waves $m, l, l+k-m$ with partial wave k, this is 4-wave-mixing.

• assumption: the two input waves $k = 0, 1$ remain dominant, hence

$$2(\partial_t + \partial_z)f_k = \left[g'n(1-i\alpha)\frac{1+\varepsilon s_k}{(1+\varepsilon\bar{s})^2} - 1\right] f_k \quad (k = 0, 1)$$

or for the intensities $s_k = f_k^* f_k$

$$(\partial_t + \partial_z)s_k = \left[\frac{g'n}{1+\varepsilon s} - 1\right] s_k - \frac{\varepsilon s_0 s_1}{(1+\varepsilon s)^2} \quad (k = 0, 1)$$

• 4wm reduces each intensity by the same amount
$\Rightarrow$ the lower one is the loser.

• transformation to a moving frame with $\tau = t - z$, $p_k(\tau, z) = s_k(\tau + z, z)$, $\rho(\tau, z) = n(\tau + z, z)$:

$$\partial_z p_k(\tau, z) = \left[g' \rho(\tau, z) \frac{1 + \varepsilon p_k(\tau, z)}{(1 + \varepsilon p(\tau, z))^2} - 1 \right] p_k(\tau, z)$$

Randbedingung: $p_k(\tau, 0) = |f_{k0}(\tau)|^2$

$$\partial_\tau \rho(\tau, z) = j - \rho(\tau, z) \left[\frac{1}{\tau_c} + \frac{g' p(\tau, z)}{1 + \varepsilon p(\tau, z)} \right]$$

"Anfangs"bedingung: $\rho(0, z) = n(t, z)\Big|_{t=z}$

– two ordinary differential equations only,
– photons: τ a parameter,   carriers: z a parameter
– satanic detail: $n(t = z, z)$ to be known.

• stationary: $\rho = 1/g'$, $p_0 + p_1 = j - \rho/\tau_c$,
and $p_0 \cdot p_1 = 0$, i.e. bistable.

• quasistationary approximation: $|\partial_\tau \rho| \ll j$
numerical solution.

wave mixing: modeling in time domain

I propose

- to withdraw the present form of gain saturation and
- instead to add to β the nonlinear contribution

$$\delta\beta^{nl} = -\Gamma \sum_{m=\text{ch,shb}} \frac{g'_m}{2} (N - N_m)\,(i + \alpha_m)\,\varepsilon_m S_m.$$

with the averaged optical intensity

$$S_m = \int_{-\infty}^{t} \frac{dt'}{\tau_m} e^{-(t-t')/\tau_m} S(t').$$

the cpp-contribution comes from the linear $\beta(N)$

problem for large detuning: gain dispersion

If a broad spectrum contributes to $S(t)$, the coefficients in front of S_m depend on the individual frequencies of the partial waves.