Do waves compete or cooperate in a saturated SOA?

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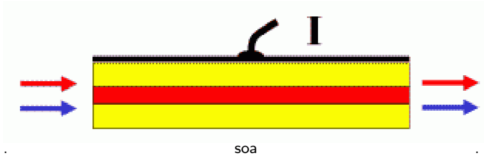
1 Assumptions



Two primary cw waves injected into the SOA.
Frequency separation (detuning) ω.
Intensities such that everywhere saturation.
Equal gain for both waves, under these conditions.
Traveling wave equation for the slowly varying amplitude $E(z,t)$ of the total field :

$$E' := (\partial_z + v_g^{-1}\partial_t)E = \frac{1}{2}\left\{\tilde{g}(1-\varepsilon|E|^2) - \alpha\right\}E, \qquad \text{b.c.: } E(0,t) = E_0 + E_1 e^{i\omega t}. \tag{1}$$

(v_g group velocity, $\tilde{g} = g - 2i\beta$ with $g =$ gain, $\beta =$ real propagation constant, $\varepsilon =$ nonlinear gain saturation coefficient, $\alpha =$ optical losses).

$\omega\tau_{rek} \gg 1 \Rightarrow$ Negligible carrier population pulsations, i.e., $\tilde{g} =$const.

$$E = \sum_m f_m e^{im\omega(t-z/v_g)} \qquad \text{Slowly varying partial wave amplitudes: } |f'_m| \ll \omega|f_m| \tag{2}$$

2 Conclusions

Inserting decomposition (2) into (1), averaging $\langle\cdots\rangle$ over one beating period $2\pi/\omega$, restricting $\langle E^*EE\rangle$ to highest order $f_0^* f_0 f_0 + f_1^* f_0 f_1 + f_1^* f_1 f_0$ in the primary waves $m = 0, 1$:

$$f'_0 = \frac{1}{2}\left\{\tilde{g}(1 - \varepsilon\sum_m |f_m|^2) - \alpha - \tilde{g}\varepsilon|f_1|^2\right\} f_0, \tag{3}$$

inserting $f_0 = \sqrt{P_0}e^{i\varphi_0}$, separating intensity and phase, gives for intensity

$$(\partial_z + v_g^{-1}\partial_t)\ln(P_0) = g(1 - \varepsilon\sum_m P_m) - \alpha - g\varepsilon P_1. \qquad \text{primary wave intensity equation} \tag{4}$$

Interchanging indexes 0 and 1 gives the equation for P_1. Subtracting both equations:

$$\boxed{(\partial_z + v_g^{-1}\partial_t)\ln\left(\frac{P_0}{P_1}\right) = g\varepsilon\,(P_0 - P_1)} \tag{5}$$

- if somewhere $P_0/P_1 > 0$, then it further increases and vice versa. The smaller partial wave is suppressed. We have competition and not cooperation of the two primary waves.

Asymptotics for $P_{min} \ll P_{max} \approx P_{sat}$: $\boxed{P_{min} \sim e^{-\alpha\varepsilon P_{sat} z}}$.

P_{sat} photon density in the saturated SOA.

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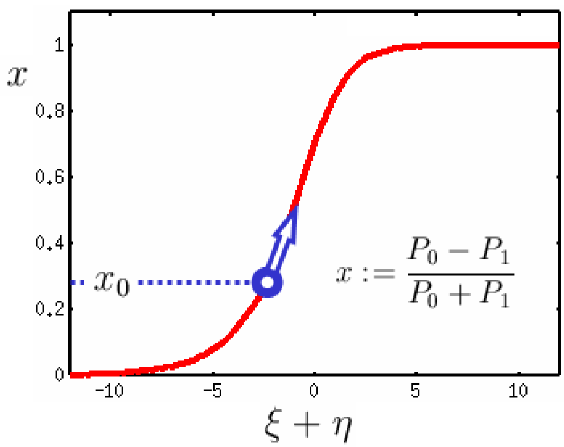
3 Stationary solution of (5)

Annahmen: $\partial_t = 0$, $P_0 + P_1$ =const.$= P_{sat}$, $g \approx \alpha$.

New variables

$$x := \frac{P_0 - P_1}{P_0 + P_1}, \xi = \alpha z \varepsilon P_{sat} \quad \Rightarrow \quad \boxed{\partial_\xi \ln(\frac{1+x}{1-x}) = x} \quad \text{universal equation} \tag{6}$$

$$\text{general solution:} \quad \boxed{\frac{P_0 - P_1}{P_0 + P_1} = x = \frac{1}{\sqrt{1 + e^{-(\xi+\eta)}}} = \frac{x_0}{\sqrt{x_0^2 + (1 - x_0^2)e^{-\xi}}}} \quad \xi = \alpha z \varepsilon P_{sat}. \tag{7}$$



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This general solution is shown in the Figure. It increases monotonically with normalized coordinate ξ. The only influence of the initial power ratio parameter $x_0 = x(z = 0)$ is the starting point on this curve.

The biggest changes appear when stating at $x_0 = 0.5$, i.e. $P_0 = 3P_1$. In this case, noticable changes should already appear along a range

$$\boxed{L_\varepsilon = \frac{1}{\varepsilon P_{sat} \ \alpha}} \tag{8}$$

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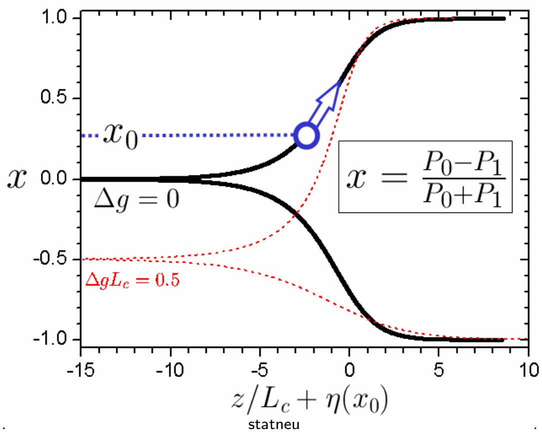
4 Stationary solution of (5) with gain difference

Up to now it was assumed that the two modes have identical gain. In praxis this can be adjusted only approximately. Therefore, I look now on the influence of a gain difference Δg between the two modes.

In this case, the only change of Equ. (5) is the additive term Δg on the right hand side. Keeping the Assumptions $\partial_t = 0$, $P_0 + P_1$ =const.= P_{sat}, $g \approx \alpha$, the solution changes to

$$\boxed{\frac{z}{L_c} = \eta(x) - \eta(x_0) \qquad \text{with the function} \quad \eta(x) = \frac{\ln\left(\frac{|\delta+x|}{1+x}\right)}{1-\delta} + \frac{\ln\left(\frac{|\delta+x|}{1-x}\right)}{1+\delta}} \tag{9}$$

$$x = \frac{P_0 - P_1}{P_0 + P_1}, \qquad \delta = \Delta g L_c, \qquad L_c = \frac{1}{\alpha \varepsilon P_{sat}}.$$



This solution is shown in the Figure for $\Delta g = 0$ (thick black) and $\Delta g L_c = 0.5$. The only influence of the initial power ratio parameter $x_0 = x(z = 0)$ is the starting point on this curve. For $x_0 > -\Delta_g L_c$, the solution moves up and approaches 1 for $\eta \to \infty$. Otherwise, it moves down approaching -1. In case without gain difference, this means suppression of the smaller initial power. With gain difference, the critical ratio x_0 for suppression is given by $-\Delta_g L_c$.

For $\Delta g \neq 0$, the suppression becomes less pronounced in that branche in the Fig. belonging to the mode with the lower gain.

No suppression effect for $|\Delta g| L_c > 1$!!!

5 Estimate of the characteristic length L_c

First we write $L_c = \frac{1}{\alpha} \frac{P_\varepsilon}{P_{sat}}$. Here is $P_\varepsilon = \hbar\omega v_g w d/\varepsilon \approx 1$W the nonlinear gain saturation power that should not be confused with the saturation power P_{sat} of the SOA. The latter one is typically 100mW, yielding the estimate $L_c \approx 10/\alpha \approx 5$ mm if assuming $\alpha = 20\text{cm}^{-1}$.

Shorter L_c require higher losses and/or lower SOA saturation power and/or lower P_ε. However, SOA saturation depends on α.

Therefore a second, different estimate. From rate equations, it can be derived (derivation will be addad later)

$$L_c = \frac{1}{\alpha} + L_\varepsilon \quad \text{with} \quad L_\varepsilon = \frac{P_\varepsilon}{\hbar\omega} \frac{e}{(I - I_{tr})/L} \approx \frac{1 \text{ A/mm}}{(I - I_{tr})/L} \times 1 \text{ mm} \tag{10}$$

The first term contributes about 0.5 mm, the last one depends on how big the current can be made. With 1 A/mm we would arrive at altogether $L_c = 1.5$ mm.

Taking more realistically 5 mm, we must have $\Delta g \ll 2$ cm^{-1}. This is quite small.