TWE for Complex Coupled DFB

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I hopefully present the TWE for complex coupled DFB lasers consistent with polarisation equations and nonlinear gain saturation.

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I. INTRODUCTION

I consider a waveguide with a periodic variation of refractive index and losses / gain. Passive and active layers may contribute to these variations as e.g. sketched in Fig. 1.

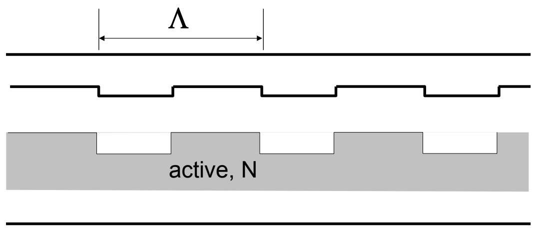

FIG. 1: Schema of the considered DFB grating with periodic thickness variations of passive and also active layers.

II. SUMMARY

The result of the following detailed considerations is:

The optical TWE keep their form with the coupling coefficients in the equations for $E_\pm$ to be replaced by

$$\boxed{\kappa^\pm = \kappa_{\rm pi} + i\kappa_{\rm pl} e^{\pm i\varphi_{\rm pl}} + \kappa_a' e^{\pm i\varphi_a} \overline{\beta}_a(N).} \tag{1}$$

$\kappa_{\rm pi}$: real contribution of the passive index grating, $\kappa_{\rm pl} e^{\pm i\varphi_{\rm pl}}$: magnitude and phase of the passive loss grating contribution, $\kappa_a' e^{\pm i\varphi_a}$: magnitude and phase of the differential contribution of the active grating. The latter one stands in front of the average active contribution to the propagation constant $\overline{\beta}_a(N)$, which may contain dispersion operator and nonlinear saturation terms.

The stimulated recombination within the carrier equation can be written as

$$\boxed{R = 2\Im m \left\{ (E_+, E_-)^* \begin{pmatrix} 1 & \kappa_a' e^{i\varphi_a} \\ \kappa_a' e^{-i\varphi_a} & 1 \end{pmatrix} \overline{\beta}_a \begin{pmatrix} E_+ \\ E_- \end{pmatrix} \right\}} \tag{2}$$

III. DERIVATION OF THE OPTICAL EQUATION

I start from the wave equation

$$\left(\partial_z^2 - \frac{1}{c^2} \partial_t^2 \varepsilon \right) E = 0. \tag{3}$$

The effective dielectric function ε of the wave guide may contain dispersion. In this case it is an operator in this time domain formula. To be concrete I shall assume in frequency domain

$$\frac{\omega^2}{c^2}\varepsilon(\omega) = \left(k + \frac{\omega - \omega_0}{c_g} + \beta(z,\omega) \right)^2, \quad k = \frac{\pi}{\Lambda} \tag{4}$$

$$\beta(z,\omega) = \beta_p(z) + \beta_a(N,\omega). \tag{5}$$

β_p: passive contribution to the propagation constant at central frequency ω_0, not connected with the resonant stimulated transitions. c_g: corresponding passive group velocity. β_a contribution from the carriers in the active layer, appears in the stimulated recombination. The dispersion of this contribution can be described with the polarisation equations. It may also depend on intensity in case of nonlinear gain and index saturation.

Let us introduce the slowly varying amplitudes $E_\pm(z,t)$:

$$E = E_+ e^{i(\omega_0 t - kz)} + E_- e^{i(\omega_0 t + kz)}. \tag{6}$$

Inserting this into Equ. 3 and disregarding higher derivatives of the slowly varying amplitudes and treating the linear disperion contribution as usual, we arrive at

$$e^{-ikz}(-i\partial_z - \frac{i}{c_g}\partial_t + \beta)E_+ + e^{+ikz}(i\partial_z - \frac{i}{c_g}\partial_t + \beta)E_- = 0$$

The dispersive contributions to β are now again the known operator (described by the polarisation equations). Multiplying with $e^{\mp ikz}$, averaging over one period Λ yields

$$\left(\frac{i}{c_g}\partial_t \pm i\partial_z \right) E_\pm = \overline{\beta}\, E_\pm + \kappa^\pm E_\mp. \tag{7}$$

$$\overline{\beta} = \frac{1}{\Lambda}\int_\Lambda \beta(z) dz \qquad \kappa^\pm = \frac{1}{\Lambda}\int_\Lambda \beta(z) e^{\mp 2ikz} dz. \tag{8}$$

IV. STIMULATED RECOMBINATION

Supposing proper normalization of the field, it is

$$R = -iE^* \beta_a E + \text{h.c.} \tag{9}$$

Inserting (6) and averaging over one period Λ gives

$$R = 2\Im m \left\{ (E_+, E_-)^* \begin{pmatrix} \beta_a & \kappa_a^+ \\ \kappa_a^- & \beta_a \end{pmatrix} \begin{pmatrix} E_+ \\ E_- \end{pmatrix} \right\}. \tag{10}$$

V. PARAMETERS

In order to find an appropriate set of parameters, let us discuss different special cases.

A. Index grating

This is the case assumed until now in LDSL. Only the real part $\Re e\beta_p$ of the passive β-contribution has a corrugation. In this case

$$\kappa^+ = (\kappa^-)^* = \kappa_{\text{pi}} = \frac{1}{\Lambda}\int_\Lambda \Re e\{\beta_p(z)\}e^{-2ikz}. \tag{11}$$

The index pi indicates that this coupling coefficient is due to a passive index grating.

Note: the position of the integration interval relative to the grating influences the phases of κ_{pi}. Shifting its center over one period, the phase rotates by 2π. There is one position with $\kappa^+ = \kappa^- = \kappa_{\text{pi}} = \text{real}$. Since the length uncertainty of a DFB section is much larger than a grating period, it is always possible to assume this constellation and an integer number of periods. This is standard for index gratings, only one real parameter κ_{pi} is necessary.

A periodic modulation of $\Re e\{\beta_p(z)\}$ occurs also in all other types of more general DFB gratings. Therefore it is useful to choose the integration interval in Eq. (8) always such that the passive index corrugation yields a real contribution κ_{pi}.

B. Loss grating

In this case also the imaginary part of β_p varies periodically due to e.g. a periodic modulation of the absorption coefficient. The active contribution β_a is constant. Accordingly, a loss-contribution to the coupling coefficient appears in addition:

$$\kappa^\pm = \kappa_p^\pm = \kappa_{\text{pi}} + i\kappa_{\text{pl}}e^{\pm i\varphi_{\text{pl}}}. \tag{12}$$

The indexes *pi* and *pl* denote here passive-index and passive-loss contributions, respectively. The index contribution is real by appropriately choosing the integration interval in $\kappa^\pm$. With the fixed zero of integration, the loss coupling coefficient has in general a magnitude κ_{pl} and a phase φ_{pl}. We need 3 real parameters now.

C. Gain grating

In this case also the thickness of the active layer is corrugated as sketched in Fig. 1. This causes a periodic modulation of the confinement factor. But also the local carrier density may show a modulation. It is not useful, however, to resolve such nanoscale variations of the carrier density. Instead, we denote by N the average carrier density in the given grating period (number of carriers within the active volume of a grating period divided by this volume) and assume

$$\beta_a(z,N) = (1+\Delta(z))\overline{\beta}_a(N). \qquad (\int_\Lambda \Delta(z)dz = 0) \tag{13}$$

$\overline{\beta}_a(N)$: average active β-contribution, $\Delta(z)$: relative local deviation from the average due to, e.g., a variation of the confinement factor. The corresponding active contribution to the coupling coefficient is

$$\kappa_a^\pm = \kappa_a' e^{\pm i\varphi_a}\overline{\beta}_a(N), \quad \text{with} \tag{14}$$

$$\kappa_a' e^{i\varphi_a} = \frac{1}{\Lambda}\int_\Lambda \Delta(z)e^{-2ikz}. \tag{15}$$

Two additional real parameters are necessary: the differential active coupling coefficient κ_a' (dimensionless) and a phase φ_a. Note that the factor $\overline{\beta}_a$ may contain dispersive polarisation contributions. It also contains both a real and an imaginary part due to the gain-index coupling (Henry factor).

With this special form of the active coupling coefficient, the stimulated recombination becomes Equ. (2).