Correlation function and time from power spectra

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After having read Born-Wolff chapter 10.2, I came to conclusions very similar to Oleg's brief introduction. Here a summary.

I. PRELIMINARIES

Be $P(t)$ a transient of optical intensities detected by the photodiode at the ESA.

Be $S(\nu)$ the power spectrum (spectral density) of $P(t)$, defined as [1]

$$S(\nu) \stackrel{def}{=} \lim_{T\to\infty} \frac{\overline{|F_T(\nu)|^2}}{2T} \stackrel{(WK)}{=} \int_{-\infty}^{\infty} d\tau G(\tau) e^{i\omega\tau}, \qquad G(\tau) = \langle P(t)P(t+\tau)\rangle, \qquad \omega = 2\pi\nu. \quad (1)$$

$F_T(\nu)$: Fourier transform of $P(t)$ truncated outside the interval $-T < t \leq T$, (WK): Wiener-Khinchin-Theorem [formula (28) in Born-Wolff], $\overline{|\cdots|^2}$: ensemble average [2], $\langle f(t)\rangle$: temporal average over t [3].

Be $P(\nu_m)$ the spectrum measured with the ESA on an equidistant grid ν_m of frequencies.

Assumption 1: $P(\nu_m)$ is a reasonable approximation of the powerspectrum of $P(t)$.

Nothing else makes sense.

Conclusion: $P(\nu_m)$ must be quadratic in the power transient $P(t)$.

This conclusion is easily checked: look to a spectrum with at least one peak well above noise background. Attanuate the input signal by x dB. Then measured spectrum should go down by 2x dB. I'm sure that one of the usable scales (hopefully dBm) fulfills this requirement [4].

Assumption 2: $P(\nu_m)$ represents the contribution of $S(\nu)$ to an grid interval $\nu_m - \nu_{m-1}$ around ν_m, normalised to a certain resolution bandwith $\Delta\nu_{\text{res}}$.

The corresponding formula is

$$P(\nu_m) = \frac{\Delta\nu_{\text{res}}}{\Delta\nu_{\text{grid}}} \int_{-\Delta\nu_{\text{grid}}/2}^{\Delta\nu_{\text{grid}}/2} S(\nu_m + \nu')d\nu'. \qquad \Delta\nu_{\text{grid}} = \nu_m - \nu_{m-1}. \quad (2)$$

It seems to me the only reasonable choice by the following reasons:

- Sharp lines in between the frequency grid points should not be lost. Thus, I must integrate over all intervals between.
- The magnitude of the spectrum should not depend on the choosen grid width. Thus, I must average over the grid intervals.
- The used unit is "mW", thus I must multiply the average with some standard frequency interval. I guess, it is the resolution band width, which can be choosen independently of the grid step.

II. CORRELATION FUNCTION

The (nonnormalised) correlation function is

$$C(\tau) \stackrel{def}{=} G(\tau) - \langle P(t) \rangle^2 = \int_{-\infty}^{\infty} S(\nu) e^{-i\omega\tau} d\nu - \langle P(t) \rangle^2. \tag{3}$$

Thus, it can be calculated from the power spectrum. Since the power spectrum is symmetric in ν, this is a real quantity. Only how to calculate the subtracted second term from the power spectra, I don't know at present. However, except in ideal coherent cases it is just $G(\tau \to \infty)$, i.e. it can be taken from the large time limit of G.

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[1] I follow chapter 10.2 of Born-Wolff.

[2] this corresponds to averaging over many repetitions of the measurement, which can be done with the ESA. Thus, $S(\nu)$ corresponds to the limit of infinitely many repetitions.

[3] i.e. $\langle f(t) \rangle = \lim_{T \to \infty} \int_{-T}^{T} f(t + t')$dt'/2T

[4] I guess, the optical power P is linearly transferred to a photo current I, which in turn is linearly related to the voltage U at some internal resistivity (50Ω?) analysed by the electronics of the ESA. The output with the choicable unit "dBm" is probably related to the electrical power consumed by the resistivity, i.e. $\sim U^2 \sim I^2 \sim P^2$, quadratic with the detected optical intensity. Let's look to the manual.