Is optical feedback by a Fabry-Perot suited for delayed control?

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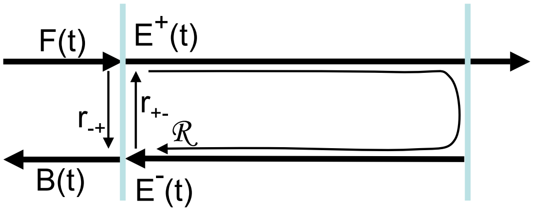

sketch of a Fabry-Perot

$r_m^{\pm}$: ampl. reflectivity $\pm$-wave mirror m
$t_m^{\pm}$: ampl. transmission $\pm$-wave mirror m
$F(t)$: incident field amplitude
$B(t)$: reflected field amplitude
$E_{\pm}(t)$: forw./backw. internal field at left mirror.

cavity propagation operator: $E_-(t) = \hat{\mathbf{R}} E_+(t) = \mathcal{R} E_+(t-\tau), \qquad \mathcal{R} = \exp(-2ic_g\beta\tau)\cdot r_2^+ \qquad (1)$

The reflected field amplitude is governed by the equations

$$\begin{aligned} B(t) &= r_1^+ F(t) + t_1^- E_-(t) & (2)\\ E_-(t) &= \hat{\mathbf{R}}[t_1^+ F(t) + r_1^- E_-(t)] = \mathcal{R}[t_1^+ F(t-\tau) + r_1^- E_-(t-\tau)]. & (3) \end{aligned}$$

Introducing $E(t) = t_1^- E_-(t)$, this is equivalent to

$$\boxed{B(t) = r_1^+ F(t) + E(t), \quad E(t) = \mathcal{R}\cdot[t_1^+ t_1^- F(t-\tau) + r_1^- E(t-\tau)].} \qquad (4)$$

τ-periodic fields

In this case, the delayed fields in Equ. (4) can be replaced by the original ones. The second equ. can be resolved for E and we get

$$B(t) = \left[r_1^+ + \frac{t_1^+ t_1^- \mathcal{R}}{1 - r_1^- \mathcal{R}}\right] F(t). \qquad (5)$$

Now it becomes important that the reflection and transmisssion coefficients of a lossless mirror have to fulfill the energy conservation equations

$$|r_m^{\pm}|^2 + |t_m^{\pm}|^2 = 1 \quad \text{and} \quad r_m^+ t_m^{-*} + t_m^+ r_m^{-*} = 0. \qquad (6)$$

This gives $t_1^+ t_1^- = -(1 - |r_1^-|^2) r_1^+ / r_1^{-*}$. Inserting this and multiplying with r_1^{-*} yields

$$\boxed{B(t) = r_1^+ \left[1 - \frac{(1-|r_1^-|^2)\mathcal{R}/r_1^{-*}}{1 - r_1^- \mathcal{R}}\right] F(t).} \qquad (7)$$

This is always zero for $\mathcal{R} = r_1^{-*}$, which allow to use it for delayed feedback control.

Remark 1:
τ-periodicity of the field means that the pulses have this period but in addition that the optical frequency fulfills $\omega\tau = m\cdot 2\pi$ with an integer m, i.e. operating in a minimum of the FP.

Remark 2:
This works for arbitrary magnitudes of reflectivity, i.e. we can use e.g. a platelet of glass or transparent semiconductor.