Modeling of noises – simple laser

H.-J. Wünsche 15.6.2001

1 Intro

I develop ideas how to check the noise performance of LDSL with a simple single mode laser.

2 Agrawal model

According to the book of Agrawal [1] (chapter 6.5, noise characteristics, pp. 258 ff.), the Langevin equations with spontaneous emission noise can be written in the form

$$\dot{N} = \frac{I}{e} - \frac{N}{\tau} - GP + F_n \tag{2.1}$$

$$\dot{P} = (G-\gamma)P + R_{sp} + F_p \tag{2.2}$$

$$\dot{\varphi} = -\omega + \frac{\alpha_H}{2}(G-\gamma) + F_\varphi. \tag{2.3}$$

N, P = carrier and photon numbers, φ = phase of the optical field, F_k = stochastic Langevin forces, ω = light frequency of the stationary state without spontaneous emission ($R_{sp} = 0, F_k = 0$), other quantities with their usual meaning. The ensemble averages of the stochastic forces obey the relations

$$\langle F_k(t)\rangle = 0 \qquad \big\langle F_k(t)F_l(t')\big\rangle = 2D_{kl}\delta(t-t') \tag{2.4}$$

$$D_{PP} = R_{sp}P, \quad D_{\varphi\varphi} = \frac{D_{PP}}{4P^2}, \quad D_{PN} = D_{NP} = -D_{PP}, \quad D_{NN} = \frac{N}{\tau} + D_{PP}. \tag{2.5}$$

All other D_{kl} vanish.

The D-coefficients are to be calculated with stationary solutions. The spontaneous emission factor $\beta = R_{sp}\tau/N$ is a very small parameter. In lowest order the stationary solutions are

$$\text{0th order:} \qquad N_0 = N_{th}, P_0 = \frac{I - I_{th}}{\gamma\tau}, \text{ with } G(N_{th}) = \gamma, I_{th} = \frac{eN_{th}}{\tau}. \tag{2.6}$$

$$\text{1st order:} \qquad N = N_{th} + n, P = P_0 + p, \text{ with } n = -\frac{\beta N_{th}}{G'\tau P_0 + \beta}, p = -\frac{n}{\gamma\tau}(1 + G'\tau P_0). \tag{2.7}$$

In addition to our present LDSL-treatment, noises appear not only in the field equations but also in the carrier equations. D_{NN} has two contributions, the first one is due to shot noise[1]. The second one equals D_{PP} because every generation of a photon by spontaneous emission removes one carrier. For the same reason, the nondiagonal diffusion coefficients are $-D_{PP}$.

3 Continuous Reformulation

I express the stochastic F_k by noncorrelated stochastic z_k with $\langle z_k\rangle = 0, \langle z_k(t)z_l(t')\rangle = \delta_{il}\delta(t-t')$ and introduce $f = \sqrt{P}e^{i\varphi}$:

$$\dot{N} = \frac{I}{e} - \frac{N}{\tau} - GP + \sqrt{\frac{2N}{\tau}}z_s - \sqrt{2R_{sp}P}z_p \tag{3.1}$$

$$\dot{f} = \frac{1}{2}(G-\gamma)(1+i\alpha_H) + \frac{R_{sp}}{2P} + \sqrt{\frac{R_{sp}}{2P}}(z_p + iz_\varphi) \tag{3.2}$$

(3.3)

[1] I have still to understand the derivation of it.

4 Descrete reformulation with lumped noises

I set $z_k = 0$ within small time steps dt and add the noises at the end in form of

$$N \rightarrow N + \sqrt{\frac{12dt \cdot 2N}{\tau}} \left[r_s - \frac{1}{2} \right] - \sqrt{12dt \cdot 2R_{sp}P} \left[r_p - \frac{1}{2} \right] \tag{4.1}$$

$$f \rightarrow f + \sqrt{\frac{12dt \cdot R_{sp}}{2P}} \left[r_p - \frac{1}{2} + i(r_\varphi - \frac{1}{2}) \right], \tag{4.2}$$

where the r_k are the usual uncorrelated random numbers equally distributed in $[0, 1]$.

Literatur

[1] G.P.Agrawal and N.K.Dutta, “Semiconductor lasers”, second edition, 1993