Derivation of Lang-Kobayashi Equations: The coupling rates

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Abstract

This is an addition to the pamphlet "Derivation of Lang-Kobayashi Equations" from July 7, 2000. I correct som sign errors and express the coupling rate in terms of the derivation of the inverse reflectivity of the solitary laser with respect to ω.

I start with Eq. (12) of the old paper:

$$-i\dot{f} = -v_g\beta' n f + iv_g\Phi_0^+(0)a(t). \qquad \text{(old 12)}$$

I corrected the sign of the β'-term and introduced v_g to have correct dimensions.

Now, I consider reflection of a harmonic wave. I set $\Psi^-(0,t) = a(t) = a_{\text{in}}e^{i\Omega t}$ for the input field and regard n as a constant. Then, the reflected amplitude will oscillate in the same manner, $\Psi^+(0,t) = f(t)\Phi_0^+(0) = a_{\text{out}}e^{i\Omega t}$. Inserting $\dot{f} = i\Omega f$ into (old 12), multiplying with $\Phi_0^+(0)$, dividing by $a_{\text{out}}e^{i\Omega t}$, yields

$$\Omega + v_g\beta' n = iv_g\Phi_0^+(0)^2\frac{a_{\text{in}}}{a_{\text{out}}} \text{ bzw. } \frac{a_{\text{out}}}{a_{\text{in}}} = \boxed{r(\Omega,n) = \frac{iv_g\Phi_0^+(0)^2}{\Omega + v_g\beta' n}} \qquad (1)$$

reflectivity of the DFB in this single-mode approximation.

$$\text{Consequence: } iv_g\Phi_0^+(0)^2 = \left(\frac{\partial q}{\partial\omega}\right)^{-1}, \qquad (2)$$

where $q = 1/r$ is the inverse of the DFB reflectivity in the given approximation.

Next I turn to delayed feedback by setting $a(t) = K\Psi^+(0, t-\tau) = K\Phi_0^+(0)f(t-\tau)$ in Eq. (old 12). Multiplying with i and using the equations derived before, one gets immediately

$$\dot{f} = -iv_g\beta' n f + \eta f(t-\tau) \quad \text{with} \quad \boxed{\eta = \frac{iK}{\partial_\omega q}} \qquad (3)$$

Up to the factor i, this is the LK coupling coefficient which we ady used in the coupled laser paper. Is the i correct or not?