Fast Estimate of the Locking Range of PhaseCOMB Self-Pulsations

ede, 08.01.03

I consider the locking of mode-beating pulsations to harmonic external signals of frequency f_{ext}. To numerically determine the locking range by stepping through f_{ext} is very time consuming. In case of the Adler model of locking (see my text from 17.11.02), this task can be solved much easier. In this case, the instantaneous frequency (= inverse temporal separation of two subsequent maxima) follows

$$\textbf{Adler model:} \quad f(t) - f_{\text{ext}} = \delta + p\sin(\varphi), \qquad \varphi(t) = \int_{t0}^{t} [f(t') - f_{\text{ext}}]\, dt' + \varphi_0. \tag{1}$$

In this case, the locking range is just $2p$, and the borders of the locking cone are $\delta \pm p$.

How to determine the parameters δ, p numerically? It is easy: Choose any f_{ext} outside of the locking range (but not too far of it), calculate f(t) over a sufficiently long time (that φ changes by more than 2π), than it holds

$$\delta = \frac{\max(f) + \min(f)}{2}, \quad p = \frac{\max(f) - \min(f)}{2}. \tag{2}$$

I checked this idea with a particular point of operation (without LSHB yet). The $f(t)$ is easily obtained from the half-height slopes by the rlab-lines

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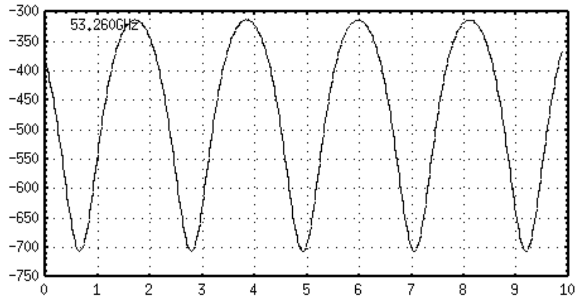
from file lock.r on sim-3r:~ede/ldsl_on_store/locking/030107/.

jittede = period(xoft[pulsnum1:pulsnum2]);
F=1e-9*jittede.frq.fr;                          // SPfrequenz in GHz
tt=[2:jittede.njit]; tt1=[1:jittede.njit-1];

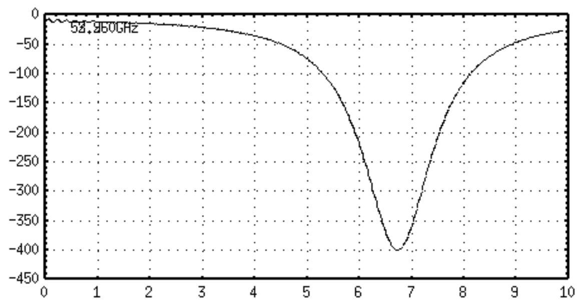
// t in ns, f-fext in MHz :
outdat=[tt1'/F,-(jittede.slope[tt;1]-jittede.slope[tt1;1])*(F*1e9*F*1e3)];
plot(outdat);
```

For simplicity, I used current modulation. Without modulation F = 52.76 GHz. I start with 500 MHz higher modulation. The resulting $f(t) - f_{\text{ext}}$ in MHz vs. t in ns is drawn righthand. Just by inspecting, I find roughly $fmax = -310$ and $fmin = -710$, i.e,. $\delta \approx -510$ and $p \approx 200$, thus locking range estimate 200, MHz each.

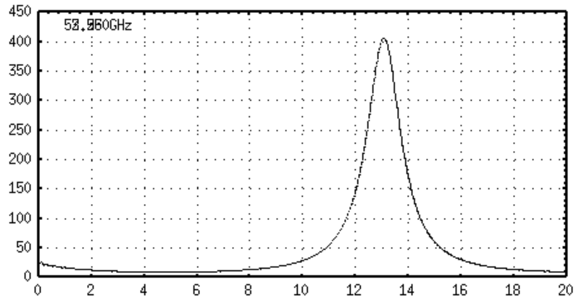


To come closer to the locking range, we change f_{ext} by nearly $fmax$ to 52.46 GHz.

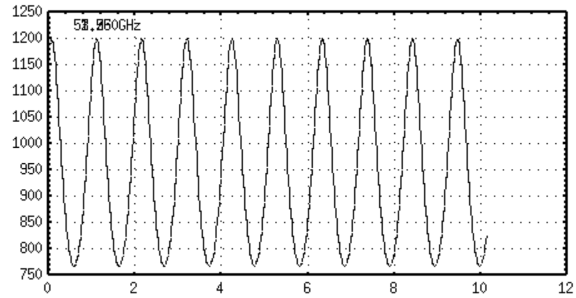
The outcome is drawn right. Indeed, we are very close to the border of the cone ! A more precise estimate of the parameters is possible now from the Fig. In particular, the opposite border of the locking cone is expected at -400 MHz.



Going to $f_{\text{ext}} = 52.07$ GHz, we indeed arrive close outside the opposite side of the cone, as drawn in the righthand Figure.



What happens if we go very far from the locking cone, say to $f_{\text{ext}} = 51$ GHz? The righthand Fig shows that this still gives resonable parameters, e.g. about 440 MHz for the locking range (modulation span of frequencies).



Conclusion

PhaseCOMB (without LSHB) locks according to the Adler model. This allows to determine the borders of the locking cone from *only one* transient with an arbitrary f_{ext} outside of the locking cone. In addition, the calculation time for the transient can be kept as short as e.g. 5 ns, because the external frequency can be taken sufficiently far away from the border of the cone.