Modeling of spontaneous emission noise

H.-J. Wünsche February 27, 2001

Dear Mindaugas,

here my first consideration concerning the spontaneous emission noise.

Continuous physics:

It is sufficient to consider one propagating wave without grating.

Using coordinates moving with the wave, the amplitude equation is

$$\partial_t f = i\Omega f + aF \quad \text{with } \langle F(t)\rangle = 0; \quad \langle F^*(t)F(t')\rangle = \delta(t-t'). \tag{1}$$

The coefficient a is to be determined such that the optical power $P(t) = \langle f^*(t)(t)\rangle$ obeys the intensity equation

$$\partial_t P = -2\Im m(\Omega)\, P + \hbar\omega w d v_g \Gamma_t\, \beta_{sp} \frac{n}{\tau_r} \tag{2}$$

with radiative life time τ_r. $\beta_{sp} \approx 10^{-5} \cdots 10^{-3}$ is the relative portion of spontaneous emission emitted into one transverse mode. It depends on the transverse mode properties and is an input parameter of our longitudinal model.

It is useful to determine the coefficient a not in the continuous model but for the discrete realization.

Descrete physics:

Normally one uses time steps, say of length δt.

The spontaneous emission is added at the end of time steps in the form

$$\delta f = a' z_i \qquad \text{with } z_i \text{ being a sequence of random complex numbers with the properties} \\ \langle z_i\rangle = 0, \quad \langle z_i z_j\rangle = \delta_{ij}. \tag{3}$$

How to choos a'? Such that the power added in one time step by spontaneous emission is according to (2)

$$\delta P_{sp} = |a'|^2 = \hbar\omega w d v_g \Gamma_t\, \beta_{sp} \frac{n}{\tau_r} \cdot \delta t. \tag{4}$$

The most simple realization in my opinion is

$$a' = \sqrt{\hbar\omega w d v_g \Gamma_t\, \beta_{sp} \frac{n}{\tau_r} \cdot \delta t}, \qquad z_i = e^{2\pi i \xi_i} \tag{5}$$

with ξ_i being random numbers equally distributed in the intervall (0,1) as from a random number generator.

These are ideas from my memory and understanding of physics. I still have to look into the literature, whether there are other ways.