Network Aspects of 2D Random Scatterer Lasers

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1 Introduction

A network is a set of objects where some pairs of the objects are connected by links (from Wiki). Seemingly, a scatterer laser (SL) as sketched in Fig.1 can be regarded as a network. The scatterers are the nodes and the light paths between scatterers are the links.

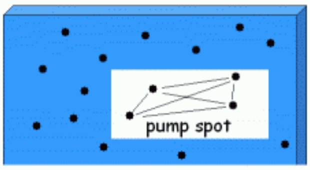

Figure 1: *Schematic top view on a planar semiconductor waveguide with scatterers (black dots). The white area illustrates a pump spot, where the waves traveling in the plane get amplified. Blue: unpumped, absorbing. Lasing starts, if the amplification between scatterers within a spot compensates the scattering losses.*

In what follows I consider the network aspects of SL in more depth. Two questions are interesting. i) Can network theory help to understand these lasers? ii) Are these lasing networks interesting for network theory? Because my knowledge of network science is rather poor, I can present amateurish considerations only. Nevertheless, I hope they can stimulate some discussion with network people in the framework of our training group.

2 The SL as a weighted network

My ansatz is to treat the SL as a weighted network. A weight w_{ij} is attributed to each link between nodes, which measures the importance of the link for the network. In case of a SL, it is natural to choose the net amplification along the link. Between two scatterers at positions $\vec{r}_i$ and $\vec{r}_j$:

$$w_{ij} = \Big| A \cdot G(k, |\vec{r}_i - \vec{r}_j|) \Big|, \quad w_{ii} = 0, \qquad\qquad k = \frac{2\pi\bar{n}}{\lambda_0} - ig/2. \tag{1}$$

A: backscattering amplitude, $G(k,r) = \exp(ikr + i\pi/4)/\sqrt{8\pi kr}$: far-field Green function of Helmholtz equation, $\bar{n}$: refractive index, λ_0: central wavelength, g: average gain along the link.

Our experimental samples exhibit weak scattering and the laser condition is well approximated by $\max w_{ij} = 1$ [2]. This condition is fulfilled by the longest link and the weights of shorter links are exponentially smaller. Thus, this choice of w_{ij} prefers those links which are most important for the lasing – it makes sense.

Above threshold, g becomes inhomogeneous and may depend on time. Accordingly, we have an evolving weighted network. It is useless to consider the evolution of all the individual weights. Appropriate summary quantifiers are required. Already only few papers provide numerous such quantifiers [3, 4, 5, 6]. Leaving the best choice for future, I restricted myself to following simple ones:

$$s_i = \sum_j w_{ij} w_{ji} \quad \text{and} \quad c_i = \frac{1}{s_i} \sum_{j,k} w_{ij} w_{jk} w_{ki}. \tag{2}$$

The first one is called strength of node i. In case of $w_{ij} = 1$ for all existing links and zero else, it is just the number of next neighbours. With weights (1), it is the summed magnitude of feedback at node i

from two-step feedback loops through all neighbours j (2-loops). The quantity c_i can be named relative cluster coefficient of node i. For $w_{ij} = 1$ or 0, this is how many triangles from i through through two other scatterers back to i (3-loops) exist in average per reachable neighbour. With weights (1), it is the summed magnitude of feedback from all 3-loops at node i, divided by s_i.

3 Application to the turn-on of an exemplary random SL

I consider the results of simulations of the turn-on of a particular SL described in [1], where also the description of the model equations and other examples are found. The present example is sketched in Fig.2. Every scatterer is linked to all 99 neighbours, yielding $\sim 10^4$ links. Much less links are seen in the graph of the weighted network of the stationary state (right). The strongest links go accross the center of the circle – they belong to the longest possible links in the circle. Nodes at the periphery are noticeably linked only with nodes on the opposite side of the circle. Therefore, the picture looks like a star.

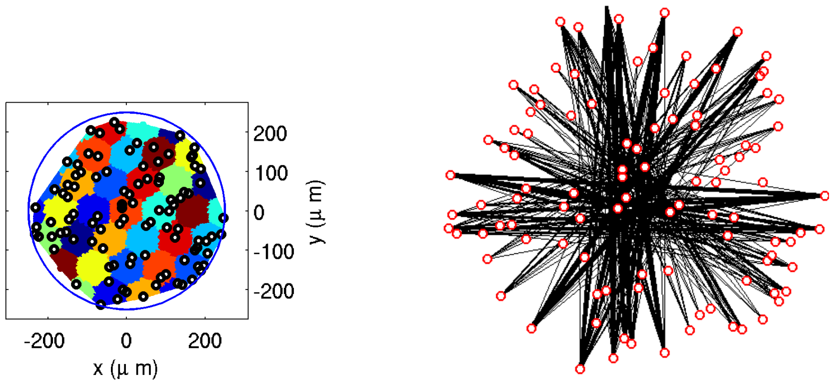

Figure 2: *Left: Sketch of the considered circular excitation spot containing 100 scatterers (circles). The colored regions represent the domains of spatially constant inversion (gain). Right: graph of the corresponding weighted network in stationary state. The width of the lines is proportional to the weight of the link according to Eq. (1). Weights below 0.1 do not appear.*

Now the dynamics of the weighted lasing network as obtained from the simulation results described in [1]. Panel a) of Fig.3 displays the temporal variation of gain g in the different domains, which together with the distances determines the weights (1). At $t \approx 40$ ps (instant b), all domains are above threshold but still have the same gain. This is the instant when the first lasing spike starts rising (not shown here, cf. Fig.4 of [1]). The network is dense, each node is strongly connected with other nodes. A bit later, at $t \approx 60$ ps, the first strong spike has depleted the gain and, accordingly, the network is practically unconnected because the weights are dropped below 0.1 (panel c). The network reconnects when the gain recovers until next spike depletes the gain again (not shown). Such RO-cycles repeat with decreasing amplitude (see the movie, address in caption) until steady state is practically reached (panel d).

The node strengths s_i in panels b to c reflect the laser physics quite well. Their maximum exceeds 1 in eras with gain above threshold, it falls below 1 in epoches of depletion, and it approaches a value ≈1 in steady state. The condition $\max s_i = 1$ is closely related to the laser condition. But it is a simplification, because i) it sums up the magnitudes of the 2-loop feedbacks and disregards phases and ii) it disregards also the contributions of multiple scattering to the feedback. Seemingly, the disregarded effects are less important in present case.

Multiple scattering being negigible, the contribution of 3-loops (lowest order of multiple) are also small.

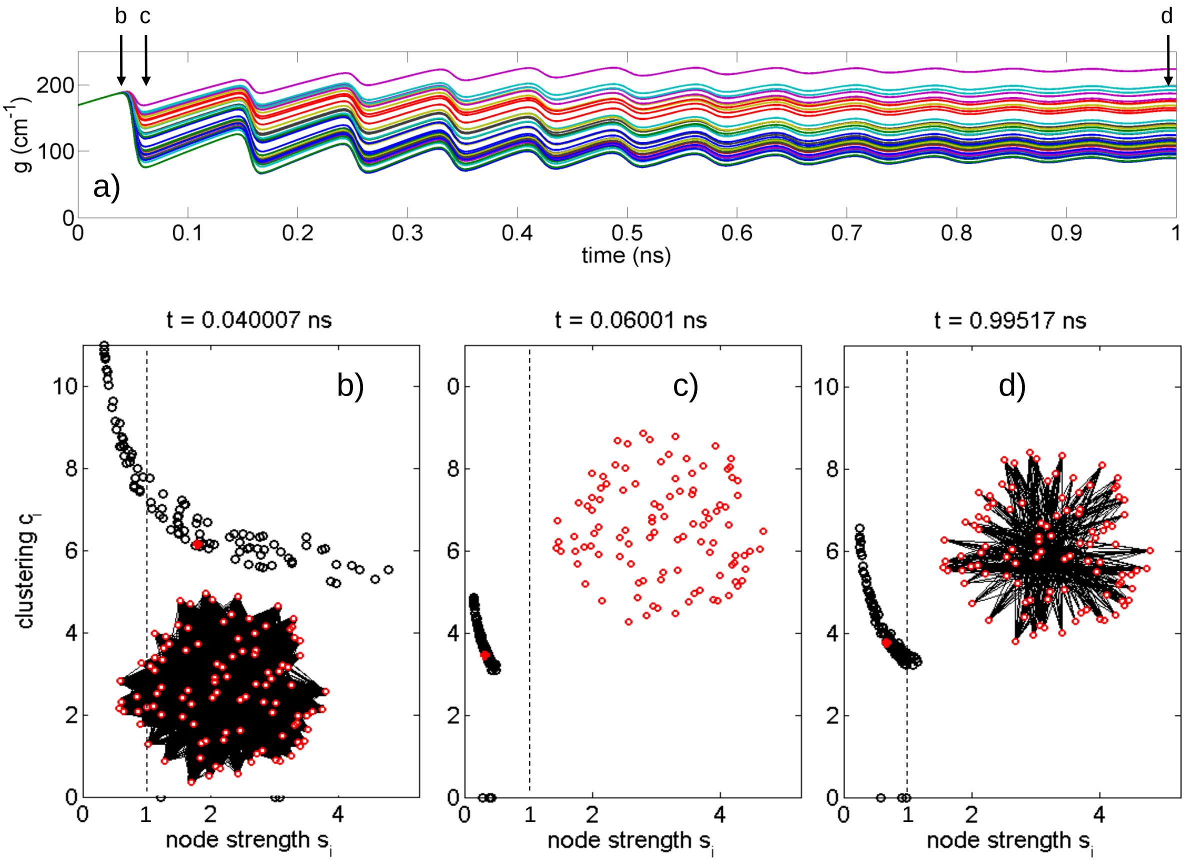

Figure 3: *Evolution of the weighted network during turn-on. a) Gain versus time. Different lines correspond to different spatial domains (cf. Fig.2, colors are different from there). b) to d): positions of the nodes in the plane of the two quantifiers node strength and correlation (cf. Eq.(2)) and graph of the weighted network. Both at the time instants labeled in panel a). b) Just before the first spike, c) just after this spike, and d) Close to stationary state. An animated gif of the dynamics is found on* http://people.physik.hu-berlin.de/~ede/sld/circle3movie30.gif

But the c_i are large due to the neglect of phases. They are no good quantifier for the physics of the lasing network.

4 Discussion and Conclusion

The described interpretation of the circularly shaped random SL as a weighted network is certainly a rather simplifying picture of the model [1]. It disregards optical phase shifts and treats the gain distribution as an external quantity, taken from numerical solutions of the full model. Nevertheless, it provides a descriptive picture how the optical connectedness of the scatterers evolves and which connections are the most important ones.

Of the two considered quantifiers node strength s_i and clustering c_i, only the first one reflects a part of the physics. Main reason for the failing of the second one is ignoring the optical phase in the choosen weights (1). Is it possible to use complex weights? Are there examples in the literature?

Which role can other quantifiers play (keeping positive weights)? As an example: shortest path length, which is a central quantitiy for the small-world aspect. I did not calculate it because another quantity seems to be more relevant for a laser: the largest round-trip gain of all possible closed pathways. Defining

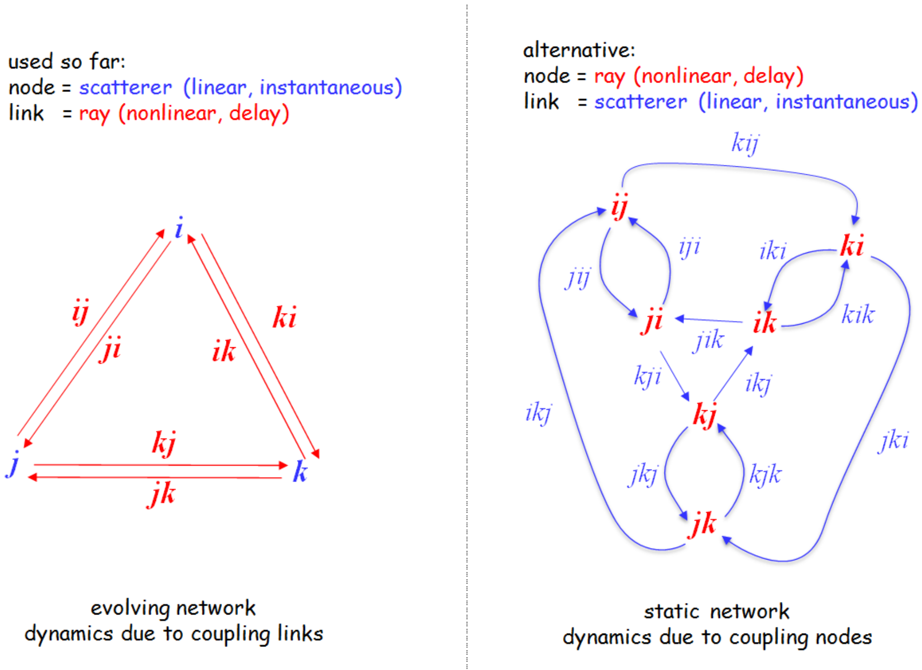

Figure 4: *Different types of networks attributed to one and the same 3-scatterer laser. i labels a scatterer, ij lables the directed optical pathway from j to i, and ijk labels the amplitude of scattering at j towards i of a wave coming from k. The rays (optical pathways) are colored red, the scatterers are colored blue.*

gain times distance as the length of a link, this corresponds to finding the longest possible closed way without repetitions. When trying simply to replace 'shortest' by 'longest' in the simple algorithm for the shortest way, I have learned it keeps by far not simple. No elegant solution seems to exist. Do the profis have an algorithm for that?

Regarding my numerical solution of the model equations as an algorithm, which in some sense (with phases ...) solves this problem – can it compete? Or can it lead to an algorithm for solving the longest path problem?

The presented network model of an SL has linear nodes (scatterer). The nonlinearity is in the links (optical pathways), more precisely in the amplification along the links, which varies due to inversion dynamics. Furthermore, self-sustaining oscillations (lasing) emerge only through the cooperation of the nonlinear elements, which as individuals do not oscillate. Therefore we call these SL 'lasing networks', which must be distinguished from 'laser networks', where individual self-sustaining oscillators are connected by passive links.

Attributing a network to a SL is not unique. A counterintuitive but interesting possible alternative network is sketched in the right part of Fig.4. Now the optical pathways are interpreted as nodes of the network and the scattering from one pathway into another one is interpreted as the link. In this case, the links are passive, static, and linear elements. All the dynamics and nonlinearity is contained in the nodes, as in conventional laser networks. So we loose the picture of an evolving network structure. However, the nodes are not self-oscillating and the lasing remains a collective phenomenon of the whole network.

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