Notes on Two-Wave Competition by Four-Wave Mixing

H.-J. Wünsche
Humboldt Universität zu Berlin, Institut für Physik
Newtonstr.15 12489 Berlin, Germany

23.08.2005

1 The Problem

I reconsider the following question. Two primary waves with different frequencies but comparable intensity are injected into a nonlinear waveguide. They create secondary waves by four-wave mixing (FWM). How does their intensity ratio evolve?

Standard FWM theory does not consider this question because it is usually assumed that one of the primary waves, the pump wave, is much more intense than the other one, the probe wave.

For the case of a saturated semiconductor optical amplifier (SOA), we have answered this question as follows in Ref. [1]. The intensity ratio increases in favour of that wave injected with higher intensity. After sufficiently long propagation, that wave with lower intensity dies exponentially out.

After having completed the proof of our paper [1], I have detected Ref. [2], which considers the same problem for a nonlinear fiber. The answer of these authors is: the intensity ratio evolves towards unity, i.e., the intensities of the two primary waves become equalized.

Obviously, the two papers are controversial. But both they present experimental evidence for their statements. Who is right?

2 The Solution

Yesterday on my travel home I found the solution: both papers are right. They (Ref. [2]) consider FWM by an index grating, whereas we (Ref. [1]) have a gain grating. In the following I give some detailed arguments.

I start with their basic equations

$$\frac{\partial A_1}{\partial z} = i\gamma_1\Bigg[\Big(|A_1|^2 + 2\sum_{j(\neq 1)}|A_j|^2\Big)A_1 + 2A_1^*A_2A_3\exp(i\Delta\beta_1 z) + A_4^*A_2^2\exp(-i\Delta\beta_2 z) + 2A_2^*A_3A_4\exp(i\Delta\beta_3 z)\Bigg], \tag{9a}$$

$$\frac{\partial A_2}{\partial z} = i\gamma_2\Bigg[\Big(|A_2|^2 + 2\sum_{j(\neq 2)}|A_j|^2\Big)A_2 + 2A_2^*A_1A_4\exp(i\Delta\beta_2 z) + A_3^*A_1^2\exp(-i\Delta\beta_1 z) + 2A_1^*A_3A_4\exp(i\Delta\beta_3 z)\Bigg], \tag{9b}$$

$$\frac{\partial A_3}{\partial z} = i\gamma_3\Bigg[\Big(|A_3|^2 + 2\sum_{j(\neq 3)}|A_j|^2\Big)A_3 + A_2^*A_1^2\exp(-i\Delta\beta_1 z) + 2A_4^*A_1A_2\exp(-i\Delta\beta_3 z)\Bigg], \tag{9c}$$

$$\frac{\partial A_4}{\partial z} = i\gamma_4\Bigg[\Big(|A_4|^2 + 2\sum_{j(\neq 4)}|A_j|^2\Big)A_4 + A_1^*A_2^2\exp(-i\Delta\beta_2 z) + 2A_3^*A_1A_2\exp(-i\Delta\beta_3 z)\Bigg], \tag{9d}$$

for the amplitudes A_k of the four waves. The indices $k = 1, 2$ corresponds to the primary waves, here. I do not need the equations in this general shape but restrict to $\Delta\beta_k = 0$ and asume negligible secondary amplitudes, i.e. $A_3, A_4 \to 0$ on the right-hand sides, which has been done in our paper, too. Furthermore, as lateron in Ref. [2], I set

$$\gamma_k = \gamma \tag{1}$$

independent of k given by the nonlinear coefficient $\chi^{(3)}$ at a certain central frequency. For the ratio of the primary amplitudes, this yields

$$\frac{\partial}{\partial z}\ln\left(\frac{A_1}{A_2}\right) = i\gamma(S_2 - S_1) \tag{2}$$

where $S_k = |A_k|^2$ is the intensity. Adding the complex conjugate gives

$$\frac{\partial}{\partial z}\ln(\frac{S_1}{S_2}) = 2\text{Im}(\gamma)(S_1 - S_2). \tag{3}$$

This equals Equation (11) of our paper when replacing index 2 by 0 and identifying $2\text{Im}(\gamma)$ with εg:

$$\frac{\partial}{\partial z}\ln\left(\frac{S_1(z,t')}{S_0(z,t')}\right) = \varepsilon g(\bar{n})\left[S_1(z,t') - S_0((z,t')\right]. \tag{11}$$

Thus, in a nonlinear fiber with $\text{Im}(\gamma) \neq 0$, the dominant effect is just what we have obtained for the saturated SOA. However, if $\text{Im}(\gamma) = 0$ as in the photonic crystal fibers considered in Ref. [2], this effect disappears and the terms of higher order in the secondary amplitudes yield what has been described there.

3 Conclusion

FWM in nonlinear waveguides causes energy transfer between the primary waves. The direction of the transfer depends on the nature of the induced grating. If the induced grating is purely index type as in fibers, the energy is transfered from the stronger wave to the weaker wave, equalizing the two amplitudes. If a gain grating is induced as in SOAs, then the energy transfer is directed reversely from the weaker wave to the stronger wave. If the grating is complex, the gain effect is expected to dominate, because the index effect is of higher order in the amplitudes of the created FWM products.

References

[1] Gero Bramann, Hans-Jürgen Wünsche, Ulrike Busolt, Christian Schmidt, Michael Schlak, Bernd Sartorius, and Hans-Peter Nolting, "Two-Wave Competition in Ultra Long Semiconductor Optical Amplifiers", to appear in IEEE Journal Quantum Electronics, October 2005

[2] Xueming Liu, Xiaoqun Zhou, and Chao Lu, "Multiple four-wave mixing self-stability in optical fibers", Phys. Rev. A **72**, 013811, July 2005.

Appendix

The above argumentation neglects contributions from carrier population pulsations (CPP). Using Eqs. (7) to (11) of [1], they can easily be included. Generally, ε is to be replaced by ϵ_{mn} given in Eq. (9) of our paper. These quantities play the role of the γ_k in Ref. [2]. In the limit of small detuning $\Delta\omega$, one gets

$$\epsilon_{mn} = \varepsilon + (1 + i\alpha_H)v_g g'\tau_c. \tag{4}$$

Here, the second contribution dominates and, by the α-factor, it is essentially imaginary. Thus, in this limit the SOA can be expected to behave like a fiber with pure index grating. This may explain why in some experiments equalization may be observed.

In the opposite limit of large detuning, as in our SOAs, one arrives at

$$\frac{\partial}{\partial z}\ln(\frac{S_1}{S_0}) = g\Big(\text{Re}(\epsilon_{10})S_1 - \text{Re}(\epsilon_{01})S_0\Big), \tag{5}$$

where with the parameters of Ref. [1],

$$\begin{aligned}\text{Re}(\epsilon_{01,10}) &= \varepsilon \pm \frac{g'\alpha_H\lambda_0^2}{2\pi n_g\Delta\lambda} \qquad (6)\\ &\approx (8 \pm \frac{30\text{nm}}{\Delta\lambda}) \times 10^{-18}\text{cm}^{-3}.\end{aligned}$$

with $\Delta\lambda$ being the wavelength difference between both primary waves. Here the α-factor contributes to the gain grating. Moreover, this formula gives an explizit expression for the dependence of the asymmetry of the effect on $\Delta\lambda$. In particular, the effect can take place only as long both real parts are greater than zero. This is the case for $\Delta\lambda > 30\text{nm}/8 \approx 4\text{nm}$ in some agreement with recent measurements.