Theo 1

galilei Trafo

$x'_i = R_{ij}(\vec{x}_j) + u_j t + a_j$

$t' = t + \tau$ | neutrales Elem., inverses, geschlossenheit, assoziativität

$\hat{e}'_j = R_{ij}\,\hat{e}_i$

Lagrange 1. Art

$g_i(\vec{x}_j) = 0$

$m \cdot \ddot{\vec{x}} = \vec{F} + \sum \lambda_i \cdot \vec{\nabla} g_i$

2maliges diff. v. g

d'Alembertsches Prinzip

$0 = \sum_i \vec{z}_i \cdot \delta \vec{x}_i$

$\sum m_i \ddot{\vec{x}}_i \delta \vec{x}_i = \sum \vec{F}_i \cdot \delta \vec{x}_i$

virtuelle Arbeit $\vec{F} \cdot \delta \vec{x}$ | $\delta \vec{x} = \frac{\partial \vec{x}}{\partial q_1} \delta q_1 + \dots$

Lagrange 2. Art | $\mathcal{L} = T - V$

$\frac{d}{dt}\frac{\partial T}{\partial \dot{q}_\alpha} - \frac{\partial T}{\partial q_\alpha} = \sum \vec{F}_i \frac{\partial \vec{x}_i}{\partial q_\alpha}$

Kraft aus V & $\frac{\partial V}{\partial \dot{q}_\alpha} \equiv 0$

$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_\alpha} - \frac{\partial \mathcal{L}}{\partial q_\alpha} = 0$

Hamilton: $H = T + V$

$H = \sum p_i \dot{q}_i - \mathcal{L}$

$p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$ | $\frac{\partial H}{\partial t} = -\frac{\partial \mathcal{L}}{\partial t}$

$\frac{\partial H}{\partial p_\alpha} = \dot{q}_\alpha$ | $\frac{\partial H}{\partial q_\alpha} = -\dot{p}_\alpha$

H-Jacobi: $\tilde{H} = H + \frac{\partial S}{\partial t} = 0$

$H(q, \frac{\partial S}{\partial q}, t) + \frac{\partial S}{\partial t} = 0$

$S(q,E,t) = W(E,q) - E \cdot t$

$p = \frac{\partial S}{\partial q}$, $q = \frac{\partial S}{\partial p}$

Arbeit

Arbeit $W = \int \vec{F}\, d\vec{x}$

Leistung: $P = \frac{dW}{dt} = \vec{F} \frac{d\vec{x}}{dt}$

Drehimpuls: $\vec{L} = \vec{x} \times \vec{p}$

$\vec{M} = \vec{r} \times \vec{F}$ Drehmoment

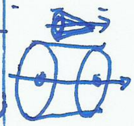
$\mathcal{L}$ / H

	$\mathcal{L}$	H
Kart.	$\frac{m}{2}(\dot{x}^2 + \dot{y}^2 + \dot{z}^2) - V(x,y,z)$	$\frac{1}{2m}(p_x^2 + p_y^2 + p_z^2) + V$
Zylinder	$\frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2) - V(r,\varphi,z)$	$\frac{1}{2m}(p_r^2 + \frac{1}{r^2}p_\varphi^2 + p_z^2) + V$
Kugel	$\frac{m}{2}(\dot{r}^2 + r^2[\dot{\theta}^2 + \sin^2\theta \cdot \dot{\varphi}^2]) - V(r,\theta,\varphi)$	$\frac{1}{2m}(p_r^2 + \frac{1}{r^2 \sin^2\theta}p_\varphi^2 + \frac{1}{r^2}p_\theta^2) + V$

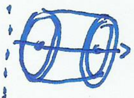
Trägheitsmomente

$J = \int r_\perp^2\, dm = \rho \int r_\perp^2\, dV$

$m r^2$

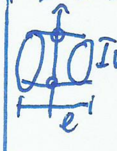


$\frac{m}{2} r^2$



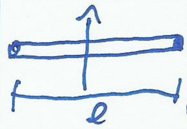
$d \ll r$

$m \cdot r^2$

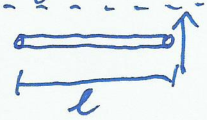


$\frac{m}{4} r^2 + \frac{m}{12} l^2$

$\frac{m}{2} r^2 + \frac{m}{12} l^2$



$\frac{m}{12} l^2$



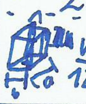
$\frac{m}{3} l^2$

$\frac{2m}{5} r^2$

$\frac{2m}{3} r^2$

$m \frac{r_1^2 + r_2^2}{2}$

$\frac{2m}{5} \frac{r_a^5 - r_i^5}{r_a^3 - r_i^3}$



$\frac{m}{12}(a^2 + b^2)$

$\frac{3}{10} m \cdot r^2$

harm. Oszi.

$m\ddot{x} = -kx - \gamma\omega\dot{x}$, $\gamma > 0$

$\mu^2 = \frac{\gamma^2}{4} - \omega_0^2 \begin{cases} = 0, & \text{A-period.} \\ > 0, & \text{Kriech} \\ < 0, & \text{Schwing} \end{cases}$

$x(t) = e^{-\frac{\gamma}{2}t}[a \cdot \cosh(\mu t) + b \cdot \sinh(\mu t)]$

Erzeugende Funktion

$\sum_k p_k \cdot \dot{q}_k - H(q,p,t) = \sum P_k \dot{Q}_k - K(Q,P,t) - \frac{d}{dt} F_i$

$F_1(q,Q) = F$	$p_i = \frac{\partial F_1}{\partial q_i}$	$P_i = -\frac{\partial F_1}{\partial Q_i}$
$F_2(q,P) = F + \sum Q_i P_i$	$p_i = \frac{\partial F_2}{\partial q_i}$	$Q_i = \frac{\partial F_2}{\partial P_i}$
$F_3(p,Q) = F - \sum q_i p_i$	$q_i = -\frac{\partial F_3}{\partial p_i}$	$P_i = -\frac{\partial F_3}{\partial Q_i}$
$F_4(p,P) = F + \sum (Q_i P_i - q_i p_i)$	$q_i = -\frac{\partial F_4}{\partial p_i}$	$Q_i = \frac{\partial F_4}{\partial P_i}$

Noether Theorem

$q_\alpha \to q'_\alpha = q_\alpha + \varepsilon \cdot \eta$

$\Rightarrow$ const. $= \sum \frac{\partial \mathcal{L}}{\partial \dot{q}_i} \eta$

$\varepsilon \cdot \vec{a} \Rightarrow \vec{P}$

$\varepsilon(\vec{n} \times \vec{x}) \Rightarrow \vec{L}$

Potentiale

$V(\vec{x}) = -\int \vec{F}(\vec{x})\, d\vec{x}$

$V(r) = -\gamma \frac{m_1 m_2}{r}$

$V(h) = -m \cdot g \cdot h$

$V(x) = \frac{k}{2} x^2$ | $\omega^2 = \frac{k}{m}$

$V(x) = \frac{m\omega^2}{2} x^2$

$\vec{\nabla} V(r) = V(r)\, \hat{e}_r$

Poisson Klammer

$\{f,g\}_{q,p} = \frac{\partial f}{\partial q}\frac{\partial g}{\partial p} - \frac{\partial g}{\partial q}\frac{\partial f}{\partial p}$

$\frac{df}{dt} = \{f,H\} + \frac{\partial f}{\partial t}$

$\{f,g\} = -\{g,f\} \Rightarrow \{f,f\} \equiv 0$

$\{c_1 f_1 + c_2 f_2, g\} = c_1\{f_1,g\} + c_2\{f_2,g\}$

$\{f, g \cdot h\} = \{f,g\}h + g\{f,h\}$

$\{f,\{g,h\}\} + \{h,\{f,g\}\} + \{g,\{h,f\}\} = 0$

$\{q_k, p_l\} = \delta_{kl}$

sin / cos

$\sin(\arctan(x)) = \frac{x}{\sqrt{1+x^2}}$ | $\sin(2x) = 2 \cdot \sin(x) \cdot \cos(x)$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\cos(\arctan(x)) = \frac{1}{\sqrt{1+x^2}}$ | $\sin(a+b) = \sin(a) \cdot \cos(b) + \cos(a) \cdot \sin(b)$

$\cosh^2(x) - \sinh^2(x) = 1$

$\cos(a+b) = \cos(a) \cdot \cos(b) - \sin(a) \cdot \sin(b)$

Euler Gleichungen:

$M_1 = I_1 \cdot \dot{\omega}_1 + (I_3 - I_2)\omega_2\omega_3$

$M_2 = I_2 \cdot \dot{\omega}_2 + (I_1 - I_3)\omega_3\omega_1$

$M_3 = I_3 \cdot \dot{\omega}_3 + (I_2 - I_1)\omega_1\omega_2$

$\omega = \frac{\dot{x}}{R}$; $\dot{\omega} = \frac{\ddot{x}}{R}$

ε

$\varepsilon_{ijk} \cdot \varepsilon^{imn} = \delta_j^m \cdot \delta_k^n - \delta_j^n \cdot \delta_k^m$

$\varepsilon_{jmn} \cdot \varepsilon^{imn} = 2 \cdot \delta_j^i$

$\varepsilon_{ijk} \cdot \varepsilon^{ijk} = 6$