Theo II EDynamik

$\vec{D} = \varepsilon_0 \varepsilon_r \vec{E}$

$\vec{H} = \frac{1}{\mu_0 \mu_r} \vec{B}$

Vektorrelationen:

$\vec{\nabla} \times (\vec{\nabla}\varphi) = 0$

$\vec{\nabla} \cdot (\vec{\nabla} \times \vec{A}) = 0$

$\vec{\nabla} \cdot \vec{A} = \oiint_V \vec{A}\, d\vec{a}$

$\vec{\nabla} \times \vec{A} = \oint_A \vec{A}\, d\vec{r}$

$\vec{\nabla} \times (\vec{\nabla} \times \vec{A}) = -\vec{\nabla} \cdot (\vec{\nabla}\vec{A}) + \vec{\nabla}(\vec{\nabla} \cdot \vec{A}) = -\Delta \vec{A} + \vec{\nabla}(\vec{\nabla} \cdot \vec{A})$

$C = \frac{1}{\sqrt{\varepsilon_0 \varepsilon_r \mu_0 \mu_r}}$

$C_0 = \frac{1}{\sqrt{\varepsilon_0 \mu_0}}$

$V_{ph} = \lambda \cdot \nu = \frac{\omega}{|k|}$

Formeln: Lorentzkraft: $\vec{F} = q \cdot (\vec{E} + \vec{v} \times \vec{B})$

$I = \frac{dq}{dt}$; $\vec{j} = \rho \cdot \vec{v}$

$\vec{f} = \rho \cdot \vec{E} + \vec{j} \times \vec{B}$

Kontinuitätsgleichung: $\frac{\partial \rho}{\partial t} + \vec{\nabla} \cdot \vec{j} = 0$

Spannung: $U = \Delta \Phi$

$U = -\int \vec{E}(\vec{r})\, d\vec{r}$

Coulombgesetz: $\vec{F} = \frac{1}{4\pi\varepsilon_0} \frac{q_1 q_2}{r^2} \hat{e}_r$

Poisson: $\Delta \phi = -\frac{\rho}{\varepsilon_0}$

Laplace: $\Delta \phi = 0$

ebene Welle: $\vec{E} = \vec{E}_0 \cos(\omega t - \vec{k} \cdot \vec{r})$

Kapazität: $C = \frac{Q}{U}$

$U_{ind} = -\frac{d\phi}{dt}$

$\omega = \omega_g = c \cdot \sqrt{k^2 + \delta^2}$

$\vec{B} = \frac{1}{c}(\hat{e}_k \times \vec{E})$

$\Delta \vec{A} = -\mu_0 \vec{j}$

$v_E = |\langle S \rangle| / \langle w \rangle$

$E = \frac{U}{d} = \frac{\sigma}{\varepsilon_0}$; $C = \varepsilon_0 \frac{A}{d}$

$(\vec{E}_2 - \vec{E}_1) \cdot \vec{n} = \frac{\sigma}{\varepsilon_0}$; $\vec{n} \times (\vec{E}_2 - \vec{E}_1) = 0$

$(\vec{B}_2 - \vec{B}_1) \cdot \vec{n} = 0$; $\vec{n} \times (\vec{B}_2 - \vec{B}_1) = \mu_0 \vec{j}$

$\Rightarrow E_{tan} = B_{norm} = 0$

Maxwell-Gleichungen

Gauß: $\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\varepsilon_0} \Rightarrow \oiint \vec{E} \cdot d\vec{A} = \frac{Q}{\varepsilon_0}$

$\vec{\nabla} \cdot \vec{B} = 0 \Rightarrow \oiint \vec{B} \cdot d\vec{A} = 0$

Faraday: $\vec{\nabla} \times \vec{E} = -\partial_t \vec{B} \Rightarrow \oint \vec{E}\, d\vec{r} = -\frac{d}{dt} \iint \vec{B}\, d\vec{A}$

Ampère: $\vec{\nabla} \times \vec{B} = \mu_0 \vec{j} + \varepsilon_0 \mu_0 \partial_t \vec{E} \Rightarrow \oint \vec{B}\, d\vec{r} = \mu_0 I + \varepsilon_0 \mu_0 \frac{d}{dt} \iint \vec{E}\, d\vec{A}$

Relationen ($\omega = 2\pi \cdot \nu$): $c = \frac{\omega}{|k|}$; $\lambda = \frac{c}{\nu}$; $V_{gr} = \frac{\partial \omega(k)}{\partial k}$

$T = \frac{1}{\nu} = \frac{2\pi}{\omega}$

$\vec{B} = \vec{\nabla} \times \vec{A}$

$\vec{E} = -\vec{\nabla}\phi - \partial_t \vec{A}$

$\vec{A}' = \vec{A} + \vec{\nabla}\phi'$

Kugelkoordinaten

$dV = r^2 \sin\vartheta\, dr\, d\varphi\, d\vartheta$

$dA = r^2 \sin\vartheta\, d\varphi\, d\vartheta$

Allgemeine:

$d\vec{A} = (\vec{r}_u \times \vec{r}_v)\, du\, dv = \vec{n}_A\, dA = \left|\frac{\partial \vec{r}}{\partial u} \times \frac{\partial \vec{r}}{\partial v}\right| du\, dv$

Delta-Fkt.

x_i: Nullstellen g

$\delta(g(x)) = \sum_{i=1} \frac{\delta(x - x_i)}{|g'(x_i)|}$

$\delta(ax) = \frac{1}{|a|} \delta(x)$

$\int_{-\infty}^{\infty} \delta(x-a)\, dx = 1$

$\int_{-\infty}^{\infty} f(x)\, \delta(x-a)\, dx = f(a)$

$\int_{-\infty}^{\infty} f(x)\, \delta^{(n)}(x)\, dx = (-1)^n f^{(n)}(0)$

Eichfreiheit

Coulombeichung: $\vec{\nabla} \cdot \vec{A} = 0$

Lorentzeichung: $\vec{\nabla} \cdot \vec{A} + \frac{1}{c^2} \partial_t \phi = 0$

Energie

$W = \frac{1}{2} C \cdot U^2$

$\vec{S} = \vec{E} \times \vec{H} = \frac{1}{\mu_0} \vec{E} \times \vec{B} = \hat{e}_k (c \varepsilon_0 |\vec{E}|^2)$

$\frac{dW}{dV} = w = \frac{1}{2}\left(\varepsilon_0 |\vec{E}|^2 + \frac{1}{\mu_0} |\vec{B}|^2\right) = \varepsilon_0 |\vec{E}|^2$

$\left(V_E = \frac{|\vec{S}|}{w}\right)$; Impulsdichte: $\vec{g}_{em} = \frac{1}{c^2} \vec{S}$

Bestimmungsgleichungen

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \iiint_V \frac{\rho(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{\vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|} dV'$

Biot-Savart:

$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \iiint_V \frac{(\vec{j}\, d\vec{V}) \times \hat{e}_{r'}}{|\vec{r}'|^2} = \frac{\mu_0}{4\pi} \int_C \frac{I\, d\vec{l} \times \vec{r}'}{|\vec{r}'|^3}$

$= \frac{\mu_0}{4\pi} \iiint_V d^3 r' \frac{(\vec{r}' - \vec{r}) \times \vec{j}(\vec{r}')}{|\vec{r} - \vec{r}'|^3}$

Kart. Multipolmomente

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\left(\frac{Q^{(0)}}{r} + \frac{\vec{Q}^{(1)} \cdot \vec{r}}{r^3} + \frac{1}{2} \sum_{i,j=1}^{3} Q_{ij}^{(2)} \frac{x_i x_j}{r^5} + \ldots\right)$

$Q^{(0)} = \iiint_V \rho(\vec{r}')\, dV'$

$\vec{Q}_i^{(1)} = \iiint_V \rho(\vec{r}')\, x_i'\, dV'$

$Q_{ij}^{(2)} = \iiint_V \rho(\vec{r}')\left(3 x_i' x_j' - (r')^2 \delta_{ij}\right) dV'$

Liénardt-Wiechert-Potentiale

$\phi(\vec{r}, t) = \frac{1}{4\pi\varepsilon_0} \frac{q}{|\vec{r} - \vec{R}_0(t')| - \frac{\vec{v}(t')}{c} \cdot (\vec{r} - \vec{R}_0(t'))}$

$\vec{A}(\vec{r}, t) = \frac{\mu_0}{4\pi} \frac{q \cdot \vec{v}(t')}{|\vec{r} - \vec{R}_0(t')| - \frac{\vec{v}(t')}{c} \cdot (\vec{r} - \vec{R}_0(t'))}$

$t' = t - \frac{|\vec{r} - \vec{R}_0(t')|}{c}$; $\vec{v}(t') = \frac{d}{dt'} \vec{R}_0(t')$

magn. Dipol

$\vec{m} = \frac{I}{2} \oint \vec{r} \times d\vec{r} = \frac{1}{2} \iiint d^3 r (\vec{r} \times \vec{j}(\vec{r}))$

$\vec{A}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{\vec{m} \times \vec{r}}{r^3}$

$\vec{B}(\vec{r}) = \frac{\mu_0}{4\pi} \frac{3\vec{r}(\vec{m} \cdot \vec{r}) - \vec{m} \cdot r^2}{r^5}$

Drehmoment: $\vec{M} = \vec{m} \times \vec{B}$

$E_{pot} = -\vec{m} \cdot \vec{B}$

$F = -\vec{\nabla}(-\vec{m} \cdot \vec{B})$

Kovariante-Maxwell-Formulierung

kontra: $a^\mu = (a_0, \vec{a})$

ko: $a_\mu = (a_0, -\vec{a})$

$a_\mu = g_{\mu\nu} a^\nu$; $g_{\mu\nu} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}$

$g^{\mu\nu} = (g^{-1})_{\mu\nu} = g_{\mu\nu}$; $\gamma = \frac{1}{\sqrt{1 - \beta^2}}$; $\beta = \frac{v}{c}$

$a_0' = \gamma(a^0 - \beta a^1)$

$a_1' = \gamma(a^1 - \beta a^0)$

$a'^2 = a^2$; $a'^3 = a^3$

$\partial^\mu = \left(\frac{1}{c}\partial_t, -\vec{\nabla}\right)$

$\partial_\mu = \left(\frac{1}{c}\partial_t, \vec{\nabla}\right)$

$\partial_\mu \partial^\mu = \square = \Delta - \frac{1}{c^2}\partial_t^2$

$L^\mu{}_\nu = \begin{pmatrix} \gamma & -\beta\gamma & 0 & 0 \\ -\beta\gamma & \gamma & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$

$a'^\mu = L^\mu{}_\nu a^\nu$; $L_\nu{}^\mu = L^{\mu\delta} g_{\delta\nu}$

$F'^{\mu\nu} = L^\mu{}_\rho L^\nu{}_\sigma F^{\rho\sigma}$

Kontinuitätsgleichung: $\partial_\mu j^\mu = 0$; $j^\mu = \begin{pmatrix} c\rho \\ \vec{j} \end{pmatrix}$

$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu = \begin{pmatrix} 0 & -E_x & -E_y & -E_z \\ E_x & 0 & -cB_z & cB_y \\ E_y & cB_z & 0 & -cB_x \\ E_z & -cB_y & cB_x & 0 \end{pmatrix}$

Lorenz-Eichung: $\partial_\mu A^\mu = 0$

$A^\mu = (\varphi, c \cdot \vec{A})$

Wellengleichung: $\partial_\mu \partial^\mu A^\nu = \frac{1}{\varepsilon_0} \frac{1}{c} j^\nu$

$F^{\mu\nu} = -F^{\nu\mu}$

Maxwellgleichungen:

$\partial_\mu \tilde{F}^{\mu\nu} = 0$

$\partial_\mu F^{\mu\nu} = \frac{1}{\varepsilon_0} \frac{1}{c} j^\nu$

$\tilde{F}^{\mu\nu} = \frac{1}{2} \varepsilon^{\mu\nu\sigma\gamma} F_{\sigma\gamma} = \begin{pmatrix} 0 & -cB_x & -cB_y & -cB_z \\ cB_x & 0 & E_z & -E_y \\ cB_y & -E_z & 0 & E_x \\ cB_z & E_y & -E_x & 0 \end{pmatrix}$

Lagrange

$\mathcal{L} = \text{const.} \cdot F_{\mu\nu} F^{\mu\nu}$; $L = \int d^3x\, \mathcal{L}(\phi_k, \partial^\nu \phi_k)$

$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) = \frac{\partial L}{\partial q_i} \rightarrow \partial^\beta\left(\frac{\partial \mathcal{L}}{\partial(\partial^\beta \phi_k)}\right) = \frac{\partial \mathcal{L}}{\partial \phi_k}$

Euler-Lagrange-Gleichung

EM in Materie

$\vec{\nabla} \times \langle \vec{E} \rangle = -\partial_t \langle \vec{B} \rangle$

$\vec{\nabla} \times \vec{H} = \vec{j}_{ext} + \vec{j}_{free} + \partial_t \vec{D}$

$\vec{\nabla} \cdot \langle \vec{B} \rangle = 0$

$\vec{\nabla} \cdot \vec{D} = \rho_{ext} + \rho_{free}$

$\vec{D}(\vec{E}) = \vec{P}_0 + \varepsilon_0 \underbrace{(1 + \chi)}_{\varepsilon_r} \vec{E}$

$\vec{D} = \varepsilon_0 \langle \vec{E} \rangle + \vec{P}$

$\vec{H} = \frac{1}{\mu_0} \langle \vec{B} \rangle + \vec{M}$

$\vec{P}(\vec{E}) = \vec{P}_0 + \varepsilon_0 \chi^{(1)} \cdot \vec{E} + \varepsilon_0 \hat{e}_i \cdot \chi_{ijk}^{(2)} \cdot E_j E_k + \ldots$

Supraleiter

$\frac{d}{dt}(\Lambda \cdot \vec{j}) = \vec{E}$; $\vec{\nabla} \times (\Lambda \cdot \vec{j}) = -\vec{B}$

$\Lambda = \frac{m}{n \cdot e^2} = \frac{2m}{\frac{n}{2}(2e)^2} \leftarrow$ Cooperpaare

Zylinderkoordinaten $\vec{\nabla}$

$\vec{\nabla} f = \left(\partial_r f, \frac{1}{r}\partial_\varphi f, \partial_z f\right)$

$\Delta f = \frac{1}{r}\partial_r(r \partial_r f) + \frac{1}{r^2}\partial_\varphi^2 f + \partial_z^2 f$

$\vec{\nabla} \cdot \vec{f} = \frac{1}{r}\partial_r(r \cdot f_r) + \frac{1}{r}\partial_\varphi f_\varphi + \partial_z f_z$

$\vec{\nabla} \times \vec{f} = \left[\frac{1}{r}\partial_\varphi f_z - \partial_z f_\varphi,\ \partial_z f_r - \partial_r f_z,\ \frac{1}{r}\partial_r(r \cdot f_\varphi) - \frac{1}{r}\partial_\varphi f_r\right]$

Kugelkoordinaten $\vec{\nabla}$

$\vec{\nabla} f = \left(\partial_r f, \frac{1}{r \cdot \sin\theta}\partial_\varphi f, \frac{1}{r}\partial_\theta f\right)$

$\Delta f = \partial_r^2(f) + \frac{2}{r}\partial_r f + \frac{1}{r^2 \sin^2\theta}\partial_\varphi^2 f + \frac{1}{r^2}\partial_\theta^2 f + \frac{\cos\theta}{r^2 \cdot \sin\theta}\partial_\theta f$

$\vec{\nabla} \cdot \vec{f} = \frac{1}{r^2}\left(\partial_r(r^2 f_r)\right) + \frac{1}{r \cdot \sin\theta}\partial_\varphi f_\varphi + \frac{1}{r \cdot \sin\theta}\left(\partial_\theta(\sin\theta\, f_\theta)\right)$

$\vec{\nabla} \times \vec{f} = \frac{1}{r \cdot \sin\theta}\left(\partial_\theta(\sin\theta\, f_\varphi) - (\partial_\varphi f_\theta)\right),\ \frac{1}{r}\partial_r(r \cdot f_\theta) - \frac{1}{r}(\partial_\theta f_r),\ \frac{1}{r \cdot \sin\theta}(\partial_\varphi f_r) - \frac{1}{r}\left(\partial_r(r \cdot f_\varphi)\right)$

Kugelflächenfkt.

$Y^{lm} = C_{l,m} \cdot P_l^m(\cos\vartheta)\, e^{im\varphi}$, $m = [-l, l]$

$\int_0^\pi \int_0^{2\pi} \sin\vartheta\, d\vartheta\, d\varphi\, Y_{lm}(\vartheta, \varphi)\, Y_{l'm'}^*(\vartheta, \varphi) = \delta_{l,l'} \delta_{m,m'}$

$\sum_{m=-l}^{l} Y_{lm}(\vartheta, \varphi)\, Y_{lm}^*(\vartheta, \varphi) = C_{l,m} \cdot \frac{2l+1}{4\pi}$

$q_{l,m} = \frac{4\pi}{2l+1} \iiint dr' \sin\vartheta\, d\vartheta\, d\varphi\, \rho(\vec{r}) \cdot r'^{l+2}\, Y_{lm}^*(\vartheta, \varphi)$

$C_{l,m} = \begin{cases} (-1)^m & m \geq 0 \\ 1 & \end{cases} \cdot \sqrt{\frac{2l+1}{4\pi}} \sqrt{\frac{(l-|m|)!}{(l+|m|)!}} (\sin\theta)^{|m|}$

$\phi(\vec{r}) = \frac{1}{4\pi\varepsilon_0} \sum_{l=0}^{\infty} \sum_{m=-l}^{l} \frac{q_{l,m}}{r^{l+1}} Y_{lm}(\theta, \varphi)$

$\sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y_{lm}(\theta, \varphi)\, Y_{lm}^*(\theta', \varphi') = \frac{1}{\sin\theta} \delta(\theta - \theta')\, \delta(\varphi - \varphi')$

Kugel: $V = \frac{4}{3}\pi \cdot r^3$; $A = 4\pi r^2$

Zylinder: $A_{mantel} = 2\pi \cdot r \cdot h$; $A_{Deckel} = 2\pi \cdot r^2$; $V = 2\pi \cdot r^2 \cdot h$

$G_{ret}(\vec{r}, t) = \frac{1}{4\pi c}\left(\delta\left(t - \frac{r}{c}\right) + \delta\left(t + \frac{r}{c}\right)\right)$