# Funktionentheorie

$\overline{(z+w)} = \bar z + \bar w$

$\overline{z\cdot w} = \bar z\cdot\bar w$

$z^{-1} = \bar z/|z|^2$

$|z\cdot w| = |z||w|$

$\langle z,w\rangle^2 = |z|^2\cdot|w|^2$

$Re(z) = \frac{1}{2}(z+\bar z)$

$Im(z) = \frac{1}{2}(z-\bar z)$

$|z|^2 = z\cdot\bar z = x^2+y^2$

## allg. Komplexe Zahlen:

$z = x+i\cdot y = r\cdot(\cos\varphi + i\sin\varphi) = r\cdot e^{i\varphi}$

$x = r\cdot\cos\varphi$, $y = r\cdot\sin\varphi$

$r = \sqrt{x^2+y^2}$, $\varphi = \arctan\frac{y}{x}$

Cauchy-Schwartz: $|\langle z,w\rangle| \leq |z|\cdot|w|$

Δ-Ungleichung: $|w+z| \leq |w|+|z|$

cos-Satz: $|w+z|^2 = |w|^2 + 2\langle w,z\rangle + |z|^2$

## Reihen:

$\sum_{k=0}^{\infty}\frac{x^k}{k!} = e^x$

$\sum_{k=1}^{\infty}\frac{1}{k^2} = \frac{\pi^2}{6}$

$\sum_{k=0}^{\infty}\frac{1}{2^k} = 2$

$\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}, \quad |q|<1$

$\sum_{k=0}^{\infty}\frac{(-1)^k}{2k+1} = \frac{\pi}{4}$

$\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{k} = \log(2)$

## Taylor:

$\sin(x) = \sum_{l=0}^{\infty}(-1)^l\frac{x^{2l+1}}{(2l+1)!}$

$\cos(x) = \sum_{l=0}^{\infty}(-1)^l\frac{x^{2l}}{(2l)!}$

$\arctan(x) = \sum_{l=0}^{\infty}(-1)^l\frac{x^{2l+1}}{2l+1}, \quad |x|<1$

$\log(x+1) = \sum_{l=1}^{\infty}(-1)^{l-1}\frac{x^l}{l}, \quad |x|<1$

Konvergenzradius: $r = \lim_{\nu\to\infty}\left|\frac{a_\nu}{a_{\nu+1}}\right| = \left|\frac{1}{\limsup\sqrt[n]{a_n}}\right|$

Periode: p

$\sin(x) = \sin(x+p)$

Log-Nebenzweig: $\log() + 2\pi i n$

$\gamma = (1-t)z_0 + t\cdot z_1, \quad t\in[0,1]$

$\gamma = c + r\cdot e^{it}, \quad t\in[a,b]: 0\leq a<b<2\pi$

$(z-a)^n = \sum_{k=0}^{n}\binom{n}{k}z^k a^{n-k}$

$\left|\int f(z)dz\right| \leq \int|f(z)|dz$

$\sqrt{z} = \pm\frac{1}{2}\left(\sqrt{|z|+x} + i\cdot\frac{y}{|y|}\sqrt{|z|-x}\right)$

## Additionstheoreme:

$\cos^2\varphi + \sin^2\varphi = 1$

$\sin(\alpha+\beta) = \sin\alpha\cdot\cos\beta + \cos\alpha\cdot\sin\beta$

$\cos(\alpha+\beta) = \cos\alpha\cdot\cos\beta - \sin\alpha\cdot\sin\beta$

$\cos^2\varphi - \sin^2\varphi = \cos(2\varphi)$ | $\cosh^2\varphi - \sinh^2\varphi = 1$

$\sin(2\varphi) = 2\cdot\sin\varphi\cdot\cos\varphi$ | $\cos(2\varphi) = 2\cdot\cos^2\varphi - 1$

## Komplexe diffbarkeit:

$f(z) = u + i\cdot v$

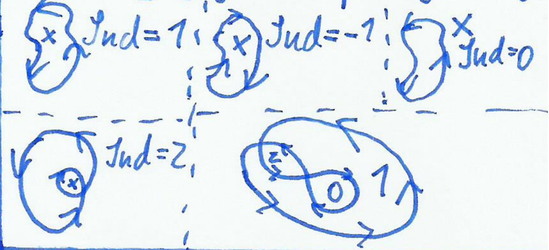
$\frac{\partial u}{\partial x} = \frac{\partial v}{\partial y}$ | $\frac{\partial u}{\partial y} = -\frac{\partial v}{\partial x}$

$\oint_\gamma \frac{f(z)}{z-a}dz = 2\pi\cdot i\cdot f(a)$, $ind_\gamma a \neq 0$

$\oint_\gamma \frac{f(z)}{(z-a)^{n+1}}dz = \frac{2\pi\cdot i}{n!}f^{(n)}(a)$, $ind_\gamma a \neq 0$

$\int_{\partial B}(z-c)^n = \begin{cases}0, & n\neq(-1)\\ 2\pi\cdot i, & n=(-1)\end{cases}$, $ind_{\partial B} c \neq 0$

$Ind_\gamma(z) := \frac{1}{2\pi i}\int_\gamma\frac{1}{\xi - z}d\xi$ | $Ind_{-\gamma} = -Ind_\gamma$




| Funktion | $d/dx$ | $\int dx$ | $\int_0^{\pi}$ | $\int_0^{2\pi}$ | $\int_{-\pi}^{\pi}$ | $\int_0^{\pi/2}$ |
|---|---|---|---|---|---|---|
| $\sin(x)$ | $+\cos(x)$ | $-\cos(x)$ | 2 | 0 | 0 | 1 |
| $\cos(x)$ | $-\sin(x)$ | $+\sin(x)$ | 0 | 0 | 0 | 1 |
| $\tan(x)$ | $1/\cos^2(x)$ | $-\log(\cos(x))$ | / | / | / | / |
| $\arcsin(x)$ | $(\sqrt{1-x^2})^{-1}$ | $\sqrt{1-x^2}+x\cdot\arcsin(x)$ | – | – | 0 | – |
| $\arccos(x)$ | $-(\sqrt{1-x^2})^{-1}$ | $-\sqrt{1-x^2}+x\cdot\arccos(x)$ | – | – | $\pi^2$ | – |
| $\arctan(x)$ | $(1+x^2)^{-1}$ | $x\cdot\arctan(x)-\frac{1}{2}\log(1+x^2)$ | – | – | 0 | – |
| $\sin^2(x)$ | $\sin(2x)$ | $\frac{1}{2}(x-\cos(x)\cdot\sin(x))$ | $\pi/2$ | $\pi$ | $\pi$ | $\pi/4$ |
| $\cos^2(x)$ | $-\sin(2x)$ | $\frac{1}{2}(x+\cos(x)\sin(x))$ | $\pi/2$ | $\pi$ | $\pi$ | $\pi/4$ |
| $\tan^2(x)$ | $2/\cos^2(x)\cdot\tan(x)$ | $-x+\tan(x)$ | / | / | / | / |
| $x\cdot\sin(x)$ | $x\cdot\cos(x)+\sin(x)$ | $-x\cdot\cos(x)+\sin(x)$ | $\pi$ | $-2\pi$ | $2\pi$ | 1 |
| $x\cdot\cos(x)$ | $\cos(x)-x\cdot\sin(x)$ | $\cos(x)+x\cdot\sin(x)$ | $-2$ | 0 | 0 | $-1+\pi/2$ |
| $x^2\cdot\sin(x)$ | – | – | $-4+\pi^2$ | $-4\pi^2$ | 0 | $-2+\pi$ |
| $x^2\cdot\cos(x)$ | – | – | $-2\pi$ | $4\pi$ | $-4\pi$ | $-2+\pi^2/4$ |
| $x\cdot\sin^2x$ | – | – | $\pi^2/4$ | $\pi^2$ | 0 | $1/4+\pi^2$ |
| $x\cdot\cos^2x$ | – | – | $\pi^2/4$ | $\pi^2$ | 0 | $-1/4+\pi^2$ |
| $\cos(x)\cdot\sin(x)$ | $\cos(2x)$ | $-1/2\cos^2(x)$ | 0 | 0 | 0 | $1/2$ |
| $x\cdot\cos(x)\cdot\sin(x)$ | – | – | $-\pi/4$ | $-\pi/2$ | $-\pi/2$ | $\pi/8$ |
| $\cos(x)\cdot\sin^2(x)$ | – | $\sin^3(x)/3$ | 0 | 0 | 0 | $1/3$ |
| $x\cdot\cos(x)\cdot\sin^2(x)$ | – | – | $-4/9$ | 0 | 0 | $-2/9+\pi/6$ |
| $\cos^2(x)\cdot\sin(x)$ | – | $-\cos^3(x)/3$ | $2/3$ | 0 | 0 | $1/3$ |
| $x\cdot\cos^2(x)\cdot\sin(x)$ | – | – | $\pi/3$ | $-2\pi/3$ | $2\pi/3$ | $2/9$ |
| $\cos^2(x)\cdot\sin^2(x)$ | $\frac{1}{2}\sin(4x)$ | – | $\pi/8$ | $\pi/4$ | $\pi/4$ | $\pi/16$ |
| $x\cdot\cos^2(x)\cdot\sin^2(x)$ | – | – | $\pi^2/16$ | $\pi^2/4$ | 0 | $\pi^2/64$ |

## Residuensatz:

$\int_\gamma f(z)dz = 2\pi\cdot i\cdot\sum_{c\in int\gamma} ind_\gamma(c)\cdot res_c(f)$

$\oint \Rightarrow \gamma \Rightarrow (-\infty,0)\to(\infty,0)\to r e^{2i\varphi}\begin{cases}r=\infty\\ \varphi\in[0,\pi]\end{cases}$

$\int_{-\infty}^{\infty}f(x)dx = 2\pi\cdot i\cdot\sum_{Im(c)\geq 0} res_c(f)$

## Residuen:

$res_c f := \frac{1}{2\pi i}\oint_{\gamma(c)}f(z)dz$

1.) $res_c(af+b\cdot g) = a\cdot res_c f + b\cdot res_c g$

2.) c einfacher Pol

$res_c f = \lim_{z\to c} f(z)(z-c)$

↳ $f = \frac{g}{h}$, $h(c)=0 \wedge h'(c)\neq 0$

$\Rightarrow res_c f = \frac{g(c)}{h'(c)}$

3.) c Pol n-ter Ordnung

$res_c f = \frac{1}{(n-1)!}\lim_{z\to c}\frac{d^{n-1}}{dz^{n-1}}([z-c]^n\cdot f(z))$

| | sin | cos | tan |
|---|---|---|---|
| 0 | 0 | 1 | 0 |
| $\pi/4$ | $1/\sqrt{2}$ | $1/\sqrt{2}$ | 1 |
| $\pi/2$ | 1 | 0 | $\pm\infty$ |
| $3\pi/4$ | $1/\sqrt{2}$ | $-1/\sqrt{2}$ | $-1$ |
| $\pi$ | 0 | $-1$ | 0 |
| $5\pi/4$ | $-1/\sqrt{2}$ | $-1/\sqrt{2}$ | 1 |
| $3\pi/2$ | $-1$ | 0 | $\pm\infty$ |
| $7\pi/4$ | $-1/\sqrt{2}$ | $1/\sqrt{2}$ | $-1$ |
| $2\pi$ | 0 | 1 | 0 |

| | Arcsin | Arccos | Arctan |
|---|---|---|---|
| $-1$ | $-\pi/2$ | $\pi$ | $-\pi/4$ |
| $-1/2$ | $-\pi/6$ | $2\pi/3$ | – |
| 0 | 0 | $\pi/2$ | 0 |
| $1/2$ | $\pi/6$ | $\pi/3$ | – |
| 1 | $\pi/2$ | 0 | $\pi/4$ |