Ma Grundlagen: Potenzgesetze: $a^p \cdot a^q = a^{p+q}$; $a^p \cdot b^p = (a \cdot b)^p$; $(a^p)^q = a^{p \cdot q}$; $a^{-p} = \frac{1}{a^p}$; $a^0 = 1$; $\sqrt[p]{a^q} = a^{q/p}$

Logarithmengesetze: $\log_a(x \cdot y) = \log_a(x) + \log_a(y)$; $\log_a\left(\frac{x}{y}\right) = \log_a(x) - \log_a(y)$; $\log_a(x^p) = p \cdot \log_a(x)$; $\log_b(y) = \frac{\log_a y}{\log_a b}$

Reihen: (∞)

$\sum_{k=0}^{\infty} \frac{x^k}{k!} = e^x$; $\sum_{k=1}^{\infty} \frac{1}{k^2} = \frac{\pi^2}{6}$; $\sum_{k=0}^{\infty} \frac{1}{2^k} = 2$; $\sum_{k=1}^{\infty} \frac{1}{2^k} = 1$; $\sum_{k=1}^{\infty} \frac{1}{k} = \infty$

$\sum_{k=0}^{\infty} q^k = \frac{1}{1-q}, |q|<1$; $\sum_{k=0}^{\infty} \frac{(-1)^k}{2k+1} = \frac{\pi}{4}$

$\sum_{k=0}^{\infty} \frac{1}{k!} = e$; $\sum_{k=1}^{\infty} \frac{(-1)^{k+1}}{k} = \ln(2)$

Reihen: (endlich)

$\sum_{k=1}^{n} k = \frac{n(n+1)}{2}$; $\sum_{k=1}^{n} 2 \cdot k = n(n+1)$; $\sum_{k=1}^{n} (2k-1) = n^2$

$\sum_{k=1}^{n} k^2 = \frac{1}{6} n(n+1)(2n+1)$; $\sum_{k=0}^{n} q^k = \frac{1-q^{n+1}}{1-q}$ $|q|<1$, $q \neq 0$; $(1+x)^\alpha = \sum_{k=0}^{\infty} \binom{\alpha}{k} x^k$, $|x|<1$

Konvergenzkriterien:

- Majorante: $|b_k| \leq |a_k|, \forall k \geq N$ $\Rightarrow b_k$ konvergiert $\Leftarrow a_k$ konvergiert
- Quotienten: $\exists q<1: \left|\frac{a_{k+1}}{a_k}\right| \leq q \quad \forall k \geq N$
- Wurzel: $\exists q<1: \sqrt[k]{|a_k|} \leq q \quad \forall k \geq N$
- Leibniz: für $\sum_k (-1)^k \cdot a_k$: $\{a_k\} \longrightarrow 0$

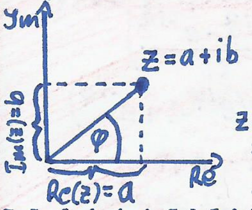
Binomialkoeffizient:

$\binom{n}{k} = \frac{n!}{k!(n-k)!}$; $(a+b)^n = \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k}$

$\frac{n!}{k!(n-k)!} = \frac{n \cdot (n-1) \cdot \ldots \cdot (n-k+1)}{k!} = \binom{n}{n-k}$; $\binom{n}{n} = 1$; $\binom{n}{n-1}$

$\binom{n+1}{k+1} = \binom{n}{k} + \binom{n}{k+1}$; $\binom{n}{0} = 1$; $\binom{n}{1} = n$; $\binom{n}{k} = 0, k>n$

Komplexe Zahlen: $i = \sqrt{-1}$; $i^2 = -1$



$|z| = \sqrt{Re^2 + Im^2}$

$z^* = \bar{z} = Re - i\,Im$

$z = a + ib$
$= r \cdot (\cos\varphi + i \cdot \sin\varphi)$
$= r \cdot e^{i\varphi}$

$Re(z) = a = r \cdot \cos\varphi$; $\varphi = \arctan\frac{Im}{Re}$
$Im(z) = b = r \cdot \sin\varphi$; $r = \sqrt{Re^2 + Im^2}$

$(\cos\varphi + i \cdot \sin\varphi)^n = \cos(n \cdot \varphi) + i \cdot \sin(n\varphi) = e^{in\varphi}$

$e^{i\varphi} = \sum_{k=0}^{\infty} \frac{(i\varphi)^k}{k!} = \text{Taylor}(\cos k) + i\,\text{Taylor}(\sin k)$

$Re(e^{i\varphi}) = \cos\varphi = \frac{e^{i\varphi} + e^{-i\varphi}}{2}$; $Im(e^{i\varphi}) = \sin\varphi = \frac{e^{i\varphi} - e^{-i\varphi}}{2i}$

$Re(z) = \frac{z + z^*}{2}$; $Im(z) = \frac{z - z^*}{2i}$; $(u+v)^* = u^* + v^*$

$(u \cdot v)^* = u^* \cdot v^*$

$(a+bi) \pm (c+di) = (a \pm c) + i(b \pm d)$; $(a+bi) \cdot (c+di) = (ac - bd) + i(ad + bc)$; $\frac{(a+bi)}{(c+di)} \frac{(c-di)}{(c-di)} = \frac{(ac+bd) + i(bc - ad)}{c^2 + d^2}$

$u \cdot u^* = |u|^2$
$|u \cdot v| = |u| \cdot |v|$
$|u + v| \leq |u| + |v|$

$\sqrt[n]{z} = \sqrt[n]{r} \cdot e^{\frac{i(\varphi + 2\pi k)}{n}}$, $k = 1 \ldots (n-1)$

$\sin = \frac{a}{c}$; $\tan = \frac{a}{b}$; $\cot = \frac{b}{a} = \frac{1}{\tan}$; $\csc = \frac{c}{a} = \frac{1}{\sin}$

$\cos = \frac{b}{c}$; $\tan = \frac{\sin}{\cos}$; $\sec = \frac{c}{b} = \frac{1}{\cos}$; $\cos(-\varphi) = \cos\varphi$; $\sin(-\varphi) = -\sin(\varphi)$; $\tan(-\varphi) = -\tan(\varphi)$

$\arcsin: [-1,1] \to [-\frac{\pi}{2}, \frac{\pi}{2}]$

Winkeltheoreme: Bew. $e^{i(\alpha+\beta)}$

$\sin(\alpha+\beta) = \sin\alpha \cdot \cos\beta + \cos\alpha \sin\beta$
$\cos(\alpha+\beta) = \cos\alpha \cdot \cos\beta - \sin\alpha \cdot \sin\beta$
$\left.\begin{array}{l}\cos^2\varphi + \sin^2\varphi = 1 \\ \cosh^2\varphi - \sinh^2\varphi = 1\end{array}\right\}$ Bew. $e^{i\varphi} \cdot e^{-i\varphi} = 1$

Winkel:

$\cos(2\varphi) = 2 \cdot \cos^2\varphi - 1$
$\sin(2\varphi) = 2 \cdot \sin\varphi \cdot \cos\varphi$
$\cos(3\varphi) = \cos\varphi \cdot (4\cos^2\varphi - 3)$
$\text{arsin}\, x = \ln(x + \sqrt{x^2+1})$

$\cosh x = \frac{1}{2}(e^x + e^{-x}) = \cosh(-x)$; $\tanh x = \frac{\sinh x}{\cosh x} = -\tanh(-x)$; $\varphi = \pi - \arcsin(x)$

$\sinh x = \frac{1}{2}(e^x - e^{-x}) = -\sinh(-x)$; $e^x = \cosh x + \sinh x$; $\arccos: [-1,1] \to [0, \pi]$

$\varphi = -\arccos(x)$

$\cos = \cosh(ix)$; $\sin = \sinh(ix)$; $\arctan: \mathbb{R} \to [-\frac{\pi}{2}, \frac{\pi}{2}]$; $\arccos(-x) = \pi - \arccos(x)$

$\text{arcos}\, x = \ln(x + \sqrt{x^2-1})$ $x \geq 1$; $\text{arctan}\, x = \frac{1}{2} \ln\frac{1+x}{1-x}$ $x \in (-1,1)$

Partialbruch Zerlegung:

- Finde Nenner Nullstellen
- bilde Partialsummen
- multiplizieren mit gesamt nenner
- Koeffizienten vergleich
- Partialwerte einsetzen

= für n-fache Nullstellen

$\frac{a}{x-x_n} + \frac{b}{(x-x_n)^2} + \ldots + \frac{c}{(x-x_n)^n}$

Differenzenquotienten:

$\lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0}$

$\frac{f(x_0 + \Delta x) - f(x_0)}{\Delta x}$

$\left(\frac{1}{g}\right)' = -\frac{g'}{g^2}$

Regeln:

$(a \cdot f)' = a \cdot f'$
$(f \pm g)' = f' \pm g'$
$(f \cdot g)' = f' \cdot g + f \cdot g'$
$\left(\frac{f}{g}\right)' = \frac{f' \cdot g - f \cdot g'}{g^2}$
$(f(g))' = f'(g) \cdot g'$

per f^{-1}

$y = f(x)$ $x = g(y)$
$y = f(g(y))$
$1 = f'(x) \cdot g'(y)$
z.B.:
$(\log x)' = \frac{1}{(e^y)'} = \frac{1}{e^y} = \frac{1}{x}$

Integrieren:

$\int a\,dx = ax$
$\int a \cdot f(x)dx = a \cdot \int f(x)dx$
$\int (f(x) + g(x))dx = \int f(x)dx + \int g(x)dx$
$\int dx\, f' \cdot g = f \cdot g - \int f \cdot g' dx$
$\int dx \frac{f'}{f} = \ln|f(x)|$

Substitution:

Typ I: $\int h' \cdot f(h) dx$
$u = h(x)$
Typ II:
$\int f(x) dx$
$\int u'(x) \cdot f(u(x))$
$x = u(x)$

Uneig. Integrale

$\lim_{a \to y} \int_a^b f(x)dx$

$\lim_{a \to y} \int_a^b f(x)dx$

falls $f(x) = n.d.$ für $x = y$

Taylor: für $x_0 \neq 0$

$\sum_{k=0}^{\infty} \frac{1}{k!} \cdot f^{(k)}(x_0) \cdot (x - x_0)^k$

für $x_0 = 0$

$\sum_{k=0}^{\infty} \frac{1}{k!} f^{(k)}(0) x^k$

$\cos(x) = \sum_{l=0}^{\infty} (-1)^l \frac{x^{2l}}{(2l)!}$; $\sin(x) = \sum_{l=0}^{\infty} (-1)^l \frac{x^{2l+1}}{(2l+1)!}$; ~~tan(x)~~ $\arctan = \sum_{l=0}^{\infty} (-1)^l \frac{x^{2l+1}}{2l+1}$, $|x|<1$

$\cosh(x) = \sum_{l=0}^{\infty} \frac{x^{2l}}{(2l)!}$; $\sinh(x) = \sum_{l=0}^{\infty} \frac{x^{2l+1}}{(2l+1)!}$; $\log(1+x) = \sum_{k=1}^{\infty} \frac{(-1)^{k-1}}{k} x^k$ $|x|<1$

$\Gamma(x) = (x-1)! = \int_0^{\infty} dt\, t^{x-1} \cdot e^{-t}$
Gammafkt.

$\lim_{x \to a}$: $x = a + h \Rightarrow \lim_{x \to h}$

$\lim_{x \to \infty} x \equiv \lim_{x \to 0} \frac{1}{x}$

$\binom{\alpha+\beta}{m} = \sum_{k=0}^{m} \binom{\alpha}{k} \binom{\beta}{m-k}$

Grenzwerte:

L'Hospital: $f(0) = g(0) = 0 \Rightarrow \lim_{x \to 0} \frac{f(x)}{g(x)} = \lim_{x \to 0} \frac{f'(x)}{g'(x)} \Leftrightarrow f(x_0) = 0, g(x_0) = 0 \wedge \lim_{x \to x_0}$

mit Reihen, z.B.: $\lim_{x \to 0} \frac{\log(1 + 2x^3)}{\sqrt{1+x^2} \cdot \sin^2 x \tan 2x} = \lim_{x \to 0} \frac{2x^3 + O(x^6)}{[1 + O(x^2)][x^2 + O(x^4)][2x + O(x^3)]} = \lim_{x \to 0} \frac{2x^3}{x^2 \cdot 2x} = 1 \Rightarrow$ 1ne Entwicklung um x_0

Koordinaten:

- Kartesisch (Fläche)
  $\vec{r} = (x, y)^T$
  $dx\,dy$
- Kartesisch (Volumen)
  $\vec{r} = (x, y, z)^T$
  $dx\,dy\,dz$
- Polar (Fläche)
  $\vec{r} = (r, \varphi)^T$; $dr \cdot r \cdot d\varphi$
  $r = \sqrt{x^2 + y^2}$
  $\varphi = \begin{cases} \arctan y/x, & x \geq 0 \\ \arctan y/x + \pi, & x < 0 \end{cases}$
  $x = r \cdot \cos\varphi$
  $y = r \cdot \sin\varphi$

- Kugel (Volumen)
  $\vec{r} = (r, \vartheta, \varphi)$
  $r = \sqrt{x^2 + y^2 + z^2}$
  $\vartheta = \arccos z/r$
  $\varphi = \arctan y/x \{+\pi\}$
  $x = r \cdot \sin\vartheta \cdot \cos\varphi$
  $y = r \cdot \sin\vartheta \cdot \sin\varphi$
  $z = r \cdot \cos\vartheta$
  $dr \cdot r^2 \cdot d\vartheta \cdot \sin\vartheta \cdot d\varphi$

Kegel (Volumen)
$\vec{r} = (r, \varphi, z)$
$r = \sqrt{x^2 + y^2}$
$\varphi = \arctan y/x \{+\pi\}$
$z = z$
$x = r \cdot \cos\varphi$
$y = r \cdot \sin\varphi$
$dr \cdot r \cdot d\varphi \cdot dz$

DGL 1:

- hom, lin, 1.Ord.: $y' = a(x) y \Rightarrow \frac{y'}{y} = a(x) \Rightarrow \int \frac{y'}{y} dx = A(x) + C = \ln y$
- inhom., lin, 1.Ordnung: $y' = a(x) y + b(x)$ $\Rightarrow$ $y = \tilde{C} \cdot e^{A(x)}$
  $\Rightarrow y_{hom} = C \cdot e^{A(x)} \Rightarrow C = C(x) \Rightarrow C(x) = \int dx\, b(x) \cdot e^{-A(x)} + C$; $y^{(n)} \ldots y^{(1)}$
- hom, lin: $a_n \cdot y^{(n)} + \ldots + a_1 \cdot y' + a_0 \cdot y = 0$ | Ansatz: $y = Ae^{\lambda x} \Rightarrow$
  1. durch '$Ae^{\lambda x}$' teilen ; 2. Koeffizientenvergleich für λ ; 3 linear kombinieren mit Ansatz.

bei $\lambda_1 = \lambda_2 \Rightarrow y = Ae^{\lambda x} + Bxe^{\lambda x}$

bei inhomogen: Konstanten abhängig von x machen $\Rightarrow$ einsetzen. nach A lösen und in Ansatz einsetzen. (Variation d. Konstanten)
$\Rightarrow y_{Partikulär}$

Gebietsintegral

$I_G = \iiint_G dx\,dy\,dz\, f(x, y, z)$

Bsp. Rechteck: $\int_a^b dx \int_c^d dy \cdot f(x, y) = V_R$ $x \in [a, b]$, $y \in [c, d]$

Schwerpunkt: $x_s = \frac{1}{V} \cdot \iiint_V dx\,dy\,dz \cdot x$

Kugel: $V = \int_0^R dr\, r^2 \cdot \int_0^{\pi} d\vartheta \cdot \sin\vartheta \cdot \int_0^{2\pi} d\varphi = \int_0^R dr \cdot r^2 \cdot \int_{-1}^{1} du \cdot \int_0^{2\pi} d\varphi$ mit $u = \cos\vartheta$, $dx = du \cdot \frac{1}{-\sin\vartheta}$

DGL 2:

• Reelen: $y''+\omega^2 y=0$
$\Rightarrow y_1=\cos(\omega x)$
$y_2=\sin(\omega x)$

Nicht linear: $y'=a(x)\cdot b(y)$
$\Rightarrow \frac{y'}{b(y)}=a \Rightarrow \int\left(\frac{1}{b(y)}\right)dx=A(x)+c$

z.B.: $\frac{y'}{y^2}=a \Rightarrow -\frac{1}{y}=A+c \Rightarrow y=\frac{1}{-A-c}$

$\ddot{x}+r\cdot\dot{x}+\omega_0^2 x=0$
$\Rightarrow \lambda_{1,2}=-\frac{r}{2}\pm\sqrt{\frac{r^2}{4}-\omega_0^2}$

1.: $r=0 \Rightarrow \lambda=\pm i\omega_0 \Rightarrow C_1\cos(\omega_0 t)+C_2\sin(\omega_0 t)$
2.: $r<2\omega_0 \Rightarrow \lambda=-\frac{r}{2}\pm i\omega_0 \Rightarrow (C_1\cos(\omega_0 t)+C_2\sin(\omega_0 t))\,e^{-\frac{r}{2}t}$
3.: $r=2\omega_0 \Rightarrow \lambda=\omega_0 \Rightarrow (C_1+C_2 t)e^{-\frac{r}{2}t}$
4.: $r>2\omega_0 \Rightarrow \lambda=-\delta_{1,2} \Rightarrow C_1 e^{-\delta_1 t}+C_2 e^{-\delta_2 t}$

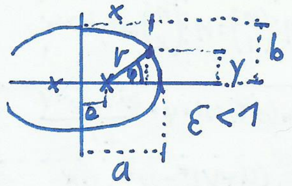
$\ddot{z}+r\dot{z}+\omega_0^2 z=f\cdot e^{i\omega t}$
Ansatz: $z_{part}=Ae^{i\omega t}$, $A\in\mathbb{C}$ $\Rightarrow A(-\omega^2+ir\omega+\omega_0^2)=f$
$\Rightarrow A=\frac{f}{\omega_0^2-\omega^2+ir\omega}$

$\omega_{res}=\sqrt{\omega_0^2-\frac{r^2}{2}}$

$|A_{res}|=\frac{|f|}{r\cdot\omega_1}$

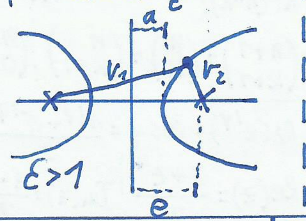
Krummlinige Koordinaten / Kegelschnitte

Ellipse: $\frac{(x-x_2)^2}{a^2}+\frac{(y-y_2)^2}{b^2}=1$
Exzentrizität: $e=\sqrt{a^2-b^2}$
Brennpunkte: $x=\pm e,\ y=0$
$r_{1;2}=\sqrt{(x\pm e)^2+y^2}$
$r_1+r_2=\text{const.}=2a$
$r_{1,2}=a\pm\frac{e}{a}\cdot x$; $\varepsilon=\frac{e}{a}$
$r(\varphi)=\frac{k}{1+\varepsilon\cdot\cos\varphi}$; $k=\frac{b^2}{a}$



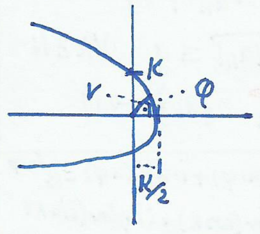
$\varepsilon<1$

Hyperbel: $\frac{(x-x_2)^2}{a^2}-\frac{(y-y_2)^2}{b^2}=1$
Exzentrizität: $e=\sqrt{a^2-b^2}$
Brennpunkte: $x=\pm e,\ y=0$
$r_{1;2}=\sqrt{(x\pm e)^2+y^2}$
$|r_1-r_2|=\text{const.}=2a$
$r_{1,2}=\left|a\pm\frac{e}{a}x\right|$; $\varepsilon=\frac{e}{a}$
$r(\varphi)=\frac{k}{1+\varepsilon\cdot\cos\varphi}$; $k=\frac{b^2}{a}$
$\varphi_\infty=\arccos\frac{-1}{\varepsilon}$



$\varepsilon>1$

rechter Ast: $x\geq a$
$r_1=+a-\frac{e}{a}x;\ r_2=a-\frac{e}{a}x$
$r_1-r_2=2a$
linker Ast: $x\leq -a$
$r_1=-a-\frac{e}{a}x;\ r_2=a-\frac{e}{a}x$
$r_1-r_2=-2a$



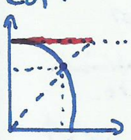
Parabel $k=\frac{b^2}{a}$
$r(\varphi)=\frac{k}{1+\cos\varphi}$
$r\cdot\cos\varphi=\frac{k}{2}+x$
$r\cdot\sin\varphi=y$
$r+\left(\frac{k}{2}+x\right)=k$
$\left(\frac{k}{2}+x\right)^2+y^2=\left(\frac{k}{2}-x\right)^2$
$\Rightarrow y^2=-2\cdot k\cdot x$

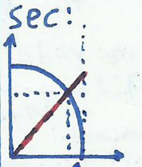
Gebrochene Winkel:
$e^{2i\varphi}=(e^{i\varphi})^2=(\cos\varphi+i\sin\varphi)^2$
$\cos(2\varphi)=2\cos^2\varphi-1$ |Re
$\sin(2\varphi)=2\sin\varphi\cdot\cos\varphi$ |Im
$\cos(3\varphi)=\cos\varphi(4\cdot\cos^2\varphi-3)$
$\sin(3\varphi)=3\cdot\sin(\varphi)-4\cdot\sin^3(\varphi)$
$\sin^2\frac{\varphi}{2}=\left(\frac{e^{i\frac{\varphi}{2}}-e^{-i\frac{\varphi}{2}}}{2i}\right)^2$
$=-\frac{1}{4}(e^{i\varphi}-2+e^{-i\varphi})$
$\Rightarrow \sin^2\frac{\varphi}{2}=-\frac{1}{4}(2\cos\varphi-2)=\frac{1}{2}(1-\cos\varphi)$
$\sin\frac{\varphi}{2}=\pm\sqrt{\frac{1-\cos\varphi}{2}}$ Analog mit cosinus, sinh, cosh...

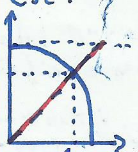
Standart Winkelfkt.-Werte:

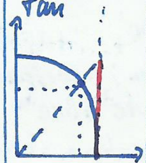
Grad	Rad	Sinus	Cosinus	Tangens
0	0	0	1	0
15	$\pi/12$	$(-1+\sqrt{3}):(2\sqrt{2})$	$(1+\sqrt{3}):(2\sqrt{2})$	$2-\sqrt{3}$
28,6	0,5	0,48	0,88	0,55
30	$\pi/6$	$1/2$	$\sqrt{3}/2$	$1/\sqrt{3}$
45	$\pi/4$	$1/\sqrt{2}$	$1/\sqrt{2}$	1
57,3	1	0,84	0,54	1,56
60	$\pi/3$	$\sqrt{3}/2$	$1/2$	$\sqrt{3}$
75	$5\pi/12$	$(1+\sqrt{3}):(2\sqrt{2})$	$(-1+\sqrt{3}):(2\sqrt{2})$	$2+\sqrt{3}$
85,9	1,5	0,997	0,07	14,1
90	$\pi/2$	1	0	$\pm\infty$
105	$7\pi/12$	$(1+\sqrt{3}):(2\sqrt{2})$	$-(-1+\sqrt{3}):(2\sqrt{2})$	$-2-\sqrt{3}$
114,6	2	0,91	-0,42	-2,19
120	$2\pi/3$	$\sqrt{3}/2$	$-1/2$	$-\sqrt{3}$
135	$3\pi/4$	$1/\sqrt{2}$	$-1/\sqrt{2}$	-1
143,2	2,5	0,6	-0,8	-0,75
150	$5\pi/6$	$1/2$	$-\sqrt{3}/2$	$-1/\sqrt{3}$
165	$11\pi/12$	$(-1+\sqrt{3}):(2\sqrt{2})$	$-(1+\sqrt{3}):(2\sqrt{2})$	$-2+\sqrt{3}$
171,9	3	0,14	-0,99	-0,14
180	π	0	-1	0

$f(x)$	$f'(x)$	$f''(x)$	$F(x)$
arcsinh(x)	$(\sqrt{1+x^2})^{-1}$	$-x/(1+x^2)^{3/2}$	/z.G.
arcosh(x)	$(\sqrt{x^2-1})^{-1}$	$-x/(-1+x^2)^{3/2}$	/z.G.
artanh(x)	$(1-x^2)^{-1}$	$2x/(x^2-1)^2$	/z.G.
arcoth(x)	$(1-x^2)^{-1}$	$2x/(x^2-1)^2$	/z.G.
arsech(x)	$-\left(x\sqrt{\frac{1-x}{1+x}}(1+x)\right)^{-1}$	/z.G.	/z.G.
arcsch(x)	$-\left(x^2\sqrt{1+\frac{1}{x^2}}\right)^{-1}$	/z.G.	/z.G.

cot:

$=\frac{1}{\tan}$

sec:

$=\frac{1}{\cos}$

csc:

$=\frac{1}{\sin}$

tan


Standart dx

$f(x)$	$f'(x)$	$f''(x)$	$F(x)+c$
C $C\in\mathbb{C}$	0	0	$C\cdot x$
x	1	0	$\frac{1}{2}x^2$
x^n	nx^{n-1}	$n(n-1)x^{n-2}$	$\frac{1}{n+1}x^{n+1}$
$\frac{1}{x}$	$-\frac{1}{x^2}$	$+\frac{2}{x^3}$	$\ln\lvert x\rvert$
$\frac{1}{x^2}$	$-\frac{2}{x^3}$	$\frac{6}{x^4}$	$-\frac{1}{x}$
$\sqrt{x}$	$\frac{1}{2\sqrt{x}}\cdot 1$	$-\frac{1}{4}\frac{1}{\sqrt{x^3}}$	$\frac{2}{3}\sqrt{x^3}$
e^x	e^x	e^x	e^x
a^x	$a^x\cdot\log(a)$ $a>0$	$a^x\cdot\log(a)^2$	$\frac{a^x}{\log(a)}$
$\log(x)$	$\frac{1}{x}$ $x\neq 0$	$-\frac{1}{x^2}$	$-x+x\cdot\log(x)$
$\sin(x)$	$\cos(x)$	$-\sin(x)$	$-\cos(x)$
$\cos(x)$	$-\sin(x)$	$-\cos(x)$	$\sin(x)$
$\tan(x)$	$\frac{1}{\cos^2(x)}=1+\tan^2(x)$	$\frac{2}{\cos^2(x)}\cdot\tan(x)$	$-\log(\cos(x))$
$\cot(x)$	$-\frac{1}{\sin^2(x)}$	$\frac{2}{\tan(x)\cdot\sin^2(x)}$	$\log(\sin(x))$
$\sec(x)$	$\sec(x)\cdot\tan(x)$	$\sec(x)^3+\sec(x)\cdot\tan(x)^2$	/z.G.
$\csc(x)$	$-\cot(x)\cdot\csc(x)$	$\cot(x)^2\cdot\csc(x)+\csc(x)^3$	/z.G.
$\arcsin(x)$	$\frac{1}{\sqrt{1-x^2}}$	$x/(1-x^2)^{3/2}$	$x\cdot\arcsin(x)+\sqrt{1-x^2}$
$\arccos(x)$	$-\frac{1}{\sqrt{1-x^2}}$	$-x/(1-x^2)^{3/2}$	$x\cdot\arccos(x)-\sqrt{1-x^2}$
$\arctan(x)$	$\frac{1}{1+x^2}$	$-2x/(1+x^2)^2$	/z.G.
$\text{arccot}(x)$	$-\frac{1}{1+x^2}$	$2x/(1+x^2)^2$	/z.G.
$\text{arcsec}(x)$	$([1-1/x^2]^{1/2}\cdot x^2)$	/z.G	/z.G.
$\text{arccsc}(x)$	$-([1-1/x^2]^{1/2}\cdot x^2)$	/z.G.	/z.G.
sinh	$\cosh(x)$	$\sinh(x)$	$\cosh(x)$
cosh	$\sinh(x)$	$\cosh(x)$	$\sinh(x)$
tanh	$(\cosh^2(x))^{-1}=\text{sech}(x)^2$	$-2\,\text{sech}(x)^2\cdot\tanh(x)^2$	$\text{Log}(\cosh(x))$
coth	$-(\sinh^2(x))^{-1}=-\text{csch}(x)^2$	$2\coth(x)\cdot\text{csch}(x)^2$	$\log(\sinh(x))$
sech	$-\text{sech}(x)\cdot\tanh(x)$	/z.G.	$2\cdot\arctan(\tanh(x/2))$
csch	$-\coth(x)\cdot\text{csch}(x)$	/z.G.	/z.G.