# Mathematisch-Naturwissenschaftliche Fakultät I Humboldt-Universität zu Berlin Institut für Physik - Physikalisches Grundpraktikum 



# Versuchsprotokoll 

Polarisation durch Reflexion (O11)

durchgeführt am 10.11.2009 mit Versuchspartner Andreas Koher

Protokoll von Sebastian Milster

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## I. Introduction

The aim of this practical was to analyse how differently polarised (orthogonal/parallel to surface) light is reflected at a plane surface of a glass lens. In order to gain comparable data, we measured the voltage induced by the reflected light for different reflection angles and for orthogonal and parallel polarized light separately. This enabled us to calculate the respective reflectivities and to plot their square roots over the angle of reflection. With the help of the graphic we were to find the Brewster angle and compare our results with the theoretical curves. At Brewster's angle none of the parallel polarized light is reflected, in other words, all of the parallel polarized light is refracted. Moreover, we will determine the refractive index of the glass lens.

The physical basics and the detailed objective for this experiment can be found in the instruction booklet (p.74-77) .

In this report you will encounter a few indices with the following meanings:
i - incident
r-reflected
o - orthogonal polarized
p - parallel polarized
m - directly measured

## II. Graphical figure of square-rooted reflectivity over angle of incidence

Before taking down any values we adjusted our voltmeter so, that it showed a voltage of $U_{0}=0 V$ for our beam not hitting (any part of) the detector. In fact, after changing the effective range of the voltmeter to the 300 mV -scale during testing the parallel polarized component, we obtained values for $U_{0}$ different from 0, which will be considered in the respective calculation. $U_{0}$ was continuously checked after every measured value.
The maximum voltage $U_{i}$ induced by the unreflected light was taken down before every series of measurements.

The actual induced voltage from our light beam is the difference $U_{r}=U_{m}-U_{0}$ between the measured value $U_{m}$ and the voltage $U_{0}$. The systematic error for every voltage measurement is $\quad u^{\prime}{ }_{U}=0,3 \% \cdot U+2 L S D$. Thus we obtain the resulting systematic error:

$$
u_{U_{r}}=\sqrt{u_{U_{m}}^{2}+u_{U_{o}}^{2}} \approx 0,3 \% \cdot U_{m}+\sqrt{2} \cdot 2 L S D
$$

The same applies to $U_{i}: \quad u_{U_{i}}=0,3 \% \cdot U_{i}+\sqrt{2} \cdot 2 L S D$

Similar to the calculations of voltage, the actual angle of refraction is determined by $\alpha_{r}=\frac{\alpha_{m}+\left(180^{\circ}-\alpha_{0}\right)}{2} . \alpha_{0}$ is the angle read from the scale for the unreflected beam. It serves an adjusting purpose and its value is $\alpha_{0}=179^{\circ}$ Every measurement has the estimated uncertainty of $u_{\alpha}^{\prime}=0,5^{\circ}$.
This propagates to: $u_{\alpha_{r}}=\frac{1}{2} \sqrt{u_{\alpha}^{\prime 2}+u_{\alpha}^{\prime 2}}=\frac{\sqrt{2}}{2} \cdot u_{\alpha}^{\prime}=\frac{\sqrt{2}}{4}$
Eventually we calculate the square root of the reflectivity as follows:

$$
\sqrt{R}=\sqrt{\frac{U_{r}}{U_{i}}} ; \quad \Rightarrow u_{\sqrt{R}}=\sqrt{\left(\frac{\partial R}{\partial U_{r}} \cdot u_{U_{r}}\right)^{2}+\left(\frac{\partial R}{\partial U_{i}} \cdot u_{U_{i}}\right)^{2}}=\sqrt{\frac{1}{4 U_{i} U_{r}} u_{U_{r}}^{2}+\frac{U_{r}}{4 U_{i}^{3}} u_{U_{i}}^{2}}
$$



## III. Brewster angle and refractive index of the glass lens

From the graphic we derived the value for Brewster's angle $\alpha_{B}$. The minimum of the red plot is approximately at an angle of $\alpha_{B}=(56 \pm 1)^{\circ}$.

Formula (6) from the booklet serves a relationship between the Brewster angle and the refractive indices of the two media air and glass: $\tan \alpha_{B}=\frac{n_{\text {glass }}}{n_{\text {air }}}$

Assuming that $n_{\text {air }}=1,00029$ and the uncertainty being negligible, we calculate the refractive index of our lens:

$$
\begin{gathered}
n_{\text {glass }}=n_{\text {air }} \cdot \tan \alpha_{B} ; \quad u_{n_{\text {glas }}}=\frac{n_{\text {air }}}{\cos ^{2} \alpha_{B}} \cdot u_{\alpha_{B}} \\
n_{\text {glass }}=1,48 \pm 0,06
\end{gathered}
$$

## IV. Comparison with theoretical curves

Now, having found the refractive index, we will be able to evaluate our figure from II. We can substitute an expression for $\alpha_{g}$, the unknown angle of refraction, from the law of refraction into the formulae (4a) and (4b). This ends up in two formulae for the square-rooted reflectivities:

Orthogonal:

$$
\sqrt{R_{o}}=\frac{\sin \left(\alpha_{r}-\arcsin \left(\frac{\sin \alpha_{r}}{n_{\text {glass }}}\right)\right)}{\sin \left(\alpha_{r}+\arcsin \left(\frac{\sin \alpha_{r}}{n_{\text {glass }}}\right)\right)}
$$

Parallel:

$$
\sqrt{R_{p}}=\frac{\tan \left(\alpha_{r}-\arcsin \left(\frac{\sin \alpha_{r}}{n_{\text {glass }}}\right)\right)}{\tan \left(\alpha_{r}+\arcsin \left(\frac{\sin \alpha_{r}}{n_{\text {glass }}}\right)\right)}
$$

For we received $n_{\text {glass }}=1,48 \pm 0,06$ in III., we will plot the two functions $\sqrt{R_{o}}$ and $\sqrt{R_{p}}$ for the mean value (blue and purple respectively) and also two for each $n_{\text {glass }}=1,42$ and $n_{\text {glass }}=1,54$. (grey). The latter represent the span of uncertainty due to the error of $n_{\text {glass }}$.

Square root of reflectivity over angle of incedence


It is obvious that all values except those close to the Brewster angle are below the theoretical curves. This implies either that the voltage $U_{r}$ is too low or the incident voltage $U_{i}$ is too high. As we determined the incident voltage only once before every series of measurements, it is most probable that the change of $U_{i}$ is an important influence we have not taken into consideration this far. As we omitted to take note of the change, we can only use the theoretical values to adjust our curves.

## V. Adjustment

We adjust the formula for $\sqrt{R}$ as follows: $\sqrt{R}=\sqrt{\frac{U_{r}}{U_{i}+\Delta U}}$
Solved for $\Delta U: \Delta U=\frac{U_{r}}{R}-U_{i}$
$U_{r}$ and $U_{i}$ are values from our measurements, $R$ is the value for the reflectivity for $n_{\text {glass }}=1,48$. Thus we get for every angle the respective difference $\Delta U$. The reason why we can do so is that the theoretical reflectivity is determined by the Brewster angle. This angle was read off at the minimum of $\sqrt{R_{p}}$, which is independent from the value of the incident voltage!

For the difference $\Delta U$ we determined the mean and added it to the incident voltage. To calculate the adjusted square-rooted reflectivity $\sqrt{R^{\prime}}$, the formula from II was used. $U_{i}$ became $U_{i}+\overline{\Delta U}$ and $u_{U_{i}}$ was replaced by $\overline{\Delta U}$ :

$$
\sqrt{R^{\prime}}=\sqrt{\frac{U_{r}}{U_{i}+\overline{\Delta U}}} ; \quad \Rightarrow u_{\sqrt{R^{\prime}}}=\sqrt{\frac{1}{4\left(U_{i}+\overline{\Delta U}\right) U_{r}} u_{U_{r}}^{2}+\frac{U_{r}}{4\left(U_{i}+\overline{\Delta U}\right)^{3}} \overline{\Delta U^{2}}}
$$

This was done for $\sqrt{R_{o}{ }^{\prime}}$ and $\sqrt{R_{p}{ }^{\prime}}$ separately. Furthermore, for parallel polarized light and for angles within the range of the Brewster angle $\alpha_{B}=(56 \pm 1)^{\circ}$,the respective differences $\Delta U$ were not used for the calculation of the mean. The result is shown in the following figure:


By adjusting the incident voltage, most points have moved closer to the theoretical curves. Especially for the orthogonal polarized light, we find a much better match. Unfortunately the error (orthogonal) has increased too, but this is due to the big difference before the adjustment:

$$
\overline{\Delta U_{o}}=-1,166 \mathrm{~V} ; \quad \overline{\Delta U_{p}}=-0,387 \mathrm{~V} ;
$$

To conclude, this adjustment shows that it might have been useful to detect the actual change during the experiment. At least two values, before and afterwards, should be taken down in order to estimate the change. Especially for the orthogonal polarized light, the mean of the difference is that big, that it is very unlikely the incident voltage being the only reason for the deviation in IV.

## VI. Summary

The results of this experiment match with the expectations from the theoretical point of view. This applies especially to the refractive index of our glass lens. Crown glass has a refractive index of approximately 1,5 . This supports the result we have obtained: $n_{\text {glass }}=1,48 \pm 0,06$.

The visualizations of the square-rooted reflectivities are not as perfect. An adjustment showed that the plots do fit the theoretical curves, but we had to sacrifice too much of the accuracy. The incident voltage not being constant is not the only disregarded influence. Although the light passed through a filter, there was not a $100 \%$ polarization of the light. The best evidence for this, is that there was no angle at which the light dot from the laser completely disappeared when analysing the parallel polarized light.

All in all, this practical is suitable for the determination of the refractive index and delivers an useful quantified overview of how light is reflected and polarised at an interface of two different media.

