Differentialoperatoren

Der Nablaoperator: $\vec{\nabla} = \frac{\partial}{\partial x}\vec{i} + \frac{\partial}{\partial y}\vec{j} + \frac{\partial}{\partial z}\vec{k}$

Der Gradient: $grad\,\vec{U}(x,y,z) = \vec{\nabla}U = \frac{\partial U}{\partial x}\vec{i} + \frac{\partial U}{\partial y}\vec{j} + \frac{\partial U}{\partial z}\vec{k}$

$$\mathrm{grad\,U}(\rho,\varphi,z) = \frac{\partial U}{\partial \rho}\vec{e}_r + \frac{1}{\rho}\frac{\partial U}{\partial \varphi}\vec{e}_\varphi + \frac{\partial U}{\partial z}\vec{k}$$

$$grad\,U(r,\varphi,\theta) = \frac{\partial U}{\partial r}\vec{e}_r + \frac{1}{r\sin\theta}\frac{\partial U}{\partial \varphi}\vec{e}_\varphi + \frac{1}{r}\frac{\partial U}{\partial \vartheta}\vec{e}_\theta$$

Die Divergenz: $div\,\vec{A}(x,y,z) = \vec{\nabla} * \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$

$$\mathrm{div}\,\vec{A}(\rho,\varphi,z) = \vec{\nabla} * \vec{A} = \frac{1}{\rho}\frac{\partial \rho A_\rho}{\partial \rho} + \frac{1}{\rho}\frac{\partial A_\varphi}{\partial \varphi} + \frac{\partial A_z}{\partial z}$$

Die Rotation: $rot\,\vec{A}(x,y,z) = \vec{\nabla}\times\vec{A} = \begin{Vmatrix} \vec{i} & \vec{j} & \vec{k} \\ & \nabla & \\ A_x & A_y & A_z \end{Vmatrix}$

$$\mathrm{rot}\,\vec{A}(x,y,z) = \vec{\nabla}\times\vec{A} = \frac{1}{\rho}\begin{Vmatrix} \vec{e}_\rho & \vec{e}_\varphi & \vec{e}_z \\ \frac{\partial}{\partial \rho} & \frac{\partial}{\partial \varphi} & \frac{\partial}{\partial z} \\ A_\rho & \rho A_\varphi & A_z \end{Vmatrix}$$

Der Laplaceoperator: $\Delta = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2}$

$$\Delta = \frac{1}{\rho}\frac{\partial}{\partial \rho}\left(\rho \frac{\partial}{\partial \rho}\right) + \frac{1}{\rho^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\Delta U = div\ grad U$$

$$\Delta \vec{A} = \Delta A_x \vec{i} + \Delta A_y \vec{j} + \Delta A_z \vec{k}$$

$$\Delta \vec{A} = grad\ div\ \vec{A} - rot\ rot\ \vec{A}$$