Neural noise and neural signals - spontaneous activity and signal transmission in models of single nerve cells

Benjamin Lindner

Theory of Complex Systems and Neurophysics

Institut für Physik
Humboldt-Universität Berlin
Integrate & fire models driven by noise and external stimuli
Rate coding of sensory signals

Experiments on stretched frog muscles

Strength of stimulus coded in mean firing rate
Rate coding of time-dependent stimuli

Leaky integrate-and-fire neuron with periodic stimulus

\[ \tau \dot{v} = -v + \mu + \varepsilon \cos(2\pi F_{\text{signal}}t) + \xi(t) \]


Leaky IF neuron: linear response to periodic stimulus

- Bias current: $\mu = 0.8$
- Noise intensity: $D = 0.1$
- Signal amplitude: $= 0.2$

Mathematical expressions:

- $\mu + \varepsilon \cos(\omega t)$
- $r_0 + \varepsilon r_1 \cos(\omega t - \phi)$
Perfect IF neuron: linear response

- Noise intensity $D = 0.02$
- Signal amplitude $= 0.2$
Perfect IF neuron: response at strong amplitude

- Noise intensity: $D=0.02$
- Signal amplitude: $=1$
Linear response theory for stochastic integrate-and-fire neurons

- for IF neurons driven by white Gaussian noise, we can calculate
  - modulation of the firing rate for a weak periodic signal $\varepsilon \cos(\omega t)$
    
    \[ r(t) = r_0 + \varepsilon |\chi| \cos(\omega t - \phi), \quad \phi = \arg(\chi) \]

- for slow signals we find
  - $\chi(0) = \frac{dr_0}{d\mu}$

- for non-periodic signals we find
  - $\tilde{r}(\omega) = \chi(\omega)\tilde{s}(\omega)$

spectral measures & MI rate
Quantifiers of signal transmission

- How strong are spike train and signal correlated?
  strength of rate modulation
- How well can the output signal be distinguished from the output noise?
  signal-to-noise ratio (SNR) / coherence / mutual information
Effect of a periodic stimulus on the spike-train power spectrum

Simple example: periodically driven LIF neuron

\[ \tau \dot{v} = -v + \mu + \varepsilon \cos(2\pi F_{\text{signal}} t) + \xi(t) \]

Output signal strength: peak height

Signal-to-noise ratio: peak height/background

Background spectrum = spontaneous power spectrum!
What is stochastic resonance?

input-output correlation
or
signal-to-noise ratio

Theory for SR in neurons

Gammaitoni et al. Rev. Mod. Phys. 1998
McDonnell & Abbott

Experimental verification

Douglass et al. Nature 1993
Levin & Miller Nature 1996
Cordo et al. Nature 1996
Stochastic resonance in sensory cells

Stochastic Resonance in the crayfish mechanoreceptor

Stochastic Resonance in cricket sensory neuron

Mechanism of stochastic resonance: optimal noise for amplitude of rate modulation
Mechanism of stochastic resonance:
optimal noise for amplitude of rate modulation

slow modulation of the input current
Mechanism of stochastic resonance: optimal noise for amplitude of rate modulation

resulting rate modulation

input current

moderate noise
Mechanism of stochastic resonance: optimal noise for amplitude of rate modulation

resulting rate modulation
Mechanism of stochastic resonance: optimal noise for amplitude of rate modulation

Optimal noise for rate modulation!

resulting rate modulation
Mechanism of stochastic resonance:
when do we get SR for the rate modulation amplitude?

- PIF model - no SR!
- LIF model - only if $\tau_{\text{abs}} > 0$, $\mu < \nu_T$
Stochastic resonance: SNR - comparison of exact theory and adiabatic weak noise expression
Exact results for the LIF neuron

\[ \tau \dot{v} = -v + \mu + \varepsilon \cos(2\pi F_{signal} t) + \xi(t) \]

Power spectrum
[Hz]

Peak height

Signal-to-noise

Linear response theory for stochastic integrate-and-fire neurons

\[ \dot{v} = -U'(v) + \varepsilon e^{-i\omega t} + \sqrt{2D} \xi(t) \]

For a weak signal: linear response

\[ P(v, t) = P_0(v) + \varepsilon e^{-i\omega t} e^{-\frac{U(v) - U(v_R)}{2D}} q(v) + \cdots \]
\[ r(t) = r_0 + \varepsilon \hat{r}(\omega) e^{-i\omega t} + \cdots \]

From the Fokker-Planck equation in linear order:

\[
Dq''(v) - F(v)q(v) = G(v) - \hat{r} e^{i\omega \tau_{\text{ref}}} \delta(v - v_R)
\]

with boundary conditions: \( q(v_T) = 0, \quad (BC1) \)  
[\(q(v)] = 0, \quad (BC2)\]
\[ \lim_{v \to -\infty} q(v) = 0, \quad (BC3)\]
\[ q'_T = \left. \frac{dq}{dv} \right|_v = v_T = -\frac{\hat{r}}{D} e_+(v_T), \quad (BC4)\]

with nonlinear functions:

\[ F(v) = \frac{(U'(v))^2}{4D} - U''(v) - i\omega \]
\[ G(v) = e_+(v)P'_0(v) = e^{\frac{U(v) - U(v_R)}{2D}} P'_0(v) \]
Linear response theory for stochastic integrate-and-fire neurons

-susceptibility for a general nonlinear IF model with potential $U(v)$

\[
\chi(\omega) = -\frac{e_-(v_T) \int_{-\infty}^{v_T} dv q(v)e_+(v)P'_0(v)}{q(v_T) - e^{i\omega \tau_{ref}} e_-(v_T)q(v_R)}
\]

where

\[
e_-(v) = e^{-\frac{U(v)-U(v_R)}{D}} \quad e_+(v) = e^{\frac{U(v)-U(v_R)}{D}}
\]

\[
P_0(v) = \frac{r_0}{D} e^{-U(v)/D} \int_v^{v_T} dx e^{U(x)/D} \Theta(x - v_R)
\]

and $q(v)$ is the solution of

\[
Dq'' - \left( \frac{(U'(v))^2}{4D} - \frac{U''(v)}{2} - i\omega \right) q = 0
\]

with \[
\lim_{v \to -\infty} q(v) = 0
\]
Linear response theory for stochastic integrate-and-fire neurons

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\]

PIF neuron

\[
\chi(\omega) = \frac{2}{v_T - v_R} \left(1 + \sqrt{1 - 2i\omega \tau_{\text{PIF}}} \right)^{-1}, \quad \tau_{\text{PIF}} = \frac{2D}{\mu^2}
\]
Leaky IF neuron: linear response

\[
\chi = \frac{r_0 i \omega_s / \sqrt{D}}{i \omega_s - 1} \frac{D_{i \omega_s - 1} \left( \frac{\mu - v_T}{\sqrt{D}} \right) - e^{\Delta} D_{i \omega_s - 1} \left( \frac{\mu - v_R}{\sqrt{D}} \right)}{D_{i \omega_s} \left( \frac{\mu - v_T}{\sqrt{D}} \right) - e^{\Delta} e^{i \omega_s \tau} D_{i \omega_s} \left( \frac{\mu - v_R}{\sqrt{D}} \right)}
\]


Question

How different are the susceptibilities of the IF models for a given spike rate and CV?
Different IF models: response to a weak stimulus

Susceptibility

Different IF models: response to a weak stimulus

Susceptibility

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Spectral coherence of various integrate-and-fire models
Spectral coherence of various integrate-and-fire models

Coherence functions always low-pass!

Perfect IF

Quadratic IF

Leaky IF

Vilela & Lindner
Summary: signal transmission in IF models

- Rate modulation by weak time-dependent stimuli can be analytically calculated for
  • PIF and LIF models with white background noise

- Rate modulation in different IF models differs by
  • magnitude & resonance behavior (none for PIF model)

- Information transfer is best for slow signal components
  • coherence of PIF, LIF, QIF is low-pass irrespective of the firing regime

- Noise can ...
  • induce resonance frequencies
  • assist the amplification of sensory stimuli via stochastic resonance
  • be an efficient signal carrier & accelerate the response to fast stimuli