Exercise Sheet 4: Scattering amplitudes in gauge theories
Discussion on Wednesday 29.05, NEW 15 2102, Prof. Dr. Jan Plefka

Exercise 2.1 – The 6-Gluon split helicity NMHV amplitude
Determine the first non-trivial next-to-maximally-helicity-violating (NMHV) amplitude $A_{\text{tree}}^6(1^+,2^+,3^+,4^-,5^-,6^-)$ from the BCFW recursion relation and our knowledge of the MHV amplitudes. In order to obtain comparable results it is advantageous to consider a shift of two helicity states $(\hat{1}^+ , \hat{6}^-)$.

Exercise 2.2 – The vanishing splitting function $\text{Split}_{\text{tree}}^+(z,a^+,b^+)$
Show by studying the factorization properties of the MHV-amplitude $A_{\text{tree}}^6(1^-,2^-,3^+,4^+,5^+,6^+)$ in the collinear limit $5||6$ that
\[ \text{Split}_{\text{tree}}^+(z,a^+,b^+) = 0. \]

Exercise 2.3 – A $\bar{q}gggg$ amplitude from collinear and soft limits
In exercise 1.4 we established the following color ordered $\bar{q}ggg$ amplitudes involving a massless quark and anti-quark using color-ordered Feynman rules:
\[
A_{\text{tree}}^4(1^-\bar{q},2^+q,3^+,4^+) = 0
\]
\[
A_{\text{tree}}^4(1^-\bar{q},2^+q,3^-,4^+) = i \frac{\langle 13 \rangle^3 \langle 23 \rangle}{\langle 12 \rangle \langle 23 \rangle \langle 34 \rangle \langle 41 \rangle}.
\]
Use these and the known splitting and soft factorization properties for gluonic legs from the lecture to derive the five-point single quark-line tree amplitude
\[ A_{\text{tree}}^5(1^-\bar{q},2^+q,3^-,4^+,5^+). \]
Check your result against all known factorization properties. Can you generalize the argument to determine the partial amplitudes
\[ A_{\text{tree}}^5(1^-\bar{q},2^+q,3^-,4^+,5^+) \text{ and } A_{\text{tree}}^n(1^-\bar{q},2^+q,3^-,4^+,...,n^+) ? \]