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TWO-LOOP RENORMALIZATION OF NON-SMOOTH STRING OPERATORS IN YANG-MILLS THEORY

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Abstract

The renormalization constants for string operators both with smooth and non-smooth non-self-intersecting open contour have been evaluated up to the order g^4.

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1. Introduction

The string operator approach to Yang-Mills theory has excited a continued interest up to the last years. Beside the formulation of pure Yang-Mills theory with the help of the basic object

$$U(x_1, x_2; C) = \mathcal{P} \exp\left[ig \int_C A_\mu(x) dx^\mu\right] \quad (1.1)$$

(the string operator) investigations of non-local gauge-invariant operators /1/,/2/ like

$$\bar{q}(x_1)\, U(x_1, x_2; C)\, q(x_2) \quad (1.2)$$

have been carried out, particularly aiming at the description of confinement in QCD. Studying string operators and related quantities different UV divergencies requiring renormalization arise. We restrict ourselves in the following to the renormalization of the string operator. Renormalizability of (1.1) has been proved by Aref'eva /3/ and Gervais/Neveu /4/. The general treatment of divergencies arising from non-smooth and self-intersecting contours in the case of loop functions

$$W(c_1, \ldots, c_n) = \langle 0 | T\; N^{-1} tr\, U(c_1) \ldots N^{-1} tr\, U(c_n) | 0 \rangle \quad (1.3)$$

is given in /5/, which has a tight connection to our problem. The auxiliary field technique for the description of the string operator has been proposed in this context by Aref'eva /3/ and Gervais/Neveu /4/. Aoyama made evaluations of the string operator renormalization constant up to the order g^4 in the case of an open smooth and non-self-intersecting path and gave his results in /6/. Two-loop results for rectangular Wilson loops (C ... rectangle)

$$W(c) = N^{-1}\, \mathrm{tr}\, U(c) \qquad (1.4)$$

have been published in /7/.

In this paper we not only present our results for the renormalization constant of the string operator up to the order g^4 in the case of an open smooth and non-self-intersecting contour but also the additional renormalization constant (up to the same order), which is necessary, if smoothness is given up. Our results agree with the renormalization group constraints, but differ from the renormalization constant derived by Aoyama /6/ in the smooth case. All evaluations have been carried out using dimensional regularization and Feynman gauge $\alpha = 1$. We have regarded the Euclidean region of the theory only.

Our presentation is the following. Chapter 2 contains a short review of the renormalization of string operators with non-self-intersecting and open path followed by the specialization up to the 2-loop level, showing the cancellation of subdivergencies and the way of the renormalization constant determination. Chapter 3 deals with the results in the smooth case, where part 1 quotes the 1-loop calculations and part 2 is devoted to the 2-loop contributions. In chapter 4 we evaluate the additional renormalization constant for non-smooth contours. Likewise in part 1 the 1-loop term is quoted and in part 2 we present our 2-loop calculations. In appendix A we give some abbreviations and plots of functions appearing in the renormalization constants. Finally appendix B deals with an integral representation, which is connected with

the treatment of the diagram containing the 3-gluon vertex.

2. Renormalization of string operators

The description of the string operator (1.1) with the help of one-dimensional fermions living on the contour is as follows. It is introduced an effective action /4/

$$S_{eff} = S_{YM} + i \int_0^1 \bar{z}(\sigma)\left(\frac{d}{d\sigma} - ig\, \dot{x}_\mu A^\mu\right) z(\sigma)\, d\sigma \qquad (2.1)$$

in such a manner, that the vacuum expectation value (V.E.V.) of (1.1) is given by

$$\langle 0 | U(c) | 0 \rangle = \langle 0 | \; z(1)\, \bar{z}(0) \, | 0 \rangle \qquad (2.2)$$

where the V.E.V. on the r.h.s. has to be taken with respect to the one-dimensional fermions and the Yang-Mills field. Note, that the theory described by S_{eff} is fully equivalent to the Yang-Mills theory (because all diagrams containing z-loops vanish identically). The renormalization of the string operator is achieved by a multiplicative renormalization of the Yang-Mills field A_μ, its gauge parameter α, the auxiliary field $z, \bar{z}$ and the coupling constant g. The renormalization constants of the auxiliary field denoted by Z_{1z} (vertex renormalization) and Z_{3z} (field renormalization $z = \sqrt{Z_{3z}}\, z_R$) are related by the Ward-Takahashi identity $Z_1/Z_3 = Z_{1z}/Z_{3z}$ to the renormalization constants of the Yang-Mills field /3/,/4/. Therefore the renormalized coupling constant of the Yang-Mills field and that of the z-A-vertex are equal. Using the Ward-Takahashi identity we write the renormalized string operator as follows

$$\langle 0|U_R(c,\alpha_R,g_R)|0\rangle = \frac{1}{Z_{3z}}\langle 0|U_0(c,Z_3\alpha_R,Z_{1z}Z_{3z}^{-1}Z_3^{-\frac{1}{2}}g_R)|0\rangle$$
$$= \frac{1}{Z_{3z}}\langle 0|P\exp[ig_R\frac{Z_{1z}}{Z_{3z}}\int_c A_R^\mu\, dx_\mu]|0\rangle \qquad (2.3)$$

(in the following we omit the index R). Formula (2.3) is correct for an open smooth and non-self-intersecting path only. If the contour has a break with the angle γ , we have to replace (2.3) by

$$\langle 0|U(c,\alpha,g)|0\rangle = \frac{1}{Z_{\bar{z}z}Z_{3z}}\langle 0|U_0(c,Z_3\alpha,Z_{1z}Z_{3z}^{-1}Z_3^{-\frac{1}{2}}g)|0\rangle \qquad (2.4)$$

where $Z_{\bar{z}z}=Z_{\bar{z}z}(\gamma)$ with $Z_{\bar{z}z}(0)=1$. The necessity to introduce this additional renormalization constant becomes clear, if we regard the path with a break as a non-smooth connection of two smooth parts /3/. Then a composite operator corresponds to this connection point, the renormalization of which leads in the non-smooth case to $Z_{\bar{z}z}\neq 1$. The generalization of this additional renormalization to a contour with some breaks is straight forward, every break has to be treated by its $Z_{\bar{z}z}$. It has been shown /3/,/4/, /8/,/9/, that there is not any dependence of the z-field renormalization constants on the global properties of the smooth and non-self-intersecting contour. Likewise $Z_{\bar{z}z}$ is a function of the break angle γ only /5/. There is no α -dependence, because $Z_{\bar{z}z}$ is a gauge-invariant quantity. This corresponds to the presence of $Z_{\bar{z}z}$ in the renormalization procedure of Wilson loops with cusps, which are gauge-invariant. Due to this fact $Z_{\bar{z}z}$ can be evaluated in a convenient gauge. For sake of convenience we chose Feynman gauge and smooth/non-smooth connected straight lines as contour in determining Z_{3z} and $Z_{\bar{z}z}$ respectively.

The V.E.V. of the renormalized smooth string operator (2.3) can now be expanded in the manner

$$\langle 0|U(c)|0\rangle = 1 + U^{(2)} + U^{(4)} + O(g^6) \qquad (2.5)$$

With respect to the r.h.s. of (2.3) this equals to

$$U^{(2)} = [\ \ - Z_{3z}^{(2)}] \qquad (2.6)$$
$$U^{(4)} = [\ \ - 2Z_{3z}^{(2)}\ \ + (Z_{3z}^{(2)})^2] + [\ \ - Z_{3z}^{(2)}\ \ ] + [\ \ + \ \ + 2Z_{1z}^{(2)}\ \ ] + [\ \ + Z_3^{(2)}(\ \ ^{\alpha=2} - 2\ \ )] - Z_{3z}^{(4)}$$
$$= [U_1^{(4)}] + [U_2^{(4)}] + [U_3^{(4)}] + [U_4^{(4)}] - Z_{3z}^{(4)} \qquad (2.7)$$

$$Z = 1 + Z^{(2)} + Z^{(4)} + O(g^6)$$

The cancellation of the subdivergencies is indicated by the brackets set correspondingly. Due to the reducibility of the diagram the divergent part of $U_1^{(4)}$ vanishes. The part $Z_3^{(2)}(\ \ ^{\alpha=2} - \ \ )$ in $U_4^{(4)}$ arises from the renormalization of the gauge parameter:

$$D_{\mu\nu}^{(0)ab}(x,\alpha(1+Z_3^{(2)}+O(g^4)))\Big|_{\alpha=1} = D_{\mu\nu}^{(0)ab}(x,1) + Z_3^{(2)}[D_{\mu\nu}^{(0)ab}(x,2) - D_{\mu\nu}^{(0)ab}(x,1)] + O(g^4).$$

The finiteness of (2.5) leads us to the requirements determining the Z_{3z} renormalization constant order by order:

$$Z_{3z}^{(2)} = pp[\ \ ] \qquad (2.8)$$
$$Z_{3z}^{(4)} = pp[U_1^{(4)} + U_2^{(4)} + U_3^{(4)} + U_4^{(4)}] \qquad (2.9)$$

With the help of the calculated terms of Z_{3z} we can then obtain the $Z_{\bar{z}z}$ terms in an analogous manner. So we get the 1-loop and 2-loop contributions by the equations

$$Z_{\bar{z}z}^{(2)} = pp[U^{(2)}] \qquad (2.10)$$
$$Z_{\bar{z}z}^{(4)} = pp[(Z_{\bar{z}z}^{(2)})^2 - Z_{\bar{z}z}^{(2)}U^{(2)} + U^{(4)}] \qquad (2.11)$$

whereas here of course $U^{(2)}$ and $U^{(4)}$ contain the corresponding to the smooth case diagrams with break in the path. Using dimensional regularization in the computations it has to be taken into account, that there only logarithmic divergencies show up. Therefore we have to have in mind, that all linear divergencies had been exponentiated out /8/.

3. Evaluation of the renormalization constant for smooth string operators

Now we are going to realize the programme for the determination of $Z_{3z}^{(2)}$ and $Z_{3z}^{(4)}$ given by (2.8), (2.9).

3.1. 1-loop level

The 1-loop contributions have been evaluated in α -gauge by Craigie/Dorn /1/. Computation of the diagram shown in figure 1

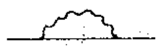

fig. 1

yields (for K s. appendix A)

$$C_F\left(\frac{g}{2\pi}\right)^2 K \frac{1}{2\varepsilon}\frac{1}{1-\varepsilon}\left[(3-\alpha)-(1-\alpha)\varepsilon\right] \qquad (3.1)$$

With (2.8) immediatly follows

$$Z_{3z}^{(2)} = C_F\left(\frac{g}{2\pi}\right)^2 \frac{3-\alpha}{2}\,\frac{1}{\varepsilon} \qquad (3.2)$$

$Z_{1z}^{(2)}$ is given by

$$Z_{1z}^{(2)} = \left(\frac{g}{2\pi}\right)^2 \frac{1}{2\varepsilon}\left[C_F(3-\alpha)-(3+\alpha)\frac{C_A}{4}\right] \qquad (3.3)$$

3.2. 2-loop level

First we evaluate all 2-loop diagrams (s. figure 2).

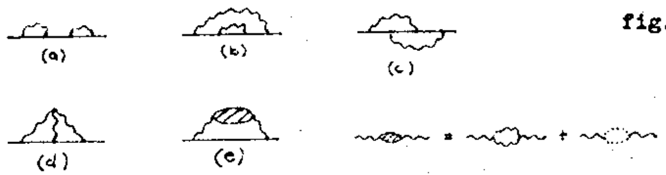

fig. 2

$$(a) = C_F^2\left(\frac{g}{2\pi}\right)^4 K^2\,\frac{1}{\varepsilon}\,\frac{1}{(1-\varepsilon)^2}\,B(1+\varepsilon,\varepsilon) \qquad (3.4)$$

$$(b) = C_F^2\left(\frac{g}{2\pi}\right)^4 K^2\,\frac{1}{2\varepsilon}\,\frac{1}{(1-\varepsilon)^2}\left(\frac{1}{\varepsilon}+\frac{1}{1-2\varepsilon}\right) \qquad (3.5)$$

$$(c) = C_F\left(C_F-\frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4 K^2\,\frac{1}{\varepsilon}\,\frac{1}{1-\varepsilon}\left(\frac{1}{2\varepsilon}\frac{1}{1-\varepsilon}+\right.$$
$$\left.-\frac{B(1+\varepsilon,\varepsilon)}{1-\varepsilon}-\frac{1}{2\varepsilon}\,\frac{1}{1-2\varepsilon}\right) \qquad (3.6)$$

Due to the structure of the 3-gluon vertex the diagram (d) vanishes in the case of a straight line contour (cf. (4.13)). With the help of the first gluon propagator correction (A.2) the diagram (e) is easy to evaluate.

$$(e) = C_A C_F\left(\frac{g}{2\pi}\right)^4 K^2\,\frac{d_1+d_2}{8(1-2\varepsilon)}\,\frac{10-3\varepsilon}{3-\varepsilon}\,\frac{1}{\varepsilon^2(2+\varepsilon)} \qquad (3.7)$$

We collect the obtained results and determine pp$[U_1^{(4)}]$, pp$[U_2^{(4)}]$, pp$[U_3^{(4)}]$, pp$[U_4^{(4)}]$ (cf. (2.7)). As we have mentioned in chapter 2 the divergent part of $U_1^{(4)}$ is equal to zero, the others are given by

$$pp[U_2^{(4)}] = C_F^2 \left(\frac{g}{2\pi}\right)^4\left(-\frac{1}{2}\frac{1}{\varepsilon^2}+\frac{1}{2\varepsilon}\right) \qquad (3.8)$$

$$pp[U_3^{(4)}] = C_F\left(C_F-\frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4\left(\frac{1}{\varepsilon^2}-\frac{1}{2\varepsilon}\right) \qquad (3.9)$$

$$pp[U_4^{(4)}] = C_F C_A \left(\frac{g}{2\pi}\right)^4\left(-\frac{5}{8}\frac{1}{\varepsilon^2}+\frac{13}{24}\frac{1}{\varepsilon}\right) \qquad (3.10)$$

According to (2.9) we are now able to determine $Z_{3z}^{(4)}$.

$$Z_{3z}^{(4)} = \left(\frac{g}{2\pi}\right)^4\left[C_F^2\frac{1}{2}\frac{1}{\varepsilon^2}+C_A C_F\left(-\frac{9}{8}\frac{1}{\varepsilon^2}+\frac{19}{24}\frac{1}{\varepsilon}\right)\right] \qquad (3.11)$$

The final result for Z_{3z} is

$$Z_{3z} = 1 + \left(\frac{g}{2\pi}\right)^2\frac{C_F}{\varepsilon} +$$
$$+\left(\frac{g}{2\pi}\right)^4\left[C_F^2\frac{1}{2}\frac{1}{\varepsilon^2}+C_A C_F\left(-\frac{9}{8}\frac{1}{\varepsilon^2}+\frac{19}{24}\frac{1}{\varepsilon}\right)\right]+O(g^6) \qquad (3.12)$$

With the help of the Ward-Takahashi identity /10/,/11/

$$\frac{Z_{1F}}{Z_{3F}} = \frac{Z_1}{Z_3} = \frac{Z_{1z}}{Z_{3z}} =$$
$$= 1-\left(\frac{g}{2\pi}\right)^2\frac{C_A}{2\varepsilon}+\left(\frac{g}{2\pi}\right)^4 C_A^2\left[\frac{17}{32}\frac{1}{\varepsilon^2}-\frac{67}{384}\frac{1}{\varepsilon}\right]+O(g^6) \qquad (3.13)$$

and the result for Z_{3z} (3.12) we obtain Z_{1z}.

$$Z_{1z} = 1 + \left(\frac{g}{2\pi}\right)^2\left(C_F - \frac{C_A}{2}\right)\frac{1}{\varepsilon} + \left(\frac{g}{2\pi}\right)^4\left[\frac{C_F^2}{2\varepsilon^2} - \frac{C_A C_F}{8}\left(\frac{13}{\varepsilon^2} - \frac{19}{3\varepsilon}\right) + C_A^2\left(\frac{17}{32}\frac{1}{\varepsilon^2} - \frac{67}{384}\frac{1}{\varepsilon}\right)\right] + O(g^6) \qquad (3.14)$$

We see from (3.12) and (3.14), that there is no term arising from the ε-expansion of the factor K. All these terms have been compensated by subdivergency subtraction terms. This agrees with the general property, that the Z-factors cannot depend on the contour length and therewith on K.

As is well known by an analysis of the anomalous dimension of an operator relations between some expansion coefficients of the corresponding renormalization constant can be found (cf. e.g. /11/). Therefore, if the terms of one order are known, it is possible to calculate the leading divergencies of the next order. So the contribution proportional to ε^{-1} always contains the essential information. Writing

$$Z_{iz} = 1 + g^2\frac{a_i}{\varepsilon} + g^4\left(\frac{b_i}{\varepsilon^2} + \frac{c_i}{\varepsilon}\right) + O(g^6)$$

$$\beta = \frac{dg^2}{d\ln\mu^2} = \beta_1 g^4 + O(g^6) \qquad \beta_1 = -\frac{11}{12}\frac{C_A}{(2\pi)^2}$$

$$\delta = \frac{d\ln\alpha}{d\ln\mu^2} = \delta_1 g^2 + O(g^4) \qquad \delta_1 = \frac{5}{12}\frac{C_A}{(2\pi)^2}$$

the constraint for the coefficient b_i reads

$$b_i = \beta_1 a_i + \frac{a_i^2}{2} + \delta_1 \left.\frac{da_i}{d\alpha}\right|_{\alpha=1} \qquad i = 1, 3 \qquad (3.15)$$

It can be easily checked, that our result (3.12) agrees with the renormalization group constraint. Aoyama /6/ has done the calculation both of Z_{1z} and Z_{3z}, but his results

do not agree with the RG considerations. Moreover, the quotient of his Z_{1z} and Z_{3z} does not fulfil the Ward-Takahashi identitiy (3.13). Therefore we regard our results as the more trustworthy ones.

4. Evaluation of the renormalization constant for non-smooth string operators

The knowledge of Z_{3z} up to g^4 permits now to evaluate the additional factor $Z_{\mathbb{Z}z}$ for the renormalization of (1.1) in the case of a non-smooth path (for some abbreviations s. appendix A).

4.1. 1-loop level

We have to evaluate the diagrams shown in figure 3. We will do this in α-gauge again, because we will need these expressions both in Feynman and $\alpha = 2$ gauge later at the 2-loop level.

fig.3

We start with (a) and (c). The procedure and the result are very analogous to the smooth case (cf. (3.1)).

$$(a) + (c) = C_F\left(\frac{g}{2\pi}\right)^2 K\left[\frac{3-\alpha}{\varepsilon} + 2 + \ln\sigma(1-\sigma)\right] + O(\varepsilon) \qquad (4.1)$$

Then we turn our attention to the remaining diagram (b).

$$(b) = -C_F g^2\mu^\varepsilon \int_\sigma^1 d\sigma_1 \int_0^\sigma d\sigma_2 \, \dot{x}_\mu(\sigma_1) \, D^{(0)}_{\mu\nu}(x(\sigma_1) - x(\sigma_2), \alpha) \, \dot{x}_\nu(\sigma_2)$$

Having introduced polar coordinates we can carry out the r-integration and after some computations we arrive at

$$(b) = -C_F\left(\frac{g}{2\pi}\right)^2\frac{K}{2}\Big[\big((1-\alpha)+2\vartheta\cot\vartheta\big)\frac{1}{\varepsilon}+(3-\alpha)(A+\tilde{A})+ + (1-\alpha)\big(2G-\tfrac{1}{2}(1+\vartheta\cot\vartheta)\big)\Big] + O(\varepsilon) \qquad (4.2)$$

Taking into account (2.10) and (3.2) we obtain now $Z^{(2)}_{\bar{z}z}$, which is given by

$$Z^{(2)}_{\bar{z}z} = C_F\left(\frac{g}{2\pi}\right)^2\frac{1-\vartheta\cot\vartheta}{\varepsilon} \qquad (4.3)$$

4.2. 2-loop level

First we evaluate the diagrams shown in figure 4 separately. Only the crucial steps of the calculations are given.

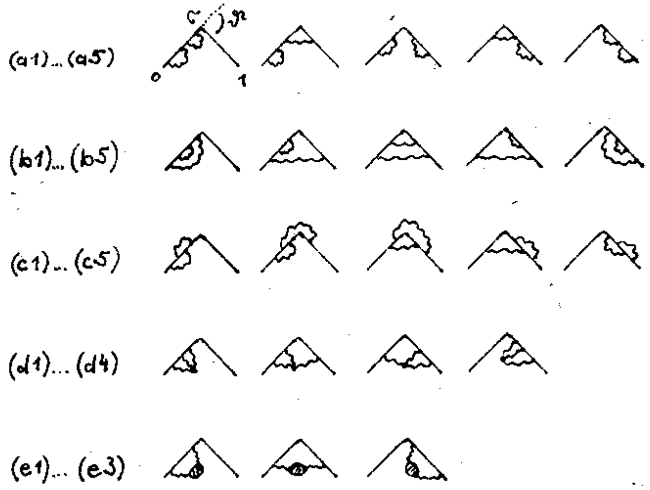

fig.4

The diagrams (a1), (a3) and (a5) each contribute a term, which partially can be read from (3.4).

$$(a1)+(a3)+(a5) = 3C_F^2\left(\frac{g}{2\pi}\right)^4K^2\left[\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\big(2+\ln\sigma(1-\sigma)\big)\right] + \text{reg.} \qquad (4.4)$$

It is more difficult to evaluate (a2) and (a4).

$$(a2) = C_F^2\left(\frac{g}{2\pi}\right)^4K^2\int_\sigma^1 d\sigma_1\int_0^\sigma d\sigma_2\; D(\sigma_1-\sigma,\sigma-\sigma_2)\int_0^{\sigma_2} d\sigma_3\int_0^{\sigma_4} d\sigma_4\,(\sigma_3-\sigma_4)^{\varepsilon-2}$$

(for D(s,t) arising from the gluon propagator connecting both legs of (a2) s. (A.6) in appendix A)

The σ_3,σ_4-integrations are quickly done. Then we introduce polar coordinates and perform the r-integration after having made an admissible expansion in ε for one term. An analogous treatment can be done for (a4). So we reach at

$$(a2)+(a4) = -2C_F^2\left(\frac{g}{2\pi}\right)^4K^2\left[\frac{\vartheta\cot\vartheta}{\varepsilon^2} + \frac{\vartheta\cot\vartheta}{\varepsilon}\left(1+\frac{\ln\sigma(1-\sigma)}{2}\right)+\frac{A+\tilde{A}}{\varepsilon}\right] + \text{reg.} \qquad (4.5)$$

The diagrams (b1) and (b5) can be read from (3.5).

$$(b1)+(b5) = C_F^2\left(\frac{g}{2\pi}\right)^4K^2\left[\frac{1}{\varepsilon^2}+\frac{1}{\varepsilon}\big(3+\ln\sigma(1-\sigma)\big)\right] + \text{reg.} \qquad (4.6)$$

(b2) and (b4) have been treated with the help of polar coordinates again. Due to pp [(b2)] = pp [(b4)] it can be written

$$(b2)+(b4) = -C_F^2\left(\frac{g}{2\pi}\right)^4K^2\left[\frac{\vartheta\cot\vartheta}{\varepsilon^2}+\frac{\vartheta\cot\vartheta}{\varepsilon} + \frac{1}{\varepsilon}(A+B+2\tilde{A})\right] + \text{reg.} \qquad (4.7)$$

Finally (b3) causes some trouble.

$$(b3) = C_F^2\left(\frac{g}{2\pi}\right)^4K^2\int_\sigma^1 d\sigma_1\int_\sigma^{\sigma_1} d\sigma_2\int_0^\sigma d\sigma_3\int_0^{\sigma_3} d\sigma_4\; D(\sigma_1-\sigma,\sigma-\sigma_4)\; D(\sigma_2-\sigma,\sigma-\sigma_3)$$

We have to evaluate an integral of the type

$$J = \int_0^{1-\sigma} ds\int_0^\sigma dt\; D(s,t)\left[\int_0^s du\int_0^t dv\; D(u,v)\right]$$

First we introduce polar coordinates into $\left[\int_0^s du\int_0^t dv\, D(u,v)\right]$ and then collect all contributions, we are interested in to determine the divergent part of the diagram. This procedure we also use to perform the last two integrations. After tedious computations we arrive at

$$J = \frac{(\vartheta\cot\vartheta)^2}{2\varepsilon^2} + \frac{\vartheta\cot\vartheta}{\varepsilon}\,(A+\tilde{A}) + \frac{2E}{\varepsilon} + \text{reg.} \tag{4.8}$$

(b3) is then given by

$$(b3) = C_F^2\left(\frac{g}{2\pi}\right)^4 K^2\left[\frac{(\vartheta\cot\vartheta)^2}{2\varepsilon^2} + \frac{\vartheta\cot\vartheta}{\varepsilon}(A+\tilde{A}) + \frac{2E}{\varepsilon}\right] + \text{reg.} \tag{4.9}$$

The straight line result (3.6) can be modified instantaneously for (c1) and (c5).

$$(c1)+(c5) = -C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4 K^2\left[\frac{2}{\varepsilon^2} + \frac{1}{\varepsilon}\left(5 + 2\ln\sigma(1-\sigma)\right)\right] + \text{reg.} \tag{4.10}$$

(c2) and (c4) contain types of integrals, which we have evaluated already in (a2) and (b2), therefore the same procedure can be used. Due to pp [(c2)] = pp [(c4)] we finally obtain

$$(c2)+(c4) = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4 K^2\left[\frac{\vartheta\cot\vartheta}{\varepsilon^2} + \frac{\vartheta\cot\vartheta}{\varepsilon} + \frac{1}{\varepsilon}\left(A + B + 2\tilde{A}\right)\right] + \text{reg.} \tag{4.11}$$

At last (c3) has to be computed.

$$(c3) = C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4 K^2 \int_\sigma^1 d\sigma_1 \int_\sigma^{\sigma_1} d\sigma_2 \int_0^\sigma d\sigma_3 \int_0^{\sigma_3} d\sigma_4\; D(\sigma_1-\sigma, \sigma-\sigma_3)\, D(\sigma_2-\sigma, \sigma-\sigma_4)$$

Using the results obtained for the integral J (4.8) the task is quickly done.

$$(c3) = -C_F\left(C_F - \frac{C_A}{2}\right)\left(\frac{g}{2\pi}\right)^4 K^2\,\frac{2E}{\varepsilon} + \text{reg.} \tag{4.12}$$

Now we are going to treat all diagrams (d1) - (d4).

$$y_i = y(\sigma_i) \qquad i = 1,2,3$$

$$(d) = (d1)+(d2)+(d3)+(d4) = C_A C_F\, g^4 \frac{\mu^{2\varepsilon}}{2}\left[\frac{\Gamma(1-\frac{\varepsilon}{2})}{4\pi^{2-\varepsilon/2}}\right]^3 \cdot$$
$$\cdot \int_0^1 d\sigma_3 \int_0^{\sigma_3} d\sigma_2 \int_0^{\sigma_2} d\sigma_1 \Big[(\dot{y}_{1\mu}\dot{y}_{2\mu})\,\dot{y}_{3\lambda}(\nabla_2-\nabla_1)_\lambda + (\dot{y}_{2\mu}\dot{y}_{3\mu})\,\dot{y}_{1\lambda}(\nabla_3-\nabla_2)_\lambda +$$
$$+ (\dot{y}_{3\mu}\dot{y}_{1\mu})\,\dot{y}_{2\lambda}(\nabla_1-\nabla_3)_\lambda\Big] \cdot \int d^{4-\varepsilon}y\, \frac{1}{[(y-y_1)^2]^{1-\varepsilon/2}[(y-y_2)^2]^{1-\varepsilon/2}[(y-y_3)^2]^{1-\varepsilon/2}}$$

The derivatives of the vertex structure have been given here in terms of contour point derivatives in order to simplify the n=(4-ε)-dimensional integral, the representation of which is given in (B.6'). Due to the use of two straight lines in the 4-dimensional space the vectors $\dot{y}_1, \dot{y}_2, \dot{y}_3, \nabla_1, \nabla_2$ and ∇_3 all lie in a 3-dimensional subspace. Then we find with the help of the Lagrange identity (A.4) a more convenient representation for the vertex structure. So we reach at the following result

$$(d) = C_A C_F\, g^4 \frac{\mu^{2\varepsilon}}{4\varepsilon}\left[\frac{\Gamma(1-\frac{\varepsilon}{2})}{4\pi^{2-\varepsilon/2}}\right]^2 \cdot$$
$$\cdot \Big[\int_\sigma^1 d\sigma_3 \int_\sigma^{\sigma_3} d\sigma_2 \int_0^{\sigma} d\sigma_1 \big((\dot{y}_1\times\dot{y}_3)_\mu(\dot{y}_2\times\nabla_2)_\mu + (\dot{y}_2\times\dot{y}_1)_\mu(\dot{y}_3\times\nabla_3)_\mu\big) +$$
$$+ \int_\sigma^1 d\sigma_3 \int_0^{\sigma} d\sigma_2 \int_0^{\sigma_2} d\sigma_1 \big((\dot{y}_1\times\dot{y}_3)_\mu(\dot{y}_2\times\nabla_2)_\mu + (\dot{y}_3\times\dot{y}_2)_\mu(\dot{y}_1\times\nabla_1)_\mu\big)\Big] \cdot$$
$$\cdot \int_0^1 dt\, \frac{1}{[((y_2-y_1)t-(y_3-y_2))^2]^{1-\varepsilon/2}}\left[\frac{[(y_3-y_1)^2]^{\varepsilon/2}}{t^{\varepsilon}} - [(y_2-y_1)^2]^{\varepsilon/2}\right] \tag{4.13}$$

The appearence of the vector product lets us understand the vanishing of the diagram (cf. chapter 3.2.) in the straight line case. We see, that (4.13) yields (d1)=(d4)=0.

Using the permutation properties of (B.6'), a reparametrization $\sigma_i' = 1-\sigma_i$ (i= 1,2,3) for (d3) and an explicit parameter representation for the contour (we locate the break of the contour in the origin of coordinates of our space) we arrive at a more compact formula, in which we substitute $\sigma_1, \sigma_2, \sigma_3$ by spherical coordinates. The r-integration is performed immediately. The remaining integrations can be shown to give finite results for the ε-region $0 < \mathrm{Re}\,\varepsilon < 1$. Performing the ε-expansion we reach at

$$(d) = C_A C_F \left(\frac{g}{2\pi}\right)^4 K^2 \, \frac{\overline{F}}{\varepsilon} + reg. \tag{4.14}$$

The diagrams (e1) and (e3) we read from (3.7).

$$(e1)+(e3) = C_A C_F \left(\frac{g}{2\pi}\right)^4 K^2 \left[\frac{5}{4}\,\frac{1}{\varepsilon^2} + \right.$$
$$\left. + \frac{1}{\varepsilon}\left(\frac{11}{4} + \frac{5}{4}\ln\sigma(1-\sigma)\right)\right] + reg. \tag{4.15}$$

Without any complication (e2) is evaluated using the first gluon propagator correction (A.2) and polar coordinates.

$$(e2) = -C_A C_F \left(\frac{g}{2\pi}\right)^4 K^2 \frac{5}{24}\left[\frac{1+2\vartheta\cot\vartheta}{\varepsilon^2} + \right.$$
$$\left. + \frac{1}{\varepsilon}\left(\frac{1-8(1-2\vartheta\cot\vartheta)}{15} + 6(A+\tilde{A}) + 4G\right)\right] + reg. \tag{4.16}$$

Now we are able to collect our results and to determine $Z_{\bar{Z}Z}^{(4)}$. This is done according to (2.9). Our final result is

$$Z_{\bar{Z}Z}^{(4)} = \left(\frac{g}{2\pi}\right)^4 \left[C_F^2 \frac{(1-\vartheta\cot\vartheta)^2}{\varepsilon^2} - C_A C_F\left(\frac{11}{12}\,\frac{(1-\vartheta\cot\vartheta)}{\varepsilon^2} + \right.\right.$$
$$\left.\left. + \frac{1}{2\varepsilon}\left(\frac{1}{2} - A + B - 2(E+F) - \frac{67}{36}(1-\vartheta\cot\vartheta)\right)\right)\right] \tag{4.17}$$

$$Z_{\bar{Z}Z} = 1 + \left(\frac{g}{2\pi}\right)^2 C_F \frac{(1-\vartheta\cot\vartheta)}{\varepsilon} + \left(\frac{g}{2\pi}\right)^4 \left[C_F^2 \frac{(1-\vartheta\cot\vartheta)^2}{\varepsilon^2} + \right.$$
$$\left. - C_A C_F\left(\frac{11}{12}\,\frac{(1-\vartheta\cot\vartheta)}{\varepsilon^2} + \frac{1}{2\varepsilon}\left(\frac{1}{2} - A + B - 2(E+F) - \frac{67}{36}(1-\vartheta\cot\vartheta)\right)\right)\right]$$
$$+ O(g^6) \tag{4.18}$$

The renormalization constant $Z_{\bar{Z}Z}$ has been proved to agree with the RG constraint (cf. (3.15)). For physical reasons we expect $Z_{\bar{Z}Z}(0)=1$. Indeed, the results of a numerical evaluation of the functions A, B, E and F (s. appendix A) confirm this expectation.

Our diagram results agree in the non-smooth case $\vartheta = \frac{\pi}{2}$ with that obtained in /7/ (Except the diagram (e), where in /7/ an incorrect gluon propagator correction has been used, and a numerical factor of about 0.5 in diagram (d); the cause of the latter difference we do not know.).

Finally we can write down the anomalous dimension of $Z_{\bar{Z}Z}$:

$$\gamma_{\bar{Z}Z} = -\left(\frac{g}{2\pi}\right)^2 C_F(1-\vartheta\cot\vartheta) + \left(\frac{g}{2\pi}\right)^4 C_F C_A\left[\frac{1}{2} - A + B + \right.$$
$$\left. - 2(E+F) - \frac{67}{36}(1-\vartheta\cot\vartheta)\right] + O(g^6) \tag{4.19}$$

Obviously (4.19) diverges for $\vartheta \to \pi$.

The short distance behaviour (including 2-loop corrections) of the Wilson loop is then given (for plots s. appendix A):

$$W\left(\frac{1}{\lambda}c, g^2\right)^{\lambda\to\infty} = W(c, \bar{g}^2(\lambda)) \left[\frac{\bar{g}^2(\lambda)}{g^2}\right]^{\frac{6}{11}\frac{C_F}{C_A}(1-\vartheta\cot\vartheta)} \cdot$$
$$\cdot \exp\left[\frac{6}{11}C_F\left(g^2-\bar{g}^2(\lambda)\right)\left(\frac{1}{2} - A + B - 2(E+F) - \frac{146}{99}(1-\vartheta\cot\vartheta)\right)\right]$$
$$\frac{d\bar{g}^2(\lambda)}{d\ln\lambda^2} = \beta(\bar{g}) \tag{4.20}$$

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Appendix A

$$\mathcal{D}^{ab}_{\mu\nu} = \mathcal{D}^{(0)ab}_{\mu\nu} + \mathcal{D}^{(2)ab}_{\mu\nu} + O(g^4) \qquad \varepsilon = (4-n)$$

(A.1) $$\mathcal{D}^{(0)ab}_{\mu\nu}(x,\alpha) = \delta^{ab}\,\mathcal{D}^{(0)}_{\mu\nu}(x,\alpha)$$

$$\mathcal{D}^{(0)}_{\mu\nu}(x,\alpha) = \frac{\Gamma(1-\frac{\varepsilon}{2})}{8\pi^{2-\varepsilon/2}}\left[(1+\alpha)\frac{\delta_{\mu\nu}}{(x^2)^{1-\varepsilon/2}} + (1-\alpha)(2-\varepsilon)\frac{x_\mu \cdot x_\nu}{(x^2)^{2-\varepsilon/2}}\right]$$

(A.2) $$\mathcal{D}^{(2)ab}_{\mu\nu}(x,1) = \delta^{ab}\,\mathcal{D}^{(2)}_{\mu\nu}(x,1)$$

$$\mathcal{D}^{(2)}_{\mu\nu}(x,1) = d\left[d_1\frac{\delta^{\mu\nu}}{(x^2)^{1-\varepsilon}} + d_2\frac{x_\mu \cdot x_\nu}{(x^2)^{2-\varepsilon}}\right] \;;\; d_1 = \frac{1+\varepsilon}{1-\varepsilon} \;;\; d_2 = 2\;;$$

$$d = \left(\frac{g}{2\pi}\right)^2 C_A\left[\Gamma(1-\tfrac{\varepsilon}{2})\right]^2 \pi^{\varepsilon-2}\frac{\mu^\varepsilon}{16}\,\frac{10-3\varepsilon}{3-\varepsilon}\,\frac{1}{2+\varepsilon}\,\frac{1}{\varepsilon}$$

(A.3) $$C_F\delta_{mn} = (T_a T_a)_{mn} \quad ; \quad C_A\delta_{ab} = f_{acd}f_{bcd}$$

$T_a \,/\, f_{abc}$ **generators/structure constants of the group respectively**

(A.4) **Lagrange identity**

$$(\vec{a}\times\vec{b})\cdot(\vec{c}\times\vec{d}) = (\vec{a}\cdot\vec{c})(\vec{b}\cdot\vec{d}) - (\vec{b}\cdot\vec{c})(\vec{a}\cdot\vec{d})$$

(A.5) $$K = \mu^\varepsilon L^\varepsilon \pi^{\varepsilon/2}\,\Gamma(1-\tfrac{\varepsilon}{2})$$

L **... length of the contour**

(A.6) $$\mathcal{D}(s,t) = \frac{\cos\vartheta}{(s^2+t^2+2st\cos\vartheta)^{1-\varepsilon/2}}$$

and in polar coordinates

$$\mathcal{D}(r\cos\varphi, r\sin\varphi) = r^{\varepsilon-2}\,\frac{\cos\vartheta}{N(\varphi)^{1-\varepsilon/2}}$$

$$N(\varphi) = (1+\cos\vartheta\sin 2\varphi)$$

(A.7) $$A = A(\vartheta) = \cos\vartheta\int_0^{\frac{\pi}{4}}\frac{\ln N(\varphi)}{N(\varphi)}\,d\varphi$$

(A.8) $$\mathcal{B} = \mathcal{B}(\vartheta) = \cos\vartheta\int_0^{\pi/2}\frac{\ln\cos\varphi}{N(\varphi)}\,d\varphi$$

(A.9) $$E = E(\vartheta) = \cos^2\vartheta\int_0^{\pi/4}d\psi\,\frac{1}{N(\psi)}\left[\int_0^{\psi}d\varphi\,\frac{\ln\tan\varphi}{N(\varphi)}\right]$$

(A.10) $$F = F(\vartheta) = \frac{\sin^2\vartheta}{4}\int_{1/\sqrt{2}}^{1}dz\,\frac{1}{\sqrt{1-z^2}}\int_0^1 dx\,x\int_0^1 dt\,t$$

$$\left\{\frac{\ln\left[\frac{(x\cos\vartheta+\sqrt{1-x^2}\,z)^2+(x\sin\vartheta)^2}{t^2(1-x^2)(z-\sqrt{1-z^2})^2}\right]}{\left[(\sqrt{1-x^2}(\sqrt{1-z^2}-z)t + x\cos\vartheta + z\sqrt{1-x^2})^2+(x\sin\vartheta)^2\right]^2}\right.$$

$$\left.-\frac{\ln\left[\frac{(x\cos\vartheta+\sqrt{1-x^2}\sqrt{1-z^2})^2+(x\sin\vartheta)^2}{t^2(1-x^2)(z-\sqrt{1-z^2})^2}\right]}{\left[(\sqrt{1-x^2}(z-\sqrt{1-z^2})t + x\cos\vartheta + \sqrt{1-z^2}\sqrt{1-x^2})^2+(x\sin\vartheta)^2\right]^2}\right\}$$

$\tilde{A}, G$ **... some finite integrals**

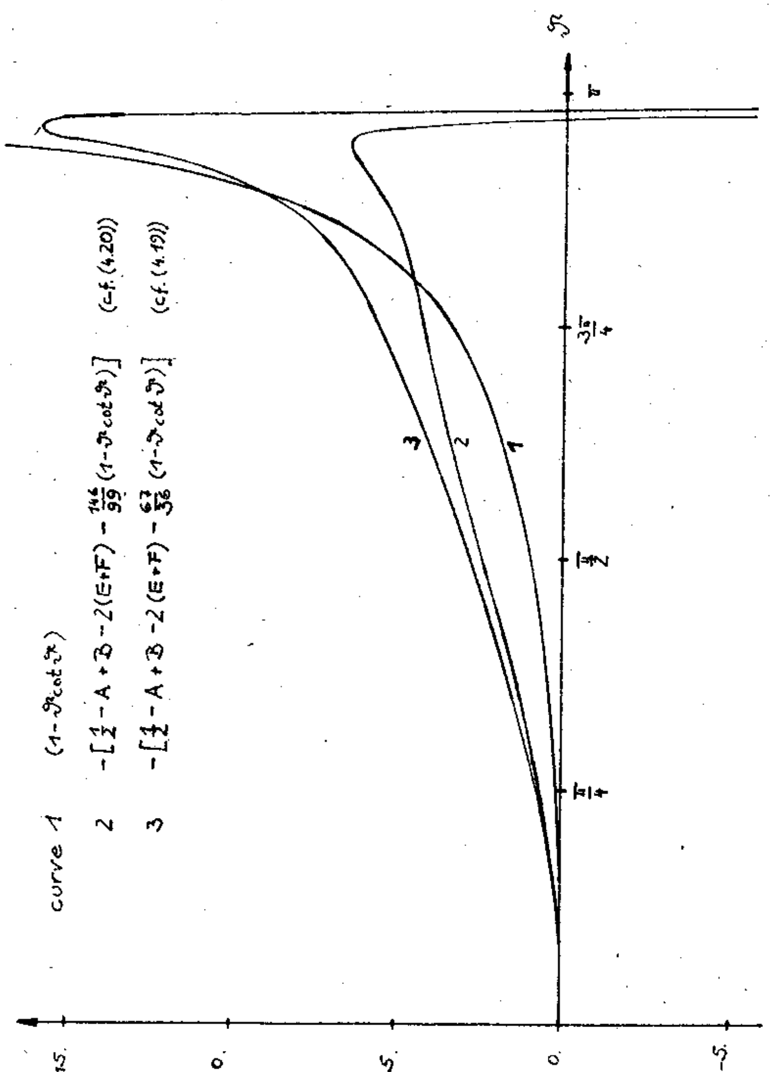

fig.5

Appendix B

In the following we want to derive a representation of the integral

$$I = \int d^n y \, \frac{1}{[(y-y_1)^2]^{n/2-1}[(y-y_2)^2]^{n/2-1}[(y-y_3)^2]^{n/2-1}} \qquad \text{(B.1)}$$

$$y_1, y_2, y_3 \in \mathcal{R}^n$$

which occurs in the evaluation of the diagram containing the 3-gluon vertex. First we find two properties of I. Operating with the n-dimensional Laplacian with respect to y_1 on I we obtain /12/:

$$\Delta_1 I = -\frac{c_n}{[(y_2-y_1)^2]^{n/2-1}[(y_3-y_1)^2]^{n/2-1}} \quad ; \quad c_n = \frac{4\pi^{n/2}}{\Gamma(\frac{n}{2}-1)} \qquad \text{(B.2)}$$

Such a relation can also be obtained with respect to y_2 and y_3. If we choose e.g. $y_2 = y_3$ in (B.1), it can be easily solved. With the help of

$$\frac{1}{[(y-y_2)^2]^{n-2}} = \Delta_2 \left[\frac{1}{2(3-n)(4-n)} \, \frac{1}{[(y-y_2)^2]^{n-3}} \right]$$

and Green's formula of the second kind

$$\int d^n y \, v(y) \Delta u(y) = \int d^n y \, u(y) \Delta v(y)$$

(the surface term vanishes for $n > 3$, this relation we have to respect in every case to prevent IR divergencies for I) we obtain

$$I = -\frac{c_n}{2(3-n)(4-n)} \, \frac{1}{[(y_2-y_1)^2]^{n-3}} \qquad \text{(B.3)}$$

So we see, that the UV divergencies of I originate from the point $y_1 = y_2 = y_3$. Now we start the general treatment of I (for some aspects cf. /13/). First we substitute y by $y+y_1$ having at this point an appearently free choice of the shift vector y_i (we take i=1). Using the expansion of $\frac{1}{[(y-c)^2]^{n/2-1}}$ in terms of Gegenbauer's polynomials

$$\frac{1}{[(y-c)^2]^{n/2-1}} = \begin{cases} \frac{1}{|c|^{n-2}} \sum_{l=0}^{\infty} C_l^{n/2-1}(\hat{y}\hat{c}) \left[\frac{|y|}{|c|}\right]^l ; & |y|<|c| \quad (*) \\ \frac{1}{|y|^{n-2}} \sum_{l=0}^{\infty} C_l^{n/2-1}(\hat{y}\hat{c}) \left[\frac{|c|}{|y|}\right]^l ; & |y|>|c| \quad (*) \end{cases} \tag{B.4}$$

$\hat{y}, \hat{c}$ unit vectors to y, c

and introducing hyperspherical coordinates we can perform the r- and Ω_n -integrations separately. In these calculations we always have to respect the inequalities ($*$) for the Gegenbauer's polynomial expansion. During the procedure we choose $|y_2-y_1|=|a|<|b|=|y_3-y_1|$ ($**$). The Ω_n -integration can be done with the help of /14/:

$$\int d\Omega_n \, C_l^{n/2-1}(\hat{y}\hat{a}) \cdot C_k^{n/2-1}(\hat{y}\hat{b}) = \delta_{lk} \frac{2\pi^{\frac{n}{2}}}{\Gamma(\frac{n}{2})} \frac{C_k^{n/2-1}(\hat{a}\hat{b})}{1+\frac{k}{n-2}} \tag{B.5}$$

After tedious computations we arrive at the following representation of I

$$I = \frac{C_n}{2(4-n)} \int_0^1 dt \frac{1}{[(at-b)^2]^{n/2-1}} \left[\frac{|b|^{4-n}}{t^{4-n}} - |a|^{4-n}\right] \tag{B.6}$$

Having used the assumption $|a|<|b|$ ($**$) to derive (B.6) now the following question arises. What is the range of validity of our representation, if the assumption ($**$) is given up (we are not interested in limitations for y_1, y_2, y_3)? Regarding the continuation of (B.6) to the unconstrained case $|a| \lesseqgtr |b|$ it can be seen, that restricting Re n to $3<\mathrm{Re}\, n<4$ (B.6) is a well defined representation

of I, from which (B.2), (B.3) can be derived again. Under the condition $3<\mathrm{Re}\, n<4$ we can optionally permute the indices i = 1,2,3 in (B.6), this is a very useful property in further diagram evaluations. After these considerations we reach at our final representation formula

$$I = \frac{2\pi^{2-\varepsilon/2}}{\varepsilon\Gamma(1-\frac{\varepsilon}{2})} \int_0^1 dt \frac{1}{[((y_{i_2}-y_{i_1})t-(y_{i_3}-y_{i_1}))^2]^{1-\varepsilon/2}} \cdot \left[\frac{[(y_{i_3}-y_{i_1})^2]^{\varepsilon/2}}{t^{\varepsilon}} - [(y_{i_2}-y_{i_1})^2]^{\varepsilon/2}\right] \tag{B.6'}$$

valid for all permutations $(i_1,i_2,i_3)=P(1,2,3)$,

$\varepsilon=(4-n)$; $0<\mathrm{Re}\,\varepsilon<1$; $y_1,y_2,y_3 \in \mathbb{R}^n$

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