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RADIATIVE CORRECTIONS TO THE CASIMIR EFFECT AT FINITE TEMPERATURE – REAL TIME FORMALISM

K. Scharnhorst, D. Robaschik and E. Wieczorek

INSTITUT FÜR HOCHENERGIEPHYSIK
AKADEMIE DER WISSENSCHAFTEN DER DDR
BERLIN-ZEUTHEN · DDR

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K. Scharnhorst
Sektion Physik der Humboldt-Universität zu Berlin, GDR

D. Robaschik
Sektion Physik der Karl-Marx-Universität Leipzig, GDR

E. Wieczorek
Institut für Hochenergiephysik der Akademie der Wissenschaften der DDR Berlin - Zeuthen, GDR

Abstract

The functional integral approach to real time quantum field theory at finite temperature is reconsidered to include boundary conditions for the electromagnetic field strength tensor on ideal conducting plates. The expression for the generating functional for real time thermodynamic Green's functions is worked out. Thereby a closed expression is obtained for the photon propagator at finite temperature respecting the appropriate boundary conditions. Within the real time formalism the well-known zero order results for energy, free energy and Casimir force are rederived.
Using a special perturbation theoretic expansion of the free energy the second order radiative corrections to the Casimir effect for different physical situations are determined.

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1. Introduction

The Casimir effect is one of the most fundamental effects in quantum field theory. Therefore, since its discovery in 1948 the effect raises interest up to now. Even in recent years a lot of work for different models has been done in order to achieve a deeper understanding.

If we consider in quantum electrodynamics two neutral conducting plates, then these two plates attract each other with a small but measurable force for small distances of the order of 10^{-4} cm. Casimir /1/ found for the pressure on the plates ($\hbar = c = 1$)

$$p(a) = -\frac{\pi^2}{240}\frac{1}{a^4} \qquad (1.1)$$

a ... distance between the plates

A physical explanation goes as follows: The electromagnetic field between the plates can be considered as a set of harmonic oscillators. As usual, quantum theoretic oscillators have zero point oscillations which contribute to the vacuum energy. The distance dependence of this energy leads to the Casimir force. This problem is treated in literature on different footings. Here we will consider the effect for finite temperature. The heat bath needs the incorporation of excited levels of the harmonic oscillators in the energy consideration. On this level the problem was studied in references /2/, /3/ and most completely in /4/. We will rederive the results of these authors in the framework of quantum electrodynamics in covariant gauge. Our treatment allows the calculation of loop corrections, especially the 1-loop contribution will be evaluated in the present paper. This work is based on a functional integral approach to quantum field theory with boundary conditions at zero temperature developed in /5/.

Recently there has been a growing interest in real time formalism for thermodynamic Green's functions /6/, /7/, /8/. The real time formalism has some advantages as compared with the well-known Matsubara technique, but up to now there remain some problems to be clarified.

So it seems to be interesting to study finite-temperature quantum field theory with boundary conditions by means of real time technique. Moreover, looking for perturbation theory the real time formalism gives hope to get closed propagators without infinite summations which are tedious to handle with in higher loop calculations.

The simultaneous study of two different boundary conditions is an attractive feature for itself. It can give deeper insight into the interplay between them both in relation to the underlying quantum structure. Therefore, boundary conditions can serve as a good tool in studying quantum field theory.

The plan of the paper is the following. In section 2 we derive a generating functional for finite-temperature quantum electrodynamics with boundary conditions (2 parallel neutral conducting plates perpendicular to the x_3-axis, appropriate boundary conditions for the field strength tensor) in

Minkowski space and give a closed propagator for the electromagnetic field under the conditions mentioned above. We use a special real time formalism, the functional integral approach to finite-temperature quantum field theory in Minkowski space given in /6/, which is tightly connected with thermo field dynamics /7/. Section 3 presents the evaluation of energy, free energy per unit area and pressure on the plates in zeroth order of perturbation theory. Section 4 is dealing with a special perturbative expansion of the free energy as well as with the calculation of the 1-loop corrections to the Casimir effect.

Finally, the appendices A and B contain some necessary formulae.

2. Finite-Temperature Quantum Electrodynamics in Minkowski Space with Boundary Conditions

In this section we will derive a generating functional Z for real time thermodynamic Green's functions respecting the appropriate boundary conditions on the plates for the electromagnetic field. Our derivation connects the work done in /5/, where the problem is treated at zero temperature in a functional integral approach, with the functional integral approach to finite-temperature quantum field theory in Minkowski space given in /6/.

Our starting point is the generating functional of quantum electrodynamics at zero temperature in a covariant gauge:

$$Z[j,\eta,\bar\eta] = \int DA_\mu D\Psi D\bar\Psi \ \exp\Big\{ i\int d^4x \Big[\tfrac{1}{2}A_\mu(x) D^{-1}(x)^{\mu\nu} A_\nu(x) + \bar\Psi(x) S^{-1}(x)\Psi(x) + e\bar\Psi(x)\not{A}(x)\Psi(x) + j^\nu(x)A_\nu(x) + \bar\eta(x)\Psi(x) + \bar\Psi(x)\eta(x)\Big]\Big\}$$

$$D^{-1}(x)^{\mu\nu} = g^{\mu\nu}\Box - \left(1-\tfrac{1}{\alpha}\right)\partial^\mu\partial^\nu$$
$$S^{-1}(x) = i\not\partial - m \qquad (2.1)$$

We apply here the standard notation /9/:

A_μ ... photon field
$\Psi, \bar\Psi$ ... electron field
α ... gauge parameter
$j, \eta, \bar\eta$ ... sources, introduced in the Z-functional.

Now we have to take into account the well-known boundary conditions for the electromagnetic field strength (a_k ... plate position)

$$n^\mu F^*_{\mu\nu}(x)\Big|_{x_3=a_k} = 0$$

$F^*_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\sigma\lambda}F^{\sigma\lambda}$, n = (0,0,0,1) ... normal vector to the plates

For this reason we introduce the δ-functions

$$\delta\left(n^\mu F^*_{\mu\nu}(x)\Big|_{x_3=a_k}\right) \qquad (2.2)$$

into the functional integral (2.1).

The δ-functions can be represented by an auxiliary functional integral, whereby the argument of the auxiliary fields $B^{k\alpha}$ is restricted to the plates. Moreover, a necessary gauge fixing term has been included.

$$\delta\left(n^\mu F^*_{\mu\nu}(x)\Big|_{x_3=a_k}\right) =$$
$$\int DB^{k\alpha} \exp\Big\{ i\int dS_k(x)\, B^{k\alpha}(\tilde{x}) H_{\alpha\mu}(\vec{x}) A^\mu(x) - \frac{1}{2\alpha}\iint dS_k(x)\, d_kS(y)\, B^{k\alpha}(\tilde{x})\Big(\frac{\partial}{\partial\tilde{x}^\alpha}\frac{\partial}{\partial\tilde{y}^\delta}D^c(x-y)\Big)B^{k\delta}(\tilde{y})\Big\},$$

$$dS_k(x) = d^4x\,\delta(x_3-a_k), \qquad \tilde{x} = (x_0,x_1,x_2)$$
$$H_{\alpha\mu}(\vec{x}) = -n^\lambda \varepsilon_{\lambda\alpha\mu\sigma}\frac{\vec\partial}{\partial x_\sigma} \qquad \alpha,\delta = 0,1,2 \qquad (2.3)$$

k denotes the plate index and is fixed here.

So we arrive at the generating functional with boundary conditions at zero temperature (for convenience, the gauge fixing term for the auxiliary field has been modified slightly in comparison with (2.3)).

$$Z[j,\eta,\bar\eta] = \int DA_\mu DB^{k\alpha} D\Psi D\bar\Psi$$
$$\exp\Big\{ i\int d^4x \Big[\tfrac{1}{2}A_\mu(x) D^{-1}(x)^{\mu\nu}A_\nu(x) + \bar\Psi(x)S^{-1}(x)\Psi(x) + e\bar\Psi(x)\not{A}(x)\Psi(x) + j^\nu(x)A_\nu(x) + \bar\eta(x)\Psi(x) +$$

$$+\overline{\Psi}(x)\eta(x)\Big] + i\int dS_k(x) A^{\mu}(x) H_{\mu\alpha}(\vec{x}) B^{k\alpha}(\tilde{x})$$
$$-\frac{i}{2\bar{\alpha}}\iint dS_k(x)\, dS_l(y)\, B^{k\alpha}(\tilde{x})$$
$$\left(\frac{\partial}{\partial\tilde{x}^{\alpha}}\frac{\partial}{\partial\tilde{y}^{\delta}} D^c(x-y)\right) B^{l\delta}(\tilde{y})\Big\} \qquad (2.4)$$

$k, l = 1,2$
$\alpha, \delta = 0,1,2$

Here a summation over k, l is understood.

Let us consider now the transition from zero-temperature to finite-temperature quantum field theory. Quantum field theory at finite temperature is characterized by taking expectation values with the help of the trace over a complete set of states weighted by the Boltzmannfactor $e^{-\beta H}$ (β is the inverse temperature, the Boltzmann constant k is set equal to 1).

Extending the Heisenberg picture to imaginary times one can write the thermal expectation value as trace over states differing in their time arguments by $-i\beta$. Expressing thermal expectation values by means of functional integrals the fields have to be integrated along complex values of the time variable x_0. In the real time formalism the x_0-integration is performed along a special contour C (s. fig. 1) (there have to be respected some additional restrictions on the shape of the curve) /6/. Therefore, the essential step from (2.4) to a generating functional at finite temperature is to define the fields not only on the real axis but at least on the curve C. Accordingly time integrations are replaced by integrations along the curve C.

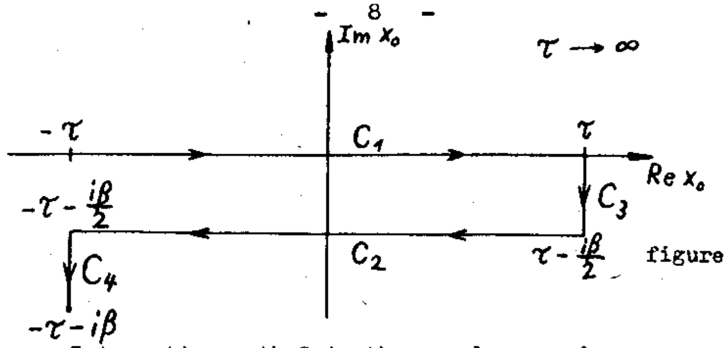

figure 1

Integration path C in the complex x_0-plane

Now of course the standard boundary conditions for finite-temperature quantum field theory ($A_\mu(x_0) = A_\mu(x_0 - i\beta), \Psi(x_0) = -\Psi(x_0 - i\beta)$ for Bose and Fermi fields respectively) have to be taken into account. In reference /6/ it is claimed, that the fields on C_1 and C_2 are important only. So one arrives at a thermal doublet representation of quantum field theory at finite temperature. Note, that the procedure described above does not change the already imposed boundary conditions on the plates. So we have got the generating functional for real time finite-temperature quantum electrodynamics with boundary conditions.

$$Z[j_1, j_2, \eta_1, \eta_2, \bar{\eta}_1, \bar{\eta}_2] = \int DA_{a\mu}\, DB^{k\alpha}_a\, D\Psi_a\, D\overline{\Psi}_a$$
$$\exp\Big\{ i\int d^4x \Big[\frac{1}{2} A^a_\mu(x) D^{-1\,\mu\nu}_{\beta\;ab}(x) A^b_\nu(x)$$
$$+\overline{\Psi}^a(x) S^{-1}_{\beta\;ab}(x)\Psi^b(x) + e\,\overline{\Psi}_1(x)\not{A}_1(x)\Psi_1(x) - e\,\overline{\Psi}_2(x)\not{A}_2(x)\Psi_2(x)$$
$$+ j^{a\nu}(x) A_{a\nu}(x) + \bar{\eta}^a(x)\Psi_a(x) + \overline{\Psi}^a(x)\eta_a(x)\Big] +$$

$$+i\int dS_k(x)\,A^{a\mu}(x)\,H_{\mu\alpha}(\vec{x})\,B_a^{k\alpha}(\tilde{x})$$
$$-\frac{i}{2\bar{\alpha}}\iint dS_k(x)\,dS_l(y)\,B_a^{k\alpha}(\tilde{x})\Big(\frac{\partial}{\partial\tilde{x}^\alpha}\frac{\partial}{\partial\tilde{y}^\delta} \tag{2.5}$$
$$D_\beta^{c\;ab}(x-y)\Big)B_b^{l\delta}(\tilde{y})\Big\}\Big\},$$

doublet index $a, b = 1,2$
plate index $k, l = 1,2$
$\alpha, \delta = 0,1,2$

For the origin of the different signs in front of the interaction terms compare /6/.

For later calculations it is necessary to derive a simple perturbation theoretic formulation of quantum electrodynamics with boundary conditions at finite temperature. Let us proceed in a standard manner. The interaction can be simulated with the help of functional derivatives acting on the remaining Gaussian functional integral. Performing the Gaussian integrations over $A_{a\mu}, \Psi_a, \bar{\Psi}_a$ we arrive at the following expression:

$$Z[j_1, j_2, \eta_1, \eta_2, \bar{\eta}_1, \bar{\eta}_2] =$$
$$\exp\Big\{-e\int d^4x\Big[\frac{\delta}{\delta\eta_1(x)}\gamma^\nu\frac{\delta}{\delta j_1^\nu(x)}\frac{\delta}{\delta\bar{\eta}_1(x)} - \frac{\delta}{\delta\eta_2(x)}\gamma^\nu\frac{\delta}{\delta j_2^\nu(x)}\frac{\delta}{\delta\bar{\eta}_2(x)}\Big]\Big\}$$
$$\exp\Big\{-\frac{i}{2}\iint d^4x\,d^4y\,j_\mu^a(x)\,D_\beta^{c\;\mu\nu}{}_{ab}(x-y)\,j_\nu^b(y)\Big\} \tag{2.6}$$
$$\exp\Big\{-i\iint d^4x\,d^4y\,\bar{\eta}^a(x)\,S_{\beta\;ab}^c(x-y)\,\eta^b(y)\Big\}$$
$$\int DB_a^{k\alpha}\exp\Big\{\frac{i}{2}\iint d^3\tilde{x}\,d^3\tilde{y}\,B_a^{k\alpha}(x)\,C_{\beta\;\alpha\delta\;kl}^{\;ab}(\tilde{x}-\tilde{y})\,B_b^{l\delta}(\tilde{y})$$
$$+i\int d^3\tilde{x}\,\xi_{k\alpha}^a(\tilde{x})\,B_a^{k\alpha}(\tilde{x})\Big\}$$

with

$$\xi_{k\alpha a}(\tilde{x}) = \int d^4y\,j_\nu^b(y)\,D_\beta^{c\;\nu\lambda}{}_{ba}(y-x)\,H_{\lambda\alpha}(\overleftarrow{x})\Big|_{x_3=a_k}$$

$$C_{\beta\;\alpha\delta\;kl}^{\;ab}(\tilde{x}-\tilde{y}) = \int\frac{d^3\tilde{k}}{(2\pi)^3}\,e^{i\tilde{k}(\tilde{x}-\tilde{y})}\big[B_+(k_0)\,C_{\beta\;kl}^{\;\alpha\delta}(\tilde{k})\,B_+(k_0)\big]^{ab}$$

$$C_{\beta\;kl}^{\;\alpha\delta}(\tilde{k}) = \frac{1}{2i}\begin{pmatrix}[\tilde{g}_{\alpha\delta}\Gamma_+^2(\tilde{k}) - (1-\frac{1}{\bar{\alpha}})\tilde{k}_\alpha\tilde{k}_\delta]\,\frac{h(\tilde{k})_{kl}}{\Gamma_+(\tilde{k})} & 0 \\ 0 & [\tilde{g}_{\alpha\delta}\Gamma_-^2(\tilde{k}) - (1-\frac{1}{\bar{\alpha}})\tilde{k}_\alpha\tilde{k}_\delta]\,\frac{h^*(\tilde{k})_{kl}}{\Gamma_-(\tilde{k})}\end{pmatrix}$$

$$\Gamma_\pm(\tilde{k}) = \sqrt{k_0^2 - k_1^2 - k_2^2 \pm i\varepsilon},$$

$\tilde{k} = (k_0, k_1, k_2)$,
$a, b = 1,2$,
$k, l = 1,2$,
$\alpha, \delta = 0,1,2$

$D_{\beta\,ab}^c, S_{\beta\,ab}^c$ are the usual propagators of the photon and electron thermal doublets in the real time formalism. They can be found together with the matrices $B_+(k_0)$ in the appendix A. The expression $h(\tilde{k})_{kl}$ which is a special outcome from the boundary conditions can be found in the appendix A also.

Finally the auxiliary functional integration can be done, and we arrive at (2.7). In the derivation of this expression we have used an unusual representation of the causal propagator (A 2) obtained by performing the k_3-integration first (s. (A 3)).

$$Z\left[j_1, j_2, \eta_1, \eta_2, \bar{\eta}_1, \bar{\eta}_2\right] =$$
$$\exp\left\{-e\int d^4x \left[\frac{\delta}{\delta\eta_1(x)}\gamma^\nu \frac{\delta}{\delta j_1^\nu(x)}\frac{\delta}{\delta\bar{\eta}_1(x)} - \frac{\delta}{\delta\eta_2(x)}\gamma^\nu \frac{\delta}{\delta j_2^\nu(x)}\frac{\delta}{\delta\bar{\eta}_2(x)}\right]\right\}$$
$$\exp\left\{-\frac{i}{2}\iint d^4x\, d^4y\, j_\mu^a(x)\, {}^sD^{c\,\mu\nu}_{\beta\;ab}(x,y)\, j_\nu^b(y)\right\}$$
$$\exp\left\{-i\iint d^4x\, d^4y\, \bar{\eta}^a(x)\, S^c_{\beta\;ab}(x-y)\,\eta^b(y)\right\} \qquad (2.7)$$

a, b = 1,2

k, l = 1,2

This expression defines the perturbation theory. The Feynman rules remain unaltered in comparison with usual quantum electrodynamics at finite temperature, only the photon propagator has to be substituted by a new one. This propagator (2.8) consists of the sum of the standard propagator and an additional term, which depends on the boundary conditions (explicit expressions can be found in appendix A).

$${}^sD^{c\;\mu\nu}_{\beta\;ab}(x,y) = D^{c\;\mu\nu}_{\beta\;ab}(x-y) + \bar{D}^{\;\mu\nu}_{\beta\;ab}(\tilde{x}-\tilde{y}; x_3, y_3) \qquad (2.8)$$

So we obtained a closed thermal propagator (2.8) for the electromagnetic field respecting the boundary conditions. This form does not contain any infinite summation, which is a

convenient property in higher loop calculations.

Finally we give some properties of the propagator derived above.

$$\frac{\partial}{\partial x^\mu}\bar{D}^{\;\mu\nu}_{\beta\;ab}(\tilde{x}-\tilde{y}; x_3, y_3) = \frac{\partial}{\partial y^\mu}\bar{D}^{\;\mu\nu}_{\beta\;ab}(\tilde{x}-\tilde{y}; x_3, y_3) = 0 \qquad (2.9)$$

The a-dependent part of the propagator does not depend on any gauge fixing parameter. It can be shown /5/, that the contracted a-dependent part of the propagator (2.8) $\frac{1}{2} g_{\mu\nu}\bar{D}^{\;\mu\nu}_{\beta\;ab}(\tilde{x}-\tilde{y}; x_3, y_3)$ gives the a-dependent part of the propagator of a massless scalar field $\varphi(x)$ with boundary conditions $\varphi(x) = 0$ on the plates.

3. Casimir Effect at T ≠ 0 in Zeroth Order

In the following we would like to present the evaluation of the energy E(a, β) and the free energy F(a, β) per unit area as well as the pressure on the plates p(a, β) in zeroth order of perturbation theory. These quantities can be obtained with the help of the diagonal elements of the thermal expectation value of the energy-momentum tensor. The canonical energy-momentum tensor has the well-known structure

$$T_{\mu\nu}(A, \bar{\Psi}, \Psi) = \partial_\nu A^\lambda F_{\lambda\mu} - \frac{1}{\alpha}\partial_\nu A_\mu \partial_\lambda A^\lambda$$
$$- g_{\mu\nu}\left[-\frac{1}{4}F_{\lambda\sigma}F^{\lambda\sigma} - \frac{1}{2\alpha}(\partial_\lambda A^\lambda)^2\right]$$
$$+ \frac{i}{2}\left[\bar{\Psi}\gamma_\mu\partial_\nu\Psi - \partial_\nu\bar{\Psi}\gamma_\mu\Psi\right] \qquad (3.1)$$

With the help of (2.7) we give the expression for the thermal expectation value of $T_{\mu\mu}$ $\langle T_{\mu\mu}\rangle_\beta$ (no summation understood) in terms of the generating functional:

$$\langle T_{\mu\mu}\rangle_\beta = T_{\mu\mu}\left(\frac{1}{i}\frac{\delta}{\delta j_1}, \frac{1}{i}\frac{\delta}{\delta \eta_1}, \frac{1}{i}\frac{\delta}{\delta \overline{\eta}_1}\right) \left.\frac{Z[j_1, j_2, \eta_1, \eta_2, \overline{\eta}_1, \overline{\eta}_2]}{Z[0,0,0,0,0,0]}\right|_{j_a = \eta_a = \overline{\eta}_a = 0} \tag{3.2}$$

Proceeding further we obtain in zeroth order (we regard contributions arising from the a-dependent part of the propagator (2.8) only):

$$\langle T_{\left\{\substack{00\\11\\33}\right\}}\rangle_\beta = -i\left[\frac{\partial}{\partial x^0}\frac{\partial}{\partial y^0}\left\{\begin{matrix}+\\+\\-\end{matrix}\right\}\frac{\partial}{\partial x^1}\frac{\partial}{\partial y^1}\left\{\begin{matrix}+\\-\\-\end{matrix}\right\}\frac{\partial}{\partial x^2}\frac{\partial}{\partial y^2}\left\{\begin{matrix}+\\-\\+\end{matrix}\right\}\frac{\partial}{\partial x^3}\frac{\partial}{\partial y^3}\right]\overline{D}_{\beta\,11}(\tilde{x}-\tilde{y}; x_3, y_3)\Big|_{x\to y} \tag{3.3}$$

$$\overline{D}_{\beta\,11}(\tilde{x}-\tilde{y}; x_3, y_3) = \frac{1}{2i}\int\frac{d^3\tilde{k}}{(2\pi)^3}\, e^{i\tilde{k}(\tilde{x}-\tilde{y})} \left[\cosh^2\theta\; e^{i\Gamma_+(\tilde{k})|x_3-a_k|}\,\frac{[h^{-1}(\tilde{k})]_{kl}}{\Gamma_+(\tilde{k})}\, e^{i\Gamma_+(\tilde{k})|y_3-a_l|} + \sinh^2\theta\; e^{-i\Gamma_-(\tilde{k})|x_3-a_k|}\,\frac{[h^{-1}(\tilde{k})]^*_{kl}}{\Gamma_-(\tilde{k})}\, e^{-i\Gamma_-(\tilde{k})|y_3-a_l|}\right] \tag{3.4}$$

$$\cosh^2\theta = \frac{1}{1-e^{-\beta|k_0|}}\,,\quad [h^{-1}(\tilde{k})]_{kl} = -i\,\frac{[1-\delta_{kl}(1+e^{-i\Gamma_+(\tilde{k})a})]}{2\sin\Gamma_+(\tilde{k})a}$$

In this expression we have introduced a point-splitting in order to regularize divergences which could possibly arise. Effectively we have got a result, which is equivalent to that in the scalar case up to a factor 2 as it should be due to the number of degrees of freedom of the electromagnetic field. There is not any dependence on the gauge parameter α left. These facts are due to the transversal structure (cf. (2.9)) of the a-dependent part of the propagator (2.8).

First we will evaluate the Casimir force in a local approach based on equation (3.3). The Casimir force is given by the discontinuity of $\langle T_{33}\rangle_\beta$ across the boundary.

Equation (3.3) leads to

$$\langle T_{33}\rangle_\beta = i\int\frac{d^3\tilde{k}}{(2\pi)^3}\left\{\frac{\Gamma_+(\tilde{k})\,e^{i\Gamma_+(\tilde{k})a}}{\sin\Gamma_+(\tilde{k})a} + \sinh^2\theta\left[\frac{\Gamma_+(\tilde{k})\,e^{i\Gamma_+(\tilde{k})a}}{\sin\Gamma_+(\tilde{k})a} - \frac{\Gamma_-(\tilde{k})\,e^{-i\Gamma_-(\tilde{k})a}}{\sin\Gamma_-(\tilde{k})a}\right]\right\} \quad \text{between the plates},$$

$$\langle T_{33}\rangle_\beta \equiv 0 \quad \text{otherwise}. \tag{3.5}$$

It is interesting and physically acceptable, that $\langle T_{33}\rangle_\beta$ is constant between the plates and zero otherwise. With the help of (A 27) and (A 28) we obtain the final result for the Casimir force:

$$p(a,\beta) = \langle T_{33}\rangle_\beta \Big|_{\substack{\text{between}\\ \text{the plates}}} - \langle T_{33}\rangle_\beta \Big|_{\substack{\text{outside}\\ \text{the plates}}}$$

$$= -\frac{\pi^2}{45}\frac{1}{\beta^4} - \frac{\pi^2}{240}\frac{1}{a^4} + \frac{\pi}{4a^3\beta}\sum_{k=1}^{\infty}\frac{1}{k}\frac{\cosh\frac{k\pi\beta}{2a}}{\sinh^3\frac{k\pi\beta}{2a}} \qquad (3.6)$$

In the next part of this section we will evaluate the energy E(a, β), the free energy F(a, β) per unit area as well as the pressure p(a, β) on the plates with the help of the integrated (along the direction perpendicular to the plates) diagonal elements of the thermal expectation value of the energy-momentum tensor.

$$E(a,\beta) = \int_{-\infty}^{\infty} dx_3 \ \langle T_{00}\rangle_\beta \qquad (3.7)$$

$$F(a,\beta) = -\int_{-\infty}^{\infty} dx_3 \ \langle T_{11}\rangle_\beta = -\int_{-\infty}^{\infty} dx_3 \langle T_{22}\rangle_\beta \qquad (3.8)$$

$$p(a,\beta) = \frac{1}{a}\int_{-\infty}^{\infty} dx_3 \langle T_{33}\rangle_\beta \qquad (3.9)$$

Due to the well-known divergences in (3.7) - (3.9) the a-dependent part of $\int dx_3 \langle T_{\mu\mu}\rangle_\beta$ has to be taken into account only.

Relation (3.9) can be understood easily in the light of (3.5). A physical explanation for (3.8) is given in /4/. Another derivation for relation (3.8) goes as follows. In zeroth order of perturbation theory it holds

$$\langle T_\mu{}^\mu\rangle_\beta = 0 \qquad (3.10)$$

for the electromagnetic part of the energy-momentum tensor.

Due to the symmetry of our problem we have

$$\langle T_{11}\rangle_\beta = \langle T_{22}\rangle_\beta \qquad (3.11)$$

Now, integrating (3.10) over x_3 and taking into account (3.7), (3.9) and (3.11) we obtain

$$a^3 E(a,\beta) - 2a^3\int_{-\infty}^{\infty}\langle T_{11}\rangle_\beta \, dx_3 - a^4 p(a,\beta) = 0 \qquad (3.12)$$

The dimensionless quantities a^3 E(a, β), $a^3\int\langle T_{11}\rangle_\beta \, dx_3$, a^4 p(a, β) can depend on the dimensionless ratio x = a/β only. With the help of the thermodynamic relations

$$E = F + \beta\left(\frac{\partial F}{\partial \beta}\right)_V \Longrightarrow a^3 E(a,\beta) = a^3 F(a,\beta) - x\frac{\partial[a^3 F(a,\beta)]}{\partial x} \qquad (3.13)$$

$$p = -\left(\frac{\partial F}{\partial V}\right)_\beta \Longrightarrow a^4 p(a,\beta) = 3a^3 F(a,\beta) - x\frac{\partial[a^3 F(a,\beta)]}{\partial x} \qquad (3.14)$$

we get from (3.12) the desired relation (3.8).

Now, from (3.3) we obtain the integrated thermal expectation value of the diagonal elements of the energy-momentum tensor:

$$\int_{-\infty}^{\infty} dx_3 \langle T_{\mu\mu}\rangle_\beta = i \lim_{\delta\to 0}\int d^4\xi \ \delta^{(4)}(\xi)\, \partial_{(i)\xi\xi} \int dx_3 \, \bar{D}_{\beta\, 11}(\tilde{\xi};\, \xi_3 + x_3 ,\, x_3 + \delta) \qquad (3.15)$$

$$\int_{-\infty}^{\infty} dx_3 \langle T_{\mu\mu} \rangle_\beta = i \lim_{\delta \to 0} \int d^4\xi \, \delta^{(4)}(\xi) \, \partial_{(i)\xi\xi} \, \cdot$$

$$\int_{-\infty}^{\infty} dx_3 \frac{1}{2i} \int \frac{d^3\tilde{k}}{(2\pi)^3} e^{i\tilde{k}\tilde{\xi}} \Big[\cosh^2\theta \, Z(\tilde{k}; \xi_3 + x_3, x_3 + \delta) + \sinh^2\theta \, Z^*(\tilde{k}; \xi_3 + x_3, x_3 + \delta) \Big] \tag{3.16}$$

$$Z(\tilde{k}; \xi_3 + x_3, x_3 + \delta) = e^{i\Gamma_+(\tilde{k})|\xi_3 + x_3 - a_k|} \frac{[h^{-1}(\tilde{k})]_{kl}}{\Gamma_+(\tilde{k})} e^{i\Gamma_+(\tilde{k})|x_3 + \delta - a_l|},$$

$$\partial_{\left\{\begin{smallmatrix}0\\1\\3\end{smallmatrix}\right\}\xi\xi} = \frac{\partial^2}{\partial\xi_0^2} \begin{Bmatrix} + \\ + \\ - \end{Bmatrix} \frac{\partial^2}{\partial\xi_1^2} \begin{Bmatrix} + \\ - \\ - \end{Bmatrix} \frac{\partial^2}{\partial\xi_2^2} \begin{Bmatrix} + \\ - \\ + \end{Bmatrix} \frac{\partial^2}{\partial\xi_3^2} \tag{3.17}$$

Taking into account (A.24) the x_3-integration as well as the necessary differentiations can be performed. The point splitting parameter δ can be set to zero without any complication.

So we arrive at

$$\int_{-\infty}^{\infty} dx_3 \langle T_{\left\{\begin{smallmatrix}00\\11\\33\end{smallmatrix}\right\}} \rangle_\beta = -i \int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_1^2 + k_2^2 \\ k_0^2 - k_2^2 \\ 0 \end{Bmatrix} \Big[\frac{\cosh^2\theta}{\Gamma_+^2(\tilde{k})} - \frac{\sinh^2\theta}{\Gamma_-^2(\tilde{k})} \Big] + ia \int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_0^2 \\ k_1^2 \\ \Gamma^2(\tilde{k}) \end{Bmatrix}$$

$$\Big[\cosh^2\theta \frac{e^{i\Gamma_+(\tilde{k})a}}{\Gamma_+(\tilde{k}) \sin \Gamma_+(\tilde{k})a} - \sinh^2\theta \frac{e^{-i\Gamma_-(\tilde{k})a}}{\Gamma_-(\tilde{k}) \sin \Gamma_-(\tilde{k})a} \Big] \tag{3.18}$$

In appendix A we list some formulae needed for the further treatment of expression (3.16) (s. (A 25) - (A 28)). Our final result is

$$\int_{-\infty}^{\infty} dx_3 \langle T_{\left\{\begin{smallmatrix}00\\11\\33\end{smallmatrix}\right\}} \rangle_\beta = - \frac{\pi^2}{45} \begin{Bmatrix} 3 \\ 1 \\ 1 \end{Bmatrix} \frac{a}{\beta^4} - \frac{\pi^2}{720} \begin{Bmatrix} 1 \\ -1 \\ 3 \end{Bmatrix} \frac{1}{a^3}$$

$$+ \frac{1}{2\pi\beta^3} \sum_{k=1}^{\infty} \Big[\frac{1}{k^3} \begin{Bmatrix} 2 \\ 1 \\ 0 \end{Bmatrix} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh \frac{k\pi\beta}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)}{k^2} \begin{Bmatrix} 2 \\ 1 \\ 0 \end{Bmatrix} \frac{1}{\sinh^2 \frac{k\pi\beta}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)^2}{k} \begin{Bmatrix} 2 \\ 0 \\ 2 \end{Bmatrix} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh^3 \frac{k\pi\beta}{2a}} \Big] \tag{3.19}$$

Using (3.7) - (3.9) we find ($\hbar$ = c = k = 1)

$$E(a,\beta) = - \frac{\pi^2}{15} \frac{a}{\beta^4} - \frac{\pi^2}{720} \frac{1}{a^3} + \frac{1}{\pi\beta^3} \sum_{k=1}^{\infty} \Big[\frac{1}{k^3} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh \frac{k\pi\beta}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)}{k^2} \frac{1}{\sinh^2 \frac{k\pi a}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)^2}{k} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh^3 \frac{k\pi\beta}{2a}} \Big] \tag{3.20}$$

$$F(a,\beta) = \frac{\pi^2}{45} \frac{a}{\beta^4} - \frac{\pi^2}{720} \frac{1}{a^3} - \frac{1}{2\pi\beta^3} \sum_{k=1}^{\infty} \Big[\frac{1}{k^3} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh \frac{k\pi\beta}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)}{k^2} \frac{1}{\sinh \frac{k\pi\beta}{2a}} \Big] \tag{3.21}$$

$$p(a,\beta) = -\frac{\pi^2}{45}\frac{1}{\beta^4} - \frac{\pi^2}{240}\frac{1}{a^4} + \frac{\pi}{4a^3\beta}\sum_{k=1}^{\infty}\frac{1}{k}\frac{\cosh\frac{k\pi\beta}{2a}}{\sinh^3\frac{k\pi\beta}{2a}} \quad . \qquad (3.22)$$

Of course (3.22) coincides with (3.6).

By means of the relations (B 3), (B 5) it can be seen, that our results for the physical interesting quantities (3.7) - (3.9) completely agree with those obtained by other authors /2/, /3/, /4/. Of course the expressions (3.20), (3.21) and (3.22) respect the thermodynamic relations

$$E(a,\beta) = F(a,\beta) + \beta\left(\frac{\partial F(a,\beta)}{\partial \beta}\right)_a \quad , \qquad (3.23)$$

$$p(a,\beta) = -\left(\frac{\partial F(a,\beta)}{\partial a}\right)_\beta \quad . \qquad (3.24)$$

Let us shortly discuss the expression for the Casimir force (3.22). First we study the limit $\frac{a}{\beta} \ll 1$, which includes the low temperature behaviour. We obtain (up to exponentially vanishing terms)

$$p(a,\beta) \underset{\frac{a}{\beta}\ll 1}{=} -\frac{\pi^2}{240}\frac{1}{a^4}\left[1 + \frac{16}{3}\left(\frac{a}{\beta}\right)^4\right] . \qquad (3.25)$$

It is interesting, that the second term in (3.25) though independent of the plate distance a originates from the boundary conditions on the plates. In the limit $\beta \rightarrow \infty$ (3.25) reproduces the well-known Casimir force at zero temperature (1.1).

With the help of (B 5) we transform (3.22) into an equivalent form more suited for the discussion of the case $\frac{\beta}{a} \ll 1$.

$$p(a,\beta) = -\frac{1}{4\pi}\frac{1}{a^3\beta}\sum_{k=1}^{\infty}\left[\frac{1}{k^3}\frac{\cosh\frac{2k\pi a}{\beta}}{\sinh\frac{2k\pi a}{\beta}} + \frac{\left(\frac{2\pi a}{\beta}\right)}{k^2}\frac{1}{\sinh^2\frac{2k\pi a}{\beta}} + \frac{\left(\frac{2\pi a}{\beta}\right)^2}{k}\frac{\cosh\frac{2k\pi a}{\beta}}{\sinh^3\frac{2k\pi a}{\beta}}\right] . \qquad (3.26)$$

It can be seen easily, that the Casimir force in the limit $\frac{\beta}{a} \ll 1$ is given up to exponentially vanishing terms by

$$p(a,\beta) \underset{\frac{\beta}{a}\ll 1}{=} -\frac{\zeta(3)}{4\pi}\frac{1}{a^3\beta} \quad . \qquad (3.27)$$

All expressions show the decrease of the Casimir force with increasing distance between the plates. Increasing temperature obviously leads to an increase of the Casimir force.

4. 1-loop Corrections to Casimir Effect at Finite Temperature

In order to calculate radiative corrections to the Casimir effect at finite temperature we have to present a perturbative expansion of the free energy in terms of the real time formalism used by us. In a recent paper the perturbative treatment of the free energy in thermo field dynamics has been studied /10/. We will use analogous considerations for the derivation of an expansion for the free energy. Considering the time path formalism for statistical averages the free energy is given by /11/ (leaving gauge theories aside for the moment):

$$F = -\frac{1}{\beta} \ln tr \left\{ e^{-\beta H_0} T_c \, e^{-i \int_c dz \, H_I(z)} \right\} . \quad (4.1)$$

Here H_0 is the unperturbed part of the Hamiltonian and $H_I(z)$ denotes the interaction Hamiltonian in the interaction representation. T_C is the path ordering symbol whereby the subscript C is related to the definite curve C in the complex time plane (s. fig. 1).

According to /10/ equation (4.1) can be rewritten in the following manner taking into account the independence of the average on the contour point z_0

$$F = F(0) + \int_0^1 ds \, tr \left\{ e^{-\beta H_0} T_c \, H_I(z_0) \, e^{-is \int_c dz \, H_I(z)} \right\}_{con.} \quad (4.2)$$

where F(0) is the free energy for the unperturbed system.

Due to this independence we can rewrite (4.2):

$$F = F(0) + \int_0^1 ds \, tr \left\{ e^{-\beta H_0} T_c \left[\xi H_I(z_0') + (1-\xi) H_I(z_0'') \right] e^{-is \int_c dz \, H_I(z)} \right\}_{con.} \quad (4.3)$$

$z_0', z_0'' \in C$

$$F = F(0) + \frac{1}{2} \int_0^1 ds \, tr \left\{ e^{-\beta H_0} T_c \left[H_I(z_0') + H_I(z_0'') \right] e^{-is \int_c dz \, H_I(z)} \right\}_{con.} + \left(\xi - \frac{1}{2}\right) \int_0^1 ds \, tr \left\{ e^{-\beta H_0} T_c \left[H_I(z_0') - H_I(z_0'') \right] e^{-is \int_c dz \, H_I(z)} \right\}_{con.} . \quad (4.4)$$

Here ξ is an arbitrary parameter. With respect to the independence of the free energy on this arbitrary parameter the last term in (4.4) has to vanish. So we arrive at

$$F = F(0) + \frac{1}{2} \int_0^1 ds \, tr \left\{ e^{-\beta H_0} T_c \left[H_I(z_0') + H_I(z_0'') \right] e^{-is \int_c dz \, H_I(z)} \right\}_{con.} \quad (4.5)$$

where the arbitrary points z_o', $z_o'' \in C$ will now be chosen: $z_o' \in C_1$, $z_o'' \in C_2$ (s. fig. 1). Applying (4.5) to a gauge theory we use the general correspondence

$$tr\left[e^{-\beta H} O(A_{\mu a}; \Psi_a; \overline{\Psi}_a)\right] = O\left(\frac{1}{i}\frac{\delta}{\delta j_a^\mu}; \frac{1}{i}\frac{\delta}{\delta \overline{\eta}_a}; \frac{1}{i}\frac{\delta}{\delta \eta_a}\right) Z[j_1, j_2, \eta_1, \eta_2, \overline{\eta}_1, \overline{\eta}_2]\Big|_{j_a = \eta_a = \overline{\eta}_a = 0}$$

where Z is given by (2.7). The role of the interaction parameter s now plays the coupling constant e. So we get the following perturbative expansion for the free energy:

$$F = F(0) - \frac{i}{2}\int_0^e d\bar{e} \int d^3\bar{x} \left[\frac{\delta}{\delta \eta_1(x)} \gamma^\nu \frac{\delta}{\delta j_1^{\nu}(x)} \frac{\delta}{\delta \overline{\eta}_1(x)} + \frac{\delta}{\delta \eta_2(x)} \gamma^\nu \frac{\delta}{\delta j_2^{\nu}(x)} \frac{\delta}{\delta \overline{\eta}_2(x)}\right] Z[j_1, j_2, \eta_1, \eta_2, \overline{\eta}_1, \overline{\eta}_2]\Big|_{\substack{j_a = \eta_a = \overline{\eta}_a = 0 \\ \text{coupl. const. } \bar{e} \\ \text{con.}}} \tag{4.6}$$

Remember that the fields on C_1 or C_2 appear as the first or second components of the thermal doublets respectively.

Up to the order e^2 the free energy is given now by

$$F = F(0) - \frac{1}{2}\int d^3\bar{x} \int d^4y \, Im\left[\Pi_{\beta\ \mu\nu}^{\ 11}(x-y) \, {}^sD_{\beta\ 11}^{c\ \mu\nu}(x,y)\right] + O(e^4) \tag{4.7}$$

where we have taken into account the relations

$${}^sD_{\beta\ \mu\nu}^{c\ ab} = -\left[{}^sD_{\beta\ \mu\nu}^{c\ ba}\right]^*, \quad {}^sD_{\beta\ \mu\nu}^{c\ 12} = {}^sD_{\beta\ \mu\nu}^{c\ 21},$$
$$\Pi_{\beta\ \mu\nu}^{\ ab} = -\left[\Pi_{\beta\ \mu\nu}^{\ ba}\right]^*, \quad \Pi_{\beta\ \mu\nu}^{\ 12} = \Pi_{\beta\ \mu\nu}^{\ 21}. \tag{4.8}$$

(4.7) shows explicitly, that in the formalism used by us the free energy does not have any imaginary part as expected.

So we find for the free energy per unit area (restricting $F(a,\beta)$ to the a-dependent part only)

$$F(a,\beta) = F(a,\beta)_0 + F(a,\beta)_1 + O(e^4)$$
$$= F(a,\beta)_0 - \frac{1}{2} Im\left\{\int \frac{d^4k}{(2\pi)^4} \widehat{\Pi}_{\beta\ \mu\nu}^{\ 11}(k) \iint dz_3 dx_3 \, e^{ik_3(x_3 - z_3)} \widehat{D}_{\beta\ 11}^{\ \mu\nu}(\tilde{k}; z_3, x_3)\right\} + O(e^4) \,. \tag{4.9}$$

$F(a,\beta)_0$ has been evaluated already in section 3 (cf. (3.21)), so we have to regard the second term in (4.9) only.

At this stage for sake of computational feasibility we would like to look for physical relevant sectors of our parameters m, a, β. In all physically realistic situations the temperature β^{-1} is small as compared with the electron mass m, so we have the relation $m\beta \gg 1$. Due to the factors

$$\frac{\delta(k^2 - m^2)}{1 + e^{\beta[|k_0| - \mu]}}$$

(s. (A 13), (A 14)) in the temperature dependent part of

$\hat{\Pi}_{\beta\,\mu\nu}^{\;11}(k)$ the polarization tensor effectively reduces to the usual polarization tensor at zero temperature $\hat{\Pi}_{\mu\nu}(k)$. All the resulting temperature dependence of $F(a,\beta)_1$ will be generated by the photon propagator. Moreover, it can be seen, that the a-dependent part of the free energy correction in order e^2 is independent of the renormalization arbitrariness. With the help of (A 14) and (A 29) we obtain for the part in the wavy brackets in (4.9)

$$\left\{\int\frac{d^4k}{(2\pi)^4}\,\hat{\Pi}_{\beta\,\mu\nu}^{\;11}(k)\iint dz_3\,dx_3\; e^{ik_3(x_3-z_3)}\,\hat{D}_{\beta\,11}^{\;\mu\nu}(\tilde{k};z_3,x_3)\right\}$$
$$= 4\int\frac{d^4k}{(2\pi)^4}\,\hat{\Pi}_+(k^2)\Bigg[\cosh^2\theta\;\frac{\Gamma_+(\tilde{k})\left[\frac{\cos\Gamma_+(\tilde{k})a-\cos k_3a}{\sin\Gamma_+(\tilde{k})a}\right]}{[\Gamma_+^2(\tilde{k})-k_3^2]}$$
$$-\sinh^2\theta\;\frac{\Gamma_-(\tilde{k})\left[\frac{\cos\Gamma_-(\tilde{k})a-\cos k_3a}{\sin\Gamma_-(\tilde{k})a}\right]}{[\Gamma_-^2(\tilde{k})-k_3^2]}\Bigg] \quad . \qquad (4.10)$$

This expression can be rewritten by insertion of $\hat{\Pi}_+(k^2)=\hat{\Pi}_-(k^2)-\mathrm{disc}\,\hat{\Pi}(k^2)$ into the second term in (4.10). Additionally we take into account, that terms connected with the discontinuity of $\hat{\Pi}(k^2)$ can be neglected when the condition $m\beta\gg 1$ is fulfilled.

$$\{\;\} = 4\int_0^{\infty}dk_0\int\frac{d^3\bar{k}}{(2\pi)^4}\Bigg[\frac{\hat{\Pi}_+(k^2)}{1-e^{-\beta k_0}}\;\frac{\Gamma_+(\tilde{k})\left[\frac{\cos\Gamma_+(\tilde{k})a-\cos k_3a}{\sin\Gamma_+(\tilde{k})a}\right]}{[\Gamma_+^2(\tilde{k})-k_3^2]}$$
$$+\frac{\hat{\Pi}_-(k^2)}{1-e^{\beta k_0}}\;\frac{\Gamma_-(\tilde{k})\left[\frac{\cos\Gamma_-(\tilde{k})a-\cos k_3a}{\sin\Gamma_-(\tilde{k})a}\right]}{[\Gamma_-^2(\tilde{k})-k_3^2]}\Bigg]+$$

$$+4\int_{-\infty}^{0}dk_0\int\frac{d^3\bar{k}}{(2\pi)^4}\Bigg[\frac{\hat{\Pi}_+(k^2)}{1-e^{\beta k_0}}\;\frac{\Gamma_+(\tilde{k})\left[\frac{\cos\Gamma_+(\tilde{k})a-\cos k_3a}{\sin\Gamma_+(\tilde{k})a}\right]}{[\Gamma_+^2(\tilde{k})-k_3^2]}$$
$$+\frac{\hat{\Pi}_-(k^2)}{1-e^{-\beta k_0}}\;\frac{\Gamma_-(\tilde{k})\left[\frac{\cos\Gamma_-(\tilde{k})a-\cos k_3a}{\sin\Gamma_-(\tilde{k})a}\right]}{[\Gamma_-^2(\tilde{k})-k_3^2]}\Bigg] \quad . \qquad (4.11)$$

Now after performing the appropriate Wick-rotations the k_0-integration reduces to the sum of the residua of the statistical factors. So we arrive effectively at an expression well-known from Matsubara technique:

$$\{\;\} = -\frac{4i}{\beta}\sum_{n=-\infty}^{\infty}\int\frac{d^3\bar{k}}{(2\pi)^3}\;\frac{\hat{\Pi}(-k_E^2)}{k_E^2}\;\gamma(\tilde{k}_E)\left[\frac{\cosh\gamma(\tilde{k}_E)a-\cos k_3a}{\sinh\gamma(\tilde{k}_E)a}\right] ,$$

$$k_E^2=\left(\frac{2\pi n}{\beta}\right)^2+\bar{k}^2 ,$$
$$\gamma^2(\tilde{k}_E)=k_E^2-k_3^2 \quad . \qquad (4.12)$$

Subtracting an infinite a-independent term from (4.12) and using complex integration for the k_3-integral we find

$$\{\,\} = \frac{4}{\beta} \sum_{n=-\infty}^{\infty} \iint \frac{dk_1 dk_2}{(2\pi)^3} \int_0^{\infty} dk_3 \; \frac{\gamma(\tilde{k}_E)}{\gamma^2(\tilde{k}_E) - k_3^2} \cdot \frac{\left[e^{-\gamma(\tilde{k}_E)a} - e^{-k_3 a}\right]}{\sinh \gamma(\tilde{k}_E) a} \, disc\, \hat{\Pi}(k_3^2 - \gamma^2(\tilde{k}_E)) \,,$$

$$disc\, \hat{\Pi}(k^2) = \hat{\Pi}_-(k^2) - \hat{\Pi}_+(k^2) \,. \tag{4.13}$$

With the help of (A 9) we arrive at

$$\{\,\} = -\frac{2ie^2}{\beta(2\pi)^3} \sum_{n=-\infty}^{\infty} \int_{\frac{2\pi|n|a}{\beta}}^{\infty} ds\, s^2 \int_0^1 dz\, (1-z^2) \int_{\frac{2m}{\sqrt{1-z^2}}}^{\infty} dt\, \frac{1}{t\sqrt{t^2+s^2}} \left[\frac{e^{-sa} - e^{-\sqrt{t^2+s^2}\,a}}{\sinh sa}\right] . \tag{4.14}$$

For further evaluations additional approximations are necessary to obtain explicit results for physically relevant sectors of the parameters m, a, β . In every realistic experimental situation the plate distance a is large as compared with the Compton wavelength m^{-1}, therefore we have the condition $a\,m \gg 1$. So we can write (4.14) approximately as

$$\{\,\} = -\frac{2i}{(2\pi)^3} \frac{e^2}{\beta a^2} \sum_{n=-\infty}^{\infty} \int_{\frac{2\pi|n|a}{\beta}}^{\infty} d\sigma\, \sigma^2 \frac{e^{-\sigma}}{\sinh \sigma} \cdot \int_0^1 dz\, (1-z^2) \int_{\frac{2ma}{\sqrt{1-z^2}}}^{\infty} d\tau\, \tau^{-2}$$

$$= -\frac{i}{(2\pi)^2} \frac{3}{64} \frac{e^2}{\beta m a^3} \left[\zeta(3) + \sum_{n=1}^{\infty} \int_{\frac{4\pi n a}{\beta}}^{\infty} d\varrho \, \frac{\varrho^2}{e^{\varrho} - 1}\right] . \tag{4.15}$$

With the help of (A 30) we find the final result for the correction to the free energy in order e^2:

$$F(a,\beta)_1 = \frac{1}{(2\pi)^2} \frac{3}{128} \frac{e^2}{\beta m a^3} \sum_{k=1}^{\infty} \left[\frac{1}{k^3} \frac{\cosh \frac{2k\pi a}{\beta}}{\sinh \frac{2k\pi a}{\beta}} + \frac{\left(\frac{2\pi a}{\beta}\right)}{k^2} \frac{1}{\sinh^2 \frac{2k\pi a}{\beta}} + \frac{\left(\frac{2\pi a}{\beta}\right)^2}{k} \frac{\cosh \frac{2k\pi a}{\beta}}{\sinh^3 \frac{2k\pi a}{\beta}}\right] , \tag{4.16}$$

$$\beta m \gg 1,\ am \gg 1.$$

Using (B 5) we can rewrite the result (4.16) equivalently as

$$F(a,\beta)_1 = \frac{\pi^2}{720} \frac{1}{a^3} \left(\frac{9}{128\pi} \frac{e^2}{ma}\right) + \frac{\pi^2}{45} \frac{a}{\beta^4} \left(\frac{3}{128\pi} \frac{e^2}{ma}\right) - \frac{\pi}{4} \frac{1}{\beta a^2} \left(\frac{3}{128\pi} \frac{e^2}{ma}\right) \sum_{k=1}^{\infty} \frac{1}{k} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh^3 \frac{k\pi\beta}{2a}} , \tag{4.17}$$

$$\beta m \gg 1,\ am \gg 1 .$$

The correction to the Casimir force can be obtained from (4.16), (4.17) with the help of relation (3.24).

Finally we will discuss the results for the special cases $\frac{a}{\beta} \ll 1$ and $\frac{\beta}{a} \ll 1$. Considering the case $\frac{a}{\beta} \ll 1$ we have also to respect the condition $(a\, m) \gg 1$, so that we are rather looking for the region $m^{-1} \ll a \ll \beta$. This obviously leads to some kind of low temperature expansion in relation to the electron mass m as well as to the inverse plate distance a^{-1}. We get from (3.21) and (4.17) the expression (up to exponentially vanishing terms and terms not depending on the plate distance a)

$$F(a,\beta) \underset{(m^{-1} \ll a \ll \beta)}{=} -\frac{\pi^2}{720}\frac{1}{a^3}\left[1 - \frac{9}{128\pi}\frac{e^2}{ma}\right] + \frac{\pi^2}{45}\frac{a}{\beta^4} + O(e^4) \tag{4.18}$$

Correspondingly the Casimir force in the region $m^{-1} \ll a \ll \beta$ is given by

$$p(a,\beta) \underset{(m^{-1} \ll a \ll \beta)}{=} -\frac{\pi^2}{240}\frac{1}{a^4}\left[1 + \frac{16}{3}\left(\frac{a}{\beta}\right)^4 - \frac{3}{32\pi}\frac{e^2}{ma}\right] + O(e^4) \tag{4.19}$$

Now we look for the limit $\frac{\beta}{a} \ll 1$. Here we have to be careful due to the condition $m\beta \gg 1$, which has to be satisfied, too .

So we have rather to look for the region $m^{-1} \ll \beta \ll a$. This corresponds to a high temperature expansion in relation to the relevant inverse plate distance a^{-1}, but it remains a low temperature expansion in relation to the electron mass m. From (4.16) and (3.21) (for the latter (B 3), (B 4) have to be applied) we read in the region $m^{-1} \ll \beta \ll a$ the expression (up to exponentially vanishing terms)

$$F(a,\beta) \underset{(m^{-1} \ll \beta \ll a)}{=} -\frac{\zeta(3)}{8\pi}\frac{1}{\beta a^2}\left[1 - \frac{3}{64\pi}\frac{e^2}{ma}\right] + O(e^4) \tag{4.20}$$

and the Casimir force is given by

$$p(a,\beta) \underset{m^{-1} \ll \beta \ll a}{=} -\frac{\zeta(3)}{4\pi}\frac{1}{\beta a^3}\left[1 - \frac{9}{128\pi}\frac{e^2}{ma}\right] + O(e^4) . \tag{4.21}$$

One can see, that the 1-loop corrections to the Casimir force are very small and far from experimental verification. Finally we would like to remark an interesting property of (4.21). This expression looks like a classical one. It does not contain Planck's constant, though it has been derived by a quantum field theoretic calculation including radiative corrections.

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Appendix A

Here we list important conventions and formulae.

$\tilde{k} = (k_0, k_1, k_2)$

$$\Gamma_{\pm}(\tilde{k}) = \sqrt{k_0^2 - k_1^2 - k_2^2 \pm i\varepsilon}\,, \quad Im\,\Gamma_{\pm}(\tilde{k}) \gtrless 0 \qquad \text{(A 1)}$$

Massless scalar propagator at T = 0

$$D^c(x) = -\int \frac{d^4k}{(2\pi)^4} \frac{e^{ikx}}{k^2 + i\varepsilon} \qquad \text{(A 2)}$$

$$= -\frac{1}{2i}\int \frac{d^3\tilde{k}}{(2\pi)^3} \frac{e^{i\tilde{k}\tilde{x}}}{\Gamma_+(\tilde{k})} e^{i\Gamma_+(\tilde{k})|x_3|} \qquad \text{(A 3)}$$

Massless scalar propagator at T ≠ 0

$$D^c_{\beta\,ab}(x) = \int \frac{d^4k}{(2\pi)^4} e^{ikx} \left[B_+(k_0)\,\hat{D}^c_\beta(k)\,B_+(k_0)\right]_{ab} \qquad \text{(A 4)}$$

$$B_+(k_0) = \begin{pmatrix} \cosh\theta & \sinh\theta \\ \sinh\theta & \cosh\theta \end{pmatrix}, \quad \cosh^2\theta = \frac{1}{1 - e^{-\beta|k_0|}} \qquad \text{(A 5)}$$

$$\hat{D}^c_\beta(k) = \begin{pmatrix} \frac{-1}{k^2 + i\varepsilon} & 0 \\ 0 & \frac{1}{k^2 - i\varepsilon} \end{pmatrix} \qquad \text{(A 6)}$$

Electron propagator at T = 0

$$S^c(x) = -\int \frac{d^4k}{(2\pi)^4} e^{ikx} \frac{\not{k} - m}{k^2 - m^2 + i\varepsilon} \qquad \text{(A 7)}$$

Polarization operator at T = 0

$$\Pi_{\mu\nu}(x) = -ie^2 tr\left[\gamma_\mu S^c(x)\gamma_\nu S^c(-x)\right]$$

$$= \int \frac{d^4k}{(2\pi)^4} e^{ikx}\, \hat{\Pi}_{\mu\nu}(k) \qquad \text{(A 8)}$$

$$\hat{\Pi}_{\mu\nu}(k) = -\left[g_{\mu\nu}k^2 - k_\mu k_\nu\right]\hat{\Pi}(k^2)$$

$$\hat{\Pi}_{\pm}(k^2)_{ren} = \frac{e^2}{8\pi^2}\int_0^1 dz\,(1 - z^2)\ln\left[1 - (1 - z^2)\frac{k^2 \pm i\varepsilon}{4m^2}\right] \qquad \text{(A 9)}$$

Electron propagator at T ≠ 0

$$S^c_{\beta\,ab}(x) = \int \frac{d^4k}{(2\pi)^4} e^{ikx}\, \hat{S}^c_{\beta\,ab}(k)$$

$$= \int \frac{d^4k}{(2\pi)^4} e^{ikx}\left[B_-(k_0)\,\hat{S}^c_\beta(k)\,B_-^T(k_0)\right]_{ab} \qquad \text{(A 10)}$$

$$B_-(k_0) = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}, \quad \cos^2\theta = \frac{1}{1+e^{-\beta[|k_0|-\mu]}} \tag{A 11}$$

$$\hat{S}^c_\beta(k) = \begin{pmatrix} -\dfrac{\not{k}-m}{k^2-m^2+i\varepsilon} & 0 \\ 0 & -\dfrac{\not{k}-m}{k^2-m^2-i\varepsilon} \end{pmatrix} \tag{A 12}$$

$$\hat{S}^c_{\beta\,11}(k) = -(\not{k}-m)\left[\frac{1}{k^2-m^2+i\varepsilon} + 2\pi i\,\frac{\delta(k^2-m^2)}{1+e^{\beta[|k_0|-\mu]}}\right] \tag{A 13}$$

Polarization operator at T $\neq$ 0

$$\Pi_{\beta\,\mu\nu}^{\;ab}(x) = -ie^2 tr\left[\gamma_\mu S_\beta^{c\;ef}(x)\, g^a_{ef}\, \gamma_\nu S_\beta^{c\;\bar{e}\bar{f}}\, g^b_{\bar{e}\bar{f}}\right]$$

$$g^a_{bc} = \begin{cases} 1 & a=b=c=1 \\ -1 & a=b=c=2 \\ 0 & \text{otherwise} \end{cases}$$

$$= \int\frac{d^4k}{(2\pi)^4}\, e^{ikx}\, \hat{\Pi}_{\beta\,\mu\nu}^{\;ab}(k) \tag{A 14}$$

Photon propagator at T $\neq$ 0 without boundary conditions

$$D^{c\;\mu\nu}_{\beta\;ab}(x) = \int\frac{d^4k}{(2\pi)^4}\, e^{ikx}\, \hat{D}^{c\;\mu\nu}_{\beta\;ab}(k)$$

$$= \int\frac{d^4k}{(2\pi)^4}\, e^{ikx}\left[B_+(k_0)\,\hat{D}^{c\;\mu\nu}_{\beta}(k)\, B_+(k_0)\right]_{ab} \tag{A 15}$$

$$\hat{D}^{c\;\mu\nu}_\beta(k) = \begin{pmatrix} \dfrac{-1}{k^2+i\varepsilon}\left[g^{\mu\nu}-(1-\alpha)\dfrac{k^\mu k^\nu}{k^2+i\varepsilon}\right] & 0 \\ 0 & \dfrac{1}{k^2-i\varepsilon}\left[g^{\mu\nu}-(1-\alpha)\dfrac{k^\mu k^\nu}{k^2-i\varepsilon}\right] \end{pmatrix} \tag{A 16}$$

Boundary correction to the photon propagator at T $\neq$ 0

$$\bar{D}_{\beta\;ab}^{\;\mu\nu}(\tilde{x}-\tilde{y}; x_3, y_3) = \int\frac{d^3\tilde{k}}{(2\pi)^3}\, e^{i\tilde{k}(\tilde{x}-\tilde{y})}\, \hat{\bar{D}}_{\beta\;ab}^{\;\mu\nu}(\tilde{k}; x_3, y_3)$$

$$= \int\frac{d^3\tilde{k}}{(2\pi)^3}\, e^{i\tilde{k}(\tilde{x}-\tilde{y})}\left[B_+(k_0)\,\hat{\bar{D}}_\beta^{\;\mu\nu}(\tilde{k}; x_3, y_3)\, B_+(k_0)\right]_{ab} \tag{A 17}$$

$$\hat{\bar{D}}_\beta^{\;\mu\nu}(\tilde{k}; x_3, y_3) = \frac{1}{2i}\begin{pmatrix} Z^{\mu\nu}(\tilde{k}; x_3, y_3) & 0 \\ 0 & Z^{*\,\mu\nu}(\tilde{k}; x_3, y_3) \end{pmatrix} \tag{A 18}$$

$$Z^{\mu\nu}(\tilde{k}; x_3, y_3) = \left[\tilde{g}^{\mu\nu} - \frac{\tilde{k}^\mu \tilde{k}^\nu}{\Gamma_+^2(\tilde{k})}\right] e^{i\Gamma_+(\tilde{k})|x_3-a_k|}\, \frac{[h^{-1}(\tilde{k})]_{kl}}{\Gamma_+(\tilde{k})}\, e^{i\Gamma_+(\tilde{k})|y_3-a_l|}$$

$$Z^{*\,\mu\nu}(\tilde{k}; x_3, y_3) = \left[\tilde{g}^{\mu\nu} - \frac{\tilde{k}^\mu \tilde{k}^\nu}{\Gamma_-^2(\tilde{k})}\right] e^{-i\Gamma_-(\tilde{k})|x_3-a_k|}\, \frac{[h^{-1}(\tilde{k})]^*_{kl}}{\Gamma_-(\tilde{k})}\, e^{-i\Gamma_-(\tilde{k})|y_3-a_l|}$$

for $\mu, \nu \neq 3$

$$Z^{\mu\nu}(\tilde{k}; x_3, y_3) = Z^{*\,\mu\nu}(\tilde{k}; x_3, y_3) = 0$$

for $\nu = 3$
or/and $\mu = 3$

$$\bar{D}_{\beta ab}(\tilde{x}-\tilde{y}; x_3, y_3) = \frac{1}{2} g_{\mu\nu} \bar{D}_{\beta}{}^{\mu\nu}_{ab}(\tilde{x}-\tilde{y}; x_3, y_3)$$

$$Z(\tilde{k}; x_3, y_3) = \frac{1}{2} g_{\mu\nu} Z^{\mu\nu}(\tilde{k}; x_3, y_3)$$

$$Z^*(\tilde{k}; x_3, y_3) = \frac{1}{2} g_{\mu\nu} Z^{*\mu\nu}(\tilde{k}; x_3, y_3) \qquad \text{(A 20)}$$

$$h(\tilde{k})_{kl} = e^{i\Gamma_+(\tilde{k})|a_k - a_l|}, \quad h^*(\tilde{k})_{kl} = e^{-i\Gamma_-(\tilde{k})|a_k - a_l|} \qquad \text{(A 21)}$$

$$[h^{-1}(\tilde{k})]_{kl} = -i \frac{[1 - \delta_{kl}(1 + e^{-i\Gamma_+(\tilde{k})a})]}{2 \sin \Gamma_+(\tilde{k})a}$$

$$[h^{-1}(\tilde{k})]^*_{kl} = i \frac{[1 - \delta_{kl}(1 + e^{i\Gamma_-(\tilde{k})a})]}{2 \sin \Gamma_-(\tilde{k})a} \qquad \text{(A 22)}$$

$$a = |a_1 - a_2|$$

$$[h^{-1}(\tilde{k})]_{kl}\, |a_k - a_l| = -\frac{ia}{\sin \Gamma_+(\tilde{k})a} \qquad \text{(A 23)}$$

$$\int_{-\infty}^{\infty} dx_3\, Z(\tilde{k}; \xi_3 + x_3, x_3 + \delta) \underset{\delta \to 0}{=}$$

$$\frac{[h^{-1}(\tilde{k})]_{kl}}{\Gamma_+(\tilde{k})}\, e^{i\Gamma_+(\tilde{k})|a_k - a_l - \xi_3|}\left[|a_k - a_l - \xi_3| + \frac{i}{\Gamma_+(\tilde{k})}\right]$$

$$\int_{-\infty}^{\infty} dx_3\, Z^*(\tilde{k}; \xi_3 + x_3, x_3 + \delta) \underset{\delta \to 0}{=}$$

$$\frac{[h^{-1}(\tilde{k})]^*_{kl}}{\Gamma_-(\tilde{k})}\, e^{-i\Gamma_-(\tilde{k})|a_k - a_l - \xi_3|}\left[|a_k - a_l - \xi_3| - \frac{i}{\Gamma_-(\tilde{k})}\right] \qquad \text{(A 24)}$$

$$\int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_0^2 \\ k_1^2 \end{Bmatrix} \left[\frac{\cosh^2\theta}{\Gamma_+^2(\tilde{k})} - \frac{\sinh^2\theta}{\Gamma_-^2(\tilde{k})}\right] = -i\, \frac{\zeta(3)}{2\pi} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \frac{1}{\beta^3}$$

$$+ \text{ irrel. div.} \qquad \text{(A 25)}$$

$$\int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_0^2 \\ k_1^2 \end{Bmatrix} \sinh^2\theta \left[\frac{1}{\Gamma_+(\tilde{k})} + \frac{1}{\Gamma_-(\tilde{k})}\right] = \frac{\pi^2}{45} \begin{Bmatrix} 3 \\ 1 \end{Bmatrix} \frac{1}{\beta^4} \qquad \text{(A 26)}$$

$$\int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_0^2 \\ k_1^2 \end{Bmatrix} \frac{e^{i\Gamma_+(\tilde{k})a}}{\Gamma_+(\tilde{k}) \sin \Gamma_+(\tilde{k})a} = i\, \frac{\pi^2}{720} \begin{Bmatrix} 1 \\ -1 \end{Bmatrix} \frac{1}{a^4} \qquad \text{(A 27)}$$

$$\int \frac{d^3\tilde{k}}{(2\pi)^3} \begin{Bmatrix} k_0^2 \\ k_1^2 \end{Bmatrix} \sinh^2\theta \left[\frac{\cos \Gamma_+(\tilde{k})a}{\Gamma_+(\tilde{k}) \sin \Gamma_+(\tilde{k})a} - \frac{\cos \Gamma_-(\tilde{k})a}{\Gamma_-(\tilde{k}) \sin \Gamma_-(\tilde{k})a}\right]$$

$$= -\frac{i}{2\pi} \frac{1}{a\beta^3} \sum_{k=1}^{\infty} \left[\frac{1}{k^3} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh \frac{k\pi\beta}{2a}}\right.$$

$$\left. + \frac{\left(\frac{\pi\beta}{2a}\right)}{k^2} \begin{Bmatrix} 2 \\ 1 \end{Bmatrix} \frac{1}{\sinh^2 \frac{k\pi\beta}{2a}} + \frac{\left(\frac{\pi\beta}{2a}\right)^2}{k} \begin{Bmatrix} 2 \\ 0 \end{Bmatrix} \frac{\cosh \frac{k\pi\beta}{2a}}{\sinh^3 \frac{k\pi\beta}{2a}}\right] \qquad \text{(A 28)}$$

All these integrals (A 25) - (A 28) can be evaluated conveniently with the help of hyperbolic coordinates. In every case the $i\varepsilon$ -prescriptions have to be respected carefully.

$$\iint dz_3 dx_3 \; e^{ik_3(x_3-z_3)} \; \hat{\tilde{D}}_\beta{}^{\mu\nu}_{11}(\tilde{k}; z_3, x_3)$$

$$= -2 \left\{ \left[\tilde{g}^{\mu\nu} - \frac{\tilde{k}^\mu \tilde{k}^\nu}{\Gamma_+^2(\tilde{k})}\right] \frac{\Gamma_+(\tilde{k}) \cosh^2\theta}{[\Gamma_+^2(\tilde{k}) - k_3^2]^2} \left[\frac{\cos\Gamma_+(\tilde{k})a - \cos k_3 a}{\sin \Gamma_+(\tilde{k})a}\right] \right.$$

$$\left. - \left[\tilde{g}^{\mu\nu} - \frac{\tilde{k}^\mu \tilde{k}^\nu}{\Gamma_-^2(\tilde{k})}\right] \frac{\Gamma_-(\tilde{k}) \sinh^2\theta}{[\Gamma_-^2(\tilde{k}) - k_3^2]^2} \left[\frac{\cos\Gamma_-(\tilde{k})a - \cos k_3 a}{\sin \Gamma_-(\tilde{k})a}\right] \right\}$$

for $\mu, \nu \neq 3$

$= 0$ for $\nu = 3$ or/and $\mu = 3$

(A 29)

$$\sum_{n=1}^{\infty} \int_{\pi n \xi}^{\infty} d\rho \, \frac{\rho^2}{e^\rho - 1} = -\zeta(3) + \sum_{k=1}^{\infty} \left[\frac{1}{k^3} \frac{\cosh \frac{k\pi\xi}{2}}{\sinh \frac{k\pi\xi}{2}} \right.$$

$$\left. + \frac{\left(\frac{\pi\xi}{2}\right)}{k^2} \frac{1}{\sinh^2 \frac{k\pi\xi}{2}} + \frac{\left(\frac{\pi\xi}{2}\right)^2}{k} \frac{\cosh \frac{k\pi\xi}{2}}{\sinh^3 \frac{k\pi\xi}{2}} \right]$$

(A 30)

Appendix B

Following Brown/Maclay /4/ we derive some relations which are useful for the asymptotic discussion of the evaluated thermodynamic quantities (3.20) - (3.22), (4.16). The starting point is the following expression.

$$f(\xi) = \sum_{k,m=1}^{\infty} \frac{\xi^4}{[k^2 + m^2 \xi^2]^2} \qquad \text{(B 1)}$$

There holds the relation

$$f(\xi) = \xi^4 f(\xi^{-1}) \qquad \text{(B 2)}$$

Performing one summation in (B 1) we arrive at the expression

$$f(\xi) = -\frac{\pi^4}{180} \xi^4 + \frac{\pi}{4} \xi^2 \sum_{k=1}^{\infty} \left[\xi \frac{1}{k^3} \frac{\cosh k\pi\xi^{-1}}{\sinh k\pi\xi^{-1}} \right.$$

$$\left. + \frac{\pi}{k^2} \frac{1}{\sinh^2 k\pi\xi^{-1}} \right] \qquad \text{(B 3)}$$

Due to (B 2) this equals

$$f(\xi) = -\frac{\pi^4}{180} + \frac{\pi}{4} \xi \sum_{k=1}^{\infty} \left[\frac{1}{k^3} \frac{\cosh k\pi\xi}{\sinh k\pi\xi} \right.$$

$$\left. + \xi \frac{\pi}{k^2} \frac{1}{\sinh^2 k\pi\xi} \right] \qquad \text{(B 4)}$$

Taking into account (B 3) and (B 4) the expression $f(\xi)-\xi f'(\xi)$ yields the following identity

$$\frac{\pi^3}{30}\xi + \frac{\pi^3}{90}\xi^{-3} - \pi^2 \sum_{k=1}^{\infty} \frac{1}{k}\frac{\cosh k\pi\xi}{\sinh^3 k\pi\xi}$$

$$= \sum_{k=1}^{\infty}\left[\frac{1}{k^3}\frac{\cosh k\pi\xi^{-1}}{\sinh k\pi\xi^{-1}} + \frac{\pi}{k^2}\xi^{-1}\frac{1}{\sinh^2 k\pi\xi^{-1}} + \frac{\pi^2}{k}\xi^{-2}\frac{\cosh k\pi\xi^{-1}}{\sinh^3 k\pi\xi^{-1}}\right] \tag{B 5}$$

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