PHE 87-13

PROCEEDINGS

OF THE

XXI INTERNATIONAL SYMPOSIUM AHRENSHOOP

on the

Theory of Elementary Particles

Sellin, October 12-16, 1987

ed. by E. Wieczorek

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AKADEMIE DER WISSENSCHAFTEN DER DDR
BERLIN-ZEUTHEN · DDR

December 1987

NON-ABELIAN GAUGE THEORY IN A HOMOGENEOUS BACKGROUND FIELD

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Classical fields have been considered in the past either as characterizing a non-trivial ground state of QFT [1] or as external fields, i.e. as fields describing the influence of the environment on the quantum system under investigation [2]. Homogeneous background fields should rather be taken as external ones because the imaginary part of the effective potential could be understood as an indication of instability.

The aim of our contribution is to clarify the physical role of Im V_{eff} by going beyond the 1-loop level. In Part 1 we present new results on the gauge field propagator. The structure of the renormalization terms for the polarization tensor obtained in Part 2 will be used in Part 3 to study the properties of Im V_{eff} at the 2-loop level.

1. The gauge field propagator in background fields

Choosing the gauge fixing term in covariant form

$$\mathcal{L}_{gf} = -\frac{1}{2\alpha}\left(D_\mu(B)\,a_\mu\right)^2 \qquad (1.1)$$

where $B_\mu(F_{\mu\nu})$ is a solution of the classical field equations $D^{ab}_\mu(B)F^b_{\mu\nu}=0$ the Lagrangian starts with the quadratic term

$$\mathcal{L} = \frac{1}{2}\, a^a_\mu\, K^{ab}_{\mu\nu}\, a^b_\nu + \frac{1}{2}\, \bar{c}^a K^{ab} c^b \qquad (1.2)$$

where

$$\begin{aligned} \underset{x}{K}{}^{ab}_{\mu\nu} &= \left(\underset{x}{D}{}^2 \delta_{\mu\nu} + \left(\tfrac{1}{\alpha}-1\right)\underset{x}{D}_\mu \underset{x}{D}_\nu\right)^{ab} + 2g\, f^{abc} F^c_{\mu\nu}(x) \\ \underset{x}{K}{}^{ab} &= \left(\underset{x}{D}{}^2\right)^{ab} \\ \underset{x}{D}{}^{ab}_\mu(B) &= \delta^{ab}\frac{\partial}{\partial x_\mu} + g\, f^{abc} B^c_\mu(x). \end{aligned} \qquad (1.3)$$

Here and in the following Euclidean QFT is always understood. Gauge and ghost field propagators are defined by

$$\underset{x}{K}{}^{ab}_{\lambda\mu}(\alpha)\, G^{bc}_{\mu\nu}(x,y,\alpha) = -\delta^{ac}\delta_{\lambda\nu}\,\delta(x-y), \qquad \underset{x}{K}{}^{ab} G^{bc}(x,y) = -\delta^{ac}\,\delta(x-y). \qquad (1.4)$$

From the identity

$$D^{ab}_\mu\left\{\delta_{\mu\nu}(D^2)^{bc} + \left(\tfrac{1}{\alpha}-1\right)(D_\mu D_\nu)^{bc} + 2g f^{bcd} F^d_{\mu\nu}\right\} = \frac{1}{\alpha}\left(D^2 D_\nu\right)^{ac} \qquad (1.5)$$

we obtain without specifying gauge group or background field

$$\underset{x}{D}{}^{ab}_\mu\, G^{bc}_{\mu\nu}(x,y,\alpha)\, \underset{y}{D}{}^{cd}_\nu = -\alpha\,\delta^{ad}\,\delta(x-y) \qquad (1.6)$$

and

$$\underset{x}{D}{}^{ab}_\mu\, G^{bc}_{\mu\nu}(x,y,\alpha) = -\alpha\, G^{ab}(x,y)\, \underset{y}{D}{}^{bc}_\nu. \qquad (1.7)$$

Both relations have to be understood as functional equalities. Using (1.4) and (1.6) we express [3] $G^{ab}(x,y,\alpha)$ i.e. the gauge propagator for arbitrary gauge parameter α in terms of the special gauge propagator $G^{ab}(x,y,1)$

$$G^{ab}_{\mu\nu}(x,y,\alpha) = G^{ab}_{\mu\nu}(x,y,1) + (1-\alpha)\int dz\, G^{ac}_{\mu\kappa}(x,z,1)\left(\underset{z}{D}_\kappa \underset{z}{D}_\lambda\right)^{cd} G^{db}_{\lambda\nu}(z,y,1). \qquad (1.8)$$

This relation reduces the problem of finding the gauge propagator to the easier case $\alpha=1$. Here a warning should be kept in mind. In order to apply (1.8) we have to assume that $D_\mu G_{\mu\nu}(z,y,1)$ vanishes sufficiently fast for $z \to \infty$ as to allow partial integration. Therefore without knowledge of the long-distance behaviour of the special propagator $G_{\mu\nu}(x,y,1)$ expression (1.8) will remain a formal one.

Now we restrict ourselves to the gauge group SU(2) and to the special background field

$$B^a_\mu(x) = -\tfrac{1}{2} F^a_{\mu\nu} x_\nu \qquad\qquad \varepsilon^\perp_{\mu\nu} = \begin{pmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$
$$F^a_{\mu\nu} = B\,\delta^{a3}\varepsilon^\perp_{\mu\nu} \tag{1.9}$$

i.e. to a constant magnetic field with a 3rd colour component only. Gauge and ghost propagators for the special case $\alpha=1$ have been evaluated earlier /4/ with the result

$$G^{ab}(x,y) = \phi^{ab}(x,y)\,D^0(x-y) + \frac{\delta^{a3}\delta^{b3}}{4\pi^2(x-y)^2}\,, \tag{1.10}$$

$$\begin{aligned} G^{ab}_{\mu\nu}(x,y,1) = {} & \phi^{ab}(x,y)\left[\delta^{\parallel}_{\mu\nu} D^0(x-y) + \delta^{\perp}_{\mu\nu} D^P(x-y)\right] \\ & + \tilde{\phi}^{ab}(x,y)\,\varepsilon^\perp_{\mu\nu} D^m(x-y) + \frac{\delta^{a3}\delta^{b3}}{4\pi^2(x-y)^2}\,\delta_{\mu\nu} \end{aligned} \tag{1.11}$$

where

$$D^P = \frac{D^+ + D^-}{2}\,, \qquad D^m = \frac{D^+ - D^-}{2} \tag{1.12}$$

$$D^0(x-y) = \frac{1}{gB}\int\frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)} \int_0^\infty \frac{d\tau}{\cosh\tau}\, e^{-\frac{k_\parallel^2}{gB}\tau - \frac{k_\perp^2}{gB}\tanh\tau} \tag{1.13}$$

$$D^-(x-y) = \frac{1}{gB}\int\frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)} \int_0^\infty \frac{d\tau}{\cosh\tau}\, e^{-2\tau - \frac{k_\parallel^2}{gB}\tau - \frac{k_\perp^2}{gB}\tanh\tau} \tag{1.14}$$

$$\begin{aligned} D^+(x-y) = {} & \frac{1}{gB}\int\frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)}\, e^{-\frac{k_\perp^2}{gB}} \times \\ & \left\{ \int_0^\infty \frac{d\tau}{\cosh\tau}\, e^{\left(2-\frac{k_\parallel^2}{gB}\right)\tau}\left[e^{\frac{k_\perp^2}{gB}(1-\tanh\tau)} - 1 - e^{-2\tau}\right] + \frac{2}{\frac{k_\parallel^2}{gB}-1} \right\} \end{aligned} \tag{1.15}$$

and

$$\begin{aligned} \phi^{ab}(x,y) &= \delta^{ab}_\perp \cos\varrho(x,y) + \varepsilon^{ab3}\sin\varrho(x,y) \\ \tilde{\phi}^{ab}(x,y) &= -\delta^{ab}_\perp \sin\varrho(x,y) + \varepsilon^{ab3}\cos\varrho(x,y) \\ \varrho(x,y) &= \frac{gB}{2}(x_1 y_2 - x_2 y_1). \end{aligned} \tag{1.16}$$

Furthermore the notations

$$\begin{aligned} x_\perp^2 &= x_1^2 + x_2^2 \\ x_\parallel^2 &= x_3^2 + x_4^2 \qquad\qquad \delta^\perp_{\mu\nu} = \begin{pmatrix} 1 & & & \\ & 1 & & \\ & & 0 & \\ & & & 0 \end{pmatrix} \qquad\qquad \delta^\parallel_{\mu\nu} = \begin{pmatrix} 0 & & & \\ & 0 & & \\ & & 1 & \\ & & & 1 \end{pmatrix} \\ \tilde{x}_\mu &= \varepsilon^\perp_{\mu\nu} x_\nu \\ \delta^{ab}_\perp &= \delta^{a1}\delta^{b1} + \delta^{a2}\delta^{b2} \end{aligned} \tag{1.17}$$

have been used. By the way, ϕ^{ab} is nothing else but the phase factor $\exp ig\int_x^y d\xi_\mu B_\mu$ in the adjoint representation, taken along the straight line connecting x and y. The propagators (1.13-15) appear in a real proper-time representation in contrast to earlier attempts /5/. The pole term in (1.15) which is due to the negative modes of the gauge kernel has been specified as a distribution with non-zero imaginary part in accordance with the imaginary part in the 1-loop effective action. A detailed discussion of the propagators (1.10), (1.11) has been given in /3/.

At short distances one is interested in the leading and in the background-dependent non-leading terms. We note the following results

$$G^{ab}_{\mu\nu}(x,y,1) \approx \left(\phi^{ab} + \delta^{a3}\delta^{b3}\right)\frac{\delta_{\mu\nu}}{4\pi^2(x-y)^2} - \tilde{\phi}^{ab}\varepsilon^\perp_{\mu\nu}\frac{gB}{8\pi^2}\ln\left(\frac{gB}{4}(x-y)^2\right) + \text{f.p.} \tag{1.18}$$

$$G^{ab}(x,y) \sim \left(\phi^{ab} + \delta^{a3}\delta^{b3}\right) \frac{1}{4\pi^2 (x-y)^2} \quad + f.p. \tag{1.19}$$

for $(x-y)^2 \to 0$. This result could have been also obtained directly from a Schwinger-DeWitt analysis /6/ without knowing the propagators completely. The large-distance behaviour is given by

$$G^{ab}_{\mu\nu}(x,y,1) \sim \frac{1 \mp i}{8\pi^{3/2}} (gB)^{3/4} \left[(x_{\parallel}-y_{\parallel})^2\right]^{-1/4} e^{\mp i \sqrt{gB(x_{\parallel}-y_{\parallel})^2} - \frac{gB}{4}(x_{\perp}-y_{\perp})^2}$$
$$\times \left[\phi^{ab}\delta^{\perp}_{\mu\nu} + \tilde{\phi}^{ab}\varepsilon^{\perp}_{\mu\nu}\right] + \frac{\delta^{a3}\delta^{b3}\delta_{\mu\nu}}{4\pi^2(x-y)^2} \tag{1.20}$$

$$G^{ab}(x,y) \sim \frac{(gB)^{3/4}}{2\cdot(2\pi)^{3/2}} \left[(x_{\parallel}-y_{\parallel})^2\right]^{-1/4} e^{-\sqrt{gB(x_{\parallel}-y_{\parallel})^2} - \frac{gB}{4}(x_{\perp}-y_{\perp})^2} \phi^{ab}$$
$$+ \frac{\delta^{a3}\delta^{b3}}{4\pi^2(x-y)^2} \tag{1.21}$$

for $(x-y)^2 \to \infty$. It is the behaviour (1.20) which decides whether the construction (1.8) for the general propagator makes sense or not. A careful investigation shows that the slowly decreasing term of (1.20) is projected to zero by D_μ so that (1.8) is well-defined. The second term in (1.8) can be rewritten in the form /3/

$$\int dz \left[G_{\alpha\lambda}(x,z,1) \underset{z}{D_\lambda} \underset{z}{D_\mu} G_{\mu\nu}(z,y,1)\right]^{ae} =$$
$$\phi^{ae}(x,y)\, \mathrm{Re}\, Q_{\alpha\nu}(x-y) + \tilde{\phi}^{ae}(x,y)\, \mathrm{Im}\, Q_{\alpha\nu}(x-y) - \delta^{a3}\delta^{e3} \int \frac{d^4k}{(2\pi)^4} e^{ik(x-y)} \frac{k_\alpha k_\nu}{k^4} \tag{1.22}$$

with

$$Q_{\alpha\nu}(x-y) = \int d\chi\, e^{i\frac{gB}{2}\xi\tilde{\eta}} \left(\frac{\partial}{\partial \xi_\alpha} - \frac{igB}{2}\tilde{\xi}_\alpha\right) D^0(\xi) \left(\frac{\partial}{\partial \eta_\nu} + \frac{igB}{2}\tilde{\eta}_\nu\right) D^0(\eta)$$
$$\xi = \frac{x-y}{2} + \chi, \qquad \eta = \frac{x-y}{2} - \chi. \tag{1.23}$$

Concerning (1.22) two points should be emphasized. At first, the final form of the general propagator is almost translation invariant (i.e. of the form phase factor times function of

the difference coordinate) which could not have been anticipated from the left-hand side of (1.22). Furthermore, the α-dependent part does not contain any imaginary part. In other words, the imaginary part in (1.11) cannot be modified by the gauge fixing term in (1.3). This fact is in accordance with the point of view which connects $\mathrm{Im}\, G_{\mu\nu}$ with a physical quantity namely the quantum instability of homogeneous magnetic field /5/.

2. Renormalization of the polarization tensor

Because of the lack of full translation invariance of the propagators all Feynman diagrams will be studied in x-space. The x-space vertices are of the same algebraic structure as in the case $B_\mu = 0$ except that derivatives are to be understood as covariant ones. Prior to any explicit calculation we note the general structure of the polarization tensor

$$\Pi^{ab}_{\mu\nu}(x,y) = \phi^{ab}(x,y)\, \Pi^{(1)}_{\mu\nu}(x-y) + \tilde{\phi}^{ab}(x,y) \quad \Pi^{(2)}_{\mu\nu}(x-y) + \delta^{a3}\delta^{b3}\, \Pi^{(3)}_{\mu\nu}(x-y)$$
$$\Pi^{(1)}_{\nu\mu} = \Pi^{(1)}_{\mu\nu}, \quad \Pi^{(2)}_{\nu\mu} = -\Pi^{(2)}_{\mu\nu} \tag{2.1}$$

which follows from (1.10), (1.11) and the relations

$$\underset{x}{D}{}^{ab}_{\mu} \left\{\phi^{bc}(x,y)\, f(x-y)\right\} = \phi^{ac}\partial_\mu f - gB\, \tilde{\phi}^{ac}(\tilde{x}_\mu - \tilde{y}_\mu) f$$
$$\underset{x}{D}{}^{ab}_{\mu} \left\{\tilde{\phi}^{bc}(x,y)\, f(x-y)\right\} = \tilde{\phi}^{ac}\partial_\mu f + gB\, \phi^{ac}(\tilde{x}_\mu - \tilde{y}_\mu) f. \tag{2.2}$$

Taking into account the complexity of the 3-gluon vertex as well as the unusual form of the gauge propagators the determination of $\Pi^{ab}_{\mu\nu}$ turns out to be a rather complicated task. However, if one is interested in the structure of the renormalization terms only one may restrict the investigation to the fermionic (quark) contribution which is considerably easier than that of the pure gauge sector.

There cannot be any doubt that the fermionic term independently reveals the structure of $\Pi_{\mu\nu}$ dictated by gauge invariance in presence of a background field. We determine the ultraviolet divergent part of the fermionic contribution

$$\Pi_{\mu\nu}^{ab,(ferm)}(x,y) = -g^2\, tr\{t^a \gamma_\mu S(x,y)\, t^b \gamma_\nu S(y,x)\} \tag{2.3}$$

along the following lines. Taking into account the leading and nonleading short-distance singular parts of S(x,y) only we obtain all the uv singular parts of the product (2.3) leaving aside the finite parts, of course. We use, for the moment, a simple light-cone regularization

$$\frac{1}{(x-y)^2} \longrightarrow \frac{1}{(x-y)^2+a^2} \tag{2.4}$$

which like Pauli-Villars regularization suffers from fictitious quadratic singularities beeing absent in a proper gauge invariant regularization. Without going into details we note that usual dimensional regularization poses specific problems in connection with background fields where, due to the complicated structure of the propagators, we have to restrict the considerations to the leading and nonleading singular parts.

The fermionic propagator in the background (1.9) is determined by

$$(i\gamma\partial - gBt_3)\, S(x,y) = -\delta(x-y), \qquad t_3 = \frac{1}{2}\begin{pmatrix}1 & \\ & -1\end{pmatrix} \tag{2.5}$$

(we take massless fermions for simplicity). It can be obtained in terms of the electron propagator in a homogeneous magnetic

background field /7/

$$S(x,y) = \begin{pmatrix} e^{i\varphi/2} S^+(x-y) & 0 \\ 0 & e^{-i\varphi/2} S^-(x-y) \end{pmatrix} \tag{2.6}$$

Here $e^{i\varphi/2} S^+(x-y)$ and $e^{-i\varphi/2} S^-(x-y)$ denote the electron propagator /7/ where in the propagator equation the electric charge e has been replaced by g/2 or -g/2 respectively. The phase $\varphi(x,y)$ is given by (1.16) and $S^\pm$ by

$$\begin{aligned} S^\pm(x-y) = -\frac{2}{gB}\Big\{ &(\gamma\partial)_\perp \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)} \int_0^\infty \frac{d\tau}{\cosh^2\tau}\, e^{-\frac{2k_\parallel^2}{gB}\tau - \frac{2k_\perp^2}{gB}\tanh\tau} \\ &+ (\gamma\partial)_\parallel \int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)} \int_0^\infty \frac{d\tau}{\cosh\tau}\, e^{-\frac{2k_\parallel^2}{gB}\tau - \frac{2k_\perp^2}{gB}\tanh\tau}\, e^{\pm i\sigma_3\tau}\Big\}. \end{aligned} \tag{2.7}$$

The short-distance expansion of $S^\pm$ starts with

$$S^\pm(x-y) = -\frac{1}{2\pi^2}\left\{\frac{\gamma(x-y)}{(x-y)^4} \pm \frac{gB}{8}\,\frac{\gamma_\parallel (x-y)_\parallel}{(x-y)^2}\,\sigma_3\right\} + f.p. \tag{2.8}$$

The uv divergencies in (2.3) arise from multiplying the leading singularities but also from the mixed products. The result depends crucially on the colour indices a and b. For a=b=3 the mixed products as well as the phase factors cancel

$$\Pi_{\mu\nu}^{33}(x,y) = \frac{g^2}{2\pi^4}\,\frac{1}{(x-y)^8}\left[2(x-y)_\mu (x-y)_\nu - \delta_{\mu\nu}(x-y)^2\right]. \tag{2.9}$$

This ill-defined i.e. non-integrable expression can be transformed into standard form either using directly

$$\frac{1}{z^6} = \frac{1}{8}\,\Delta\,\frac{1}{z^4}, \qquad \frac{z_\mu z_\nu}{z^8} = \left[\frac{1}{24}\partial_\mu\partial_\nu + \frac{\delta_{\mu\nu}}{48}\Delta\right]\frac{1}{z^4} \tag{2.1o}$$

or using the regularization (2.4). For regularized expressions the relations (2.1o) are valid op to terms $O(a^{-2})$ only. Using finally

$$\left(\frac{1}{z^4}\right)_{reg.} \equiv \frac{1}{(z^2+a^2)^2} = -\pi^2 \ln(a^2)\,\delta(z) + \text{regular terms} \tag{2.11}$$

we obtain the renormalization term for (2.9)

$$\mathrm{c.t.}\ \Pi^{33}_{\mu\nu} = -\frac{2}{3}\,\frac{g^2}{16\pi^2}\ln(a^2)\,(\partial_\mu\partial_\nu - \delta_{\mu\nu}\Delta)\,\delta(x-y). \tag{2.12}$$

This standard form corresponds to the result /7/ for abelian theory (QED) in a homogeneous background. Significant deviations from the abelian case occur for (a,b = 1,2) because $t^1 = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ and $t^2 = \frac{1}{2}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}$ are non-diagonal. As a rule, the phase factors do not cancel and the mixed products cannot be neglected. We obtain

$$\begin{aligned} \mathrm{c.t.}\ \Pi^{ab}_{\mu\nu}(x,y) = &-\frac{2}{3}\,\frac{g^2}{16\pi^2}\ln(a^2)\,\{\phi^{ab}(x,y)\,[\partial_\mu\partial_\nu - \delta_{\mu\nu}\Delta] \\ &-\frac{3}{2}\,gB\,\varepsilon^{\perp}_{\mu\nu}\,\varepsilon^{ab3}\}\,\delta(x-y), \qquad (a,b=1,2). \end{aligned} \tag{2.13}$$

It should be noted that the phase factors in (2.6) did combine into the phase factor (1.16) in the adjoint representation. The counter term depends in a specific manner on the background field. Since $\tilde{\phi}^{ab}(x,y)\delta(x-y) = \varepsilon^{ab3}\delta(x-y)$ the expressions (2.12) and (2.13) are in accordance with (2.1). From the identity

$$\phi^{ab}(x,y)(\delta_{\mu\nu}\Delta - \partial_\mu\partial_\nu)\delta(x-y) = \left[\delta_{\mu\nu}(D^2)^{ab} - (D_\mu D_\nu)^{ab} + \frac{gB}{2}\,\varepsilon^{\perp}_{\mu\nu}\,\varepsilon^{ab3}\right]\delta(x-y) \tag{2.14}$$

which is valid if integrated with a test function $f^a(x)$ the counter term (2.13) can be rewritten as

$$\mathrm{c.t.}\ \Pi^{ab}_{\mu\nu} = \frac{2}{3}\,\frac{g^2}{16\pi^2}\ln a^2\left[\delta_{\mu\nu}(D^2)^{ab} - (D_\mu D_\nu)^{ab} + 2gB\,\varepsilon^{\perp}_{\mu\nu}\,\varepsilon^{ab3}\right]\delta(x-y), \quad (a,b=1,2) \tag{2.15}$$

This is a structure which could have been anticipated from general considerations /8/; (in fact the renormalization terms for $\Pi_{\mu\nu}$ are not given in /8/).

3. Two-loop corrections to Im V_{eff}

The effective potential for homogeneous background fields is well-known in 1-loop approximation /9,10/

$$V_{eff} = -\beta_1\left(\frac{gB}{2}\right)^2\ln\left(\frac{gB}{\mu^2}\right)^2 \pm \frac{iN}{16\pi}(gB)^2. \tag{3.1}$$

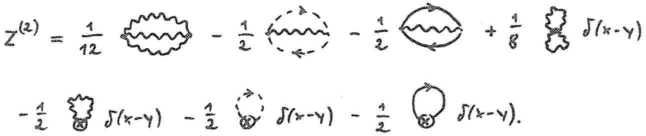
Here $\beta_1 = -(11N - 2n_f)/48\,\pi^2$ is the first coefficient of the SU(N) β function. Furthermore the limit of vanishing fermion (quark) masses is understood. $V^{(1)}_{eff}$ does not depend on the gauge parameter α within the class of gauge fixing terms $-\frac{1}{2\alpha}(D_\mu(B)a_\mu)^2$. This result has been extended to a large class of general gauge fixing terms /10/. Questions about the physical meaning of Im V_{eff} could arise from the fact that an imaginary part appears already in the Euclidean variant of the gauge theory. This is in contrast to QED in homogeneous background fields /7/.

In order to arrive at a better understanding of this imaginary part we turn to the 2-loop corrections. Our aim is to clarify whether the properties of these corrections (in particular the independence on renormalization arbitrariness and on the choice of the gauge parameter) are consistent with those of a quantity of physical relevance.

The 2-loop contributions are obtained via

$$W = -\ln\left[Z^0(1 + Z^{(2)} + \ldots)\right] = \int dx\, V_{eff} \tag{3.2}$$

from the 2-loop diagrams (including renormalization terms) for the vacuum functional Z

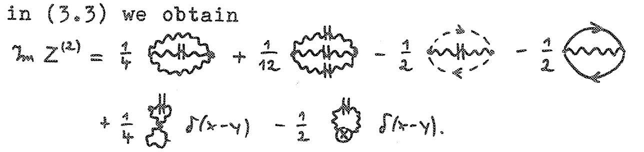


(3.3)

In (3.3) we have indicated explicitely the combinatorial weights /2/ and we understand that the integrations $\int dxdy$ have to be performed. The imaginary part of V_{eff} arises from insertions of the imaginary part of the gauge propagator. The latter will be pictured by a crossed wavy line whereas an uncrossed wavy will from now on correspond to the real part of the propagator. From (1.11), (1.15) we get

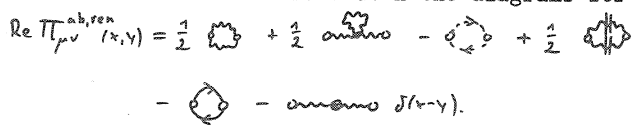
$$= \mathrm{Im}\, G^{ab}_{\mu\nu}(x,y) = \pm \frac{i\pi}{2}\left[\phi^{ab}\delta^{\perp}_{\mu\nu} + \tilde{\phi}^{ab}\varepsilon^{\perp}_{\mu\nu}\right]\int \frac{d^4k}{(2\pi)^4}\, e^{ik(x-y)}\, e^{-\frac{k_\perp^2}{gB}}\, \delta(k_\parallel^2 - gB). \qquad (3.4)$$

Taking into account the equivalence of the gauge field lines in (3.3) we obtain



(3.5)

It is instructive to write down the diagrams for $\mathrm{Re}\,\Pi_{\mu\nu}$ too



(3.6)

Comparing (3.5) with (3.6) it becomes obvious that the weight factors combine in such a manner that we can write

$$\mathrm{Im}\, V^{(2)}_{eff} = -\frac{1}{V_4}\int dx\, dy \left\{ \frac{1}{2}\mathrm{tr}\left[\mathrm{Re}\,\Pi^{ren}_{\mu\nu}\, \mathrm{Im}\, G_{\mu\nu}\right] - \frac{1}{6} \right\}. \qquad (3.7)$$

Because of (2.1) and (3.4) the integrand in (3.7) is translation invariant which allows to factor out the infinite space-time volume V_4.

What about a renormalization arbitrariness of $\mathrm{Im}\, V^{(2)}_{eff}$?
In accordance with (2.13) $\mathrm{Re}\,\Pi^{ren}_{\mu\nu}(x,y)$ is only determined up to a term

$$\mathrm{const.} * \left\{ \phi^{ab}(x,y)\left[\delta_{\mu\nu}\Delta - \partial_\mu\partial_\nu\right] - \frac{3}{2} gB\, \varepsilon^{\perp}_{\mu\nu}\, \varepsilon^{ab3} \right\} \delta(x-y). \qquad (3.8)$$

Inserting the contribution (3.8) for $\mathrm{Re}\,\Pi^{ren}_{\mu\nu}$ together with the expression (3.4) for $\mathrm{Im}\, G_{\mu\nu}$ into (3.7) the result is zero. In other words, the renormalization arbitrariness apparent in $\Pi^{ren}_{\mu\nu}$ drops out in $\mathrm{Im}\, V_{eff}$ which therefore turns out to be - contrary to our earlier expectations /4/ - a uniquely determined quantity.

In this respect the result is similar to that for QED in homogeneous electric background fields. For QED it is well-known that $\mathrm{Im}\, V_{eff}$ corresponds to an (in principle) measurable quantity namely the pair production rate per space-time unit. So $\mathrm{Im}\, V_{eff}$ must be uniquely determined provided the usual renormalizations of QED have been done /11/.

Our result is in accordance with a similar interpretation of $\mathrm{Im}\, V_{eff}$ for the non-abelian theory.

4. Conclusions

The construction of the gauge field propagator has been reduced to the case $\alpha=1$ (Feynman gauge) which, in general, is easier to handle. For the non-trivial example of a homogeneous background field this construction is free of infrared problems. The main conclusion to be drawn is that the imaginary part of the gauge propagator (due to negative modes) does not depend on the gauge fixing term.

The structure of the renormalization part for the polarization tensor has been determined paying special attention to the background-dependent terms. It is equivalent to a generalized (covariant) gradient invariant structure but not identical with that.

At the 2-loop level Im V_{eff} is a uniquely determined quantity independent of the renormalization arbitrariness in the polarization tensor. This is in accordance with understanding Im V_{eff} as a physical quantity.

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