Looking at the gluon moment of the nucleon with dynamical twisted mass fermions

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Outline

- computing $\langle x \rangle_g$ with the Feynman-Hellmann theorem
- direct way of computing the gluon moment $\langle x \rangle_g$ with a disconnected three-point function
- current simulation setup
- status of computations
- future plans
• DIS: existence of partons in the nucleon
• $f_p(x)$ parton distribution function
• probability of finding a parton to have a momentum fraction of $x$
• first moment of parton distribution function: $\langle x \rangle_p = \int x f_p(x) dx$
  → average momentum contribution to the nucleon momentum
• $\sum_p \langle x \rangle_p = 1$
• experimentally: $\langle x \rangle_{u+d} \approx 0.4$
  → There is something missing.
solution: gluon pdf $f_g(x)$ and gluon moment $\langle x \rangle_g$

$\sum_q \langle x \rangle_q + \langle x \rangle_g = 1$

up to now: many computations of fermionic quantities e.g.: 
$\langle x \rangle_{u-d}$ (Talk by C. Alexandrou today)
$g_A$ (Talk by M. Constantinou today)
$\sigma_{\pi N}$ (Talk by V. Drach today)

only few attempts to compute $\langle x \rangle_g$ (with quenched fermions)
Computation of gluon moment

- We can access the gluon moment via the matrix elements of a suitable operator:

- Gluon operator:

\[ O_{\mu\nu} = -\text{tr}_c G_{\mu\rho} G_{\nu\rho} \]

- We can compute the matrix elements with a ratio of a three-point and a two-point function:

\[
\frac{\langle H(p, t) O(\tau) H(p, 0) \rangle}{\langle H(p, t) H(p, 0) \rangle} \quad 0 \ll \tau \ll t \quad (O)_{H(p) H(p)}
\]
Computation of gluon moment II

- two possible forms of the operator
- $A_i = O_{i4}$
- $B = O_{44} - \frac{1}{3} O_{jj}$ (euclidean notation for $T_{00}$)
- after form factor decomposition

\[ (A_i)_{N(p)N(p)} = -ip_i \langle x \rangle_g \]

× requires non-zero momentum

\[ (B)_{N(p)N(p)} = (m_N + \frac{2}{3E_N} \vec{p}^2) \langle x \rangle_g \]

× subtraction of two terms similar in magnitude
Feynman-Hellmann theorem

- method for computing the matrix elements following a paper of QCDSF and UKQCD Collaborations\(^a\)
- starting point: **Feynman-Hellmann theorem**
- introduce parameter \(\lambda\) into the action: \(S \to S(\lambda)\)
- then one can derive:

\[
\frac{\partial E_N}{\partial \lambda} = \left( \frac{\partial S(\lambda)}{\partial \lambda} \right)_{N(p)N(p)}
\]

- \(\vdots\): means the subtraction of the vacuum term

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Feynman-Hellmann theorem II

- we modify the Wilson plaquette action:
  \[ S(\lambda) = \frac{1}{3}\beta(1 + \lambda) \sum_i \text{tr}_c[1 - U_{i4}] + \frac{1}{3}\beta(1 - \lambda) \sum_{i<j} \text{tr}_c[1 - U_{ij}] \]

  \( \lambda = 0 \) is the standard plaquette action

- the derivative can be related to the gluon moment:
  \[ \frac{\partial E_N}{\partial \lambda} = \frac{3}{2}(\mathcal{B})_{N(p)N(p),\lambda} \]

  \[ \left. \frac{\partial E_N}{\partial \lambda} \right|_{\lambda=0} = -\frac{3}{2}(m_N + \frac{2}{3E_n}\vec{p}^2) \langle x \rangle_g \]

- no subtraction here because vacuum term of the operator is zero

- when computing at zero momentum we get
  \[ \langle x \rangle_g = \frac{2}{3am_N} \left. \frac{\partial am_N}{\partial \lambda} \right|_{\lambda=0} \]

- we need to compute the nucleon mass for different \( \lambda \) values
Setup for the Feynman-Hellman theorem

- $N_f = 2 + 1 + 1$ with twisted mass fermions
  - Iwasaki action!
- $\beta = 1.90$, $(a \approx 0.078 \text{ fm})$
- $L^3 \times T = 24^3 \times 48$
- $\mu = 0.0085$ ($m_{PS} \approx 491 \text{ MeV}$)
- at the moment $O(200)$ configurations
- $\lambda = \{-0.0333, -0.01, +0.0333\}$
- For $\lambda = 0$ we can use existing results for the nucleon.
- problem: We have to tune $\kappa$ for each $\lambda$. 
Preliminary data

- visible slope
- Is it linear?
  → we need more statistics and more $\lambda$ points
Computation with the direct method

- alternative: directly compute the matrix element with given three- and two-point functions
- For zero zero momentum:
  \[
  (B)_{N(0)N(0)} = m_N \langle x \rangle_g
  \]
- write the operator in terms of plaquettes:
  \[
  B(t) = \frac{4}{9} \frac{\beta}{a} \left( \sum_i \text{tr} \{ U_{i4}(t) \} - \sum_{i<j} \text{tr} \{ U_{ij}(t) \} \right)
  \]
- The three-point function is disconnected and can be computed by multiplying the two-point function with the inserted operator:
  \[
  \frac{\langle [N(t)N(0)]B(\tau) \rangle}{\langle N(t)N(0) \rangle} \stackrel{0 \ll \tau \ll t}{=} (B)_{N(0)N(0)}
  \]
Setup for the direct method

- $N_f = 2 + 1 + 1$ with twisted mass fermions
- $\beta = 1.95 \ (a \approx 0.078 \text{ fm})$
- $L^3 \times T = 32^3 \times 64$
- $\mu = 0.0055 \ (m_{PS} \approx 393 \text{ MeV})$
- $O(2300)$ configurations with 32 two-point functions each
  (proton and neutron two-point function for 16 different source positions)
- \(\sim 73000\) measurements
Matrix element of the unsmeared operator

- poor signal, compatible with 0
- try using a HYP smeared operator as suggested in \(^a\) (quenched work)

Matrix element of the unsmeared operator

- strong scaling of the error with HYP steps
- using more then one step of HYP smearing seems reasonable
- Our choice: We used 5 steps of HYP smearing with $\alpha_{1,2,3} = \{0.75, 0.6, 0.3\}$. 
Matrix element: HYP-smeared operator

- nice signal, possible plateau
- result needs to be normalized and renormalized
Renormalization

- Gluon operators are singlet operators.

  → mixing with quark singlet

  \[
  \left( \frac{\langle x \rangle^\text{MS}_G}{\sum_q \langle x \rangle^\text{MS}_q} \right) = Z_{2 \times 2} \left( \frac{\langle x \rangle^\text{bare}_G}{\sum_q \langle x \rangle^\text{bare}_q} \right)
  \]

  - \( \langle x \rangle^\text{bare}_G = Z_g \langle x \rangle^\text{lat}_G, \langle x \rangle^\text{bare}_q = Z_q \langle x \rangle^\text{lat}_q \)

  → \( \langle x \rangle^\text{MS}_G = \frac{Z_{\text{bare } gg}}{Z_{\text{bare } gg}} \langle x \rangle^\text{bare}_G + \left[ 1 - Z_{\text{bare } qq}^\text{MS} \right] \sum_q \langle x \rangle^\text{bare}_q \)

  - we need \( Z_g, Z_q, Z_{\text{bare } qq}^\text{MS}, Z_{\text{bare } gg}^\text{MS}, \langle x \rangle^\text{lat}_u \) and \( \langle x \rangle^\text{lat}_d \)

  different renormalization because of using a HYP smeared operator
Conclusion

- Two methods which can be used to extract $\langle x \rangle_g$:
  - Feynman-Hellmann theorem
  - visible signal with small statistics
    - configurations and two-point functions for $\lambda$ have to be computed
  - direct method:
    - two-point functions and configurations are available
    - computation of plaquettes is cheap
    - need large statistics
Future plans and problems

- Which method should be preferred?
- FHt: $\lambda$ for the improved term of the gauge action?
- How to compute all the renormalization factors?
- physical point, continuum limit
- gluon moment of other hadrons
Thank you for your attention and future discussions.