Onset of Chaos in the Weakly Dissipative Two-dimensional Complex Ginzburg-Landau Equation

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Abstract

The two-dimensional Ginzburg-Landau (GL) equation in the weakly dissipative regime (real parts of the coefficients are assumed to be small in comparison with the imaginary ones) is considered in a square cell with reflecting (Neumann) boundary conditions. Following the lines of the analysis developed earlier for the analogous 1D equation, we demonstrate that, near the threshold of the modulational instability, the GL equation can be consistently approximated by a five-dimensional dynamical system which possesses a three-dimensional attracting invariant manifold. On the manifold, the dynamics are governed by a modified Lorenz model containing an additional cubic term. By means of numerical simulations of this approximation, a diagram of dynamical regimes is constructed, in a relevant parameter space. A region of chaos is found. Unlike the previously studied case of the 1D GL equation, in the present case a blow-up is possible, depending on initial conditions.

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1 Introduction

One of the traditional approximations in the theory of pattern formation is the approximation of nonlinear models based on partial differential equations (PDE's) by essentially simpler finite-dimensional dynamical systems based on ordinary differential equations (ODE's). One of the first attempts of this sort was the work by Rabinovich and Fabrikant [1], in which a three-dimensional dynamical system was derived to model onset of "turbulence" in the complex Ginzburg-Landau (GL) equation. In the case when the finite-dimensional system produces a dynamical chaos, an issue of fundamental importance is to realize if this chaos has anything to do with the "turbulence" in the underlying PDE model. This issue remains open, e.g., in the case considered in Ref. [1], as the derivation of the dynamical system from the PDE was based in that work on a rather arbitrary truncation of the infinite-dimensional dynamical system, which could be obtained by projecting the PDE onto a certain functional basis.

A principally important result closely related to this problem, was obtained for several physically meaningful nonlinear dissipative PDE-based models with periodic boundary conditions (b.c.): it was rigorously proven that, in the infinite-dimensional phase spaces of these models, there exist *finite-dimensional* attractors. This assertion has been proved for the complex GL equation, for the Kuramoto-Sivashinsky equation, and for a few other equations (see, e.g. [2]). A circumstance which severely confines the use of the theorem guaranteeing the existence of the finite-dimensional attractor is the fact that, although one can prove the finiteness of the attractor's dimension, one has no way to construct a finite-dimensional dynamical system whose usually defined attractor would coincide with that generated by the PDE with the periodic b.c.

Nevertheless, this can be done in terms of a consistent approximation in the case when the "turbulence" appears in the PDE model above a certain parametric threshold. In this case, a natural idea, which was first put forward by Malomed and Nepomnyashchy in [3], and which will be pursued in the present work as well, is to use a small overcriticality parameter (the one which measures departure from the threshold) for constructing a regular expansion. The small overcriticality can be used to construct a systematic finite-dimensional expansion in the case when the onset of the instability is related to a finite number of well-defined eigenmodes of the corresponding linear stability analysis. These eigenmodes can be used as a natural basis for derivation of the truncated dynamical system: one should project the full PDE onto them, disregarding eventually an infinite set of all other modes, which (near the threshold) will be either heavily damped or "enslaved" by the slowly varying free eigenmodes belonging to the critical set (the set of the modes amenable for the onset of the instability).

In Ref. [3], where this general idea was put forward, it was applied to the complex GL equation in one spatial dimension. It is relevant to note that a lot of studies of various types of "turbulence" in this model has been performed by means of direct numerical simulations, using powerful computers which allow to scan systematically the corresponding parameter space (this space is, as a matter of fact, two-dimensional), see, e.g., the review paper by Cross and Hohenberg [4].

In Ref. [3], it was noticed that analysis of the onset of chaos in the GL equation can be strongly facilitated by considering the equation in a regime in which it is close to the nonlinear Schrödinger (NLS) equation, i.e., the real parts of coefficients in the GL equations, which account for its dissipative properties, are small in comparison with their imaginary parts, that are responsible for the dispersive properties of the model. If one considers the limiting case of the pure NLS equation with self-attraction (i.e., when the coefficients in front of the dispersive and nonlinear terms in this equation have the same sign), a well-known property is the modulation instability (MI) of the carrier wave (cw) solution (i.e., the one with a constant amplitude) [5]. Now, when the NLS equation is supplemented by the small dissipative terms turning it into the GL equation, the natural trick, employed in Ref. [3] is to consider the case when the unperturbed cw solution has an amplitude slightly above the threshold. As a result, it was shown that, depending on a sign of a certain combination of the parameters, one obtains (after some transformations) either the famous Lorenz equations [6], or another chaos-generating system ("modified Lorenz"), differing from the Lorenz system by a single sign. Thus, the onset of chaos in the complex GL equation (in the near-NLS regime) can be adequately approximated either by the Lorenz equations, or by the "modified Lorenz" system.

The objective of the present work is to extend the previous analysis to the two-dimensional (2D) GL equation, again in the regime when it is close to the NLS equation. The chaos in the 2D GL equation is also triggered by the MI inherited from the NLS equation. An obvious difference from the 1D case is that the instability basis will contain more eigenmodes. However, a more important difference is that, as it is well known, the MI in the 2D cubic NLS equation, on the contrary to its 1D counterpart, leads to a *weak collapse* (see, e.g., the review [7]). Therefore, one may expect that an adequate finite-dimensional approximation to the 2D GL equation will have dynamical trajectories escaping to infinity. On the other hand, since the collapse in the 2D NLS model is weak, it is also natural to expect that the same model may have compact attractors coexisting with the escaping trajectories.

It will be demonstrated that the natural finite-dimensional system approximating the GL equation (with the small dissipative terms) just above the MI threshold has a five-dimensional phase space. It will be further shown that there are three-dimensional attracting invariant manifolds in the full phase space (which, however, coexist with the attracting "collapsing" trajectories), the chaotic motion on those manifolds being governed by a modified Lorenz system.

2 The finite-dimensional model

We consider the generalized Ginzburg-Landau equation

$$iu_t + \nabla^2 u + 2|u|^2 u = i\epsilon_1 u - i\epsilon_2 |u|^2 u + i\epsilon_3 \nabla^2 u \tag{1}$$

in a closed region $0 \le x \le \pi/q_0$, $0 \le y \le \pi/q_0$ with the condition of a vanishing normal derivative on the whole boundary:

$$\partial_n u = 0. \tag{2}$$

By means of the transformation $u{=}a\exp i\phi$ where a and ϕ are real, the complex equation (1) can be represented in the form of two real equations:

$$\begin{aligned} a_t + 2\nabla a\cdot\nabla\phi + a\nabla^2\phi &= \epsilon_1 a - \epsilon_2 a^3 + \epsilon_3[\nabla^2 a - a(\nabla\phi)^2], \\ -a\phi_t + \nabla^2 a - a(\nabla\phi)^2 + 2a^3 &= -\epsilon_3(2\nabla a\cdot\nabla\phi + a\nabla^2\phi). \end{aligned} \tag{3}$$

The solution

$$a_0 = (\epsilon_1/\epsilon_2)^{1/2},\ \phi_0 = 2a_0^2 t, \tag{4}$$

describing uniform oscillations is subject to the modulational instability if q_0 is sufficiently small. The linear modes satisfying the boundary condition (2) have the spatial structure $\cos mq_0x \cos nq_0y$; m,n=0,1,2,... The corresponding growth rate is determined by the formula

$$\sigma(m,n) = -\epsilon_2 a_0^2 - \epsilon_3 q^2 \pm \sqrt{q^2(4a_0^2 - q^2) + (\epsilon_2 a_0^2)^2}, \tag{5}$$

where $q^2{=}(m^2+n^2)q_0^2$. Later on, we shall assume that $\epsilon_1 \sim \epsilon_2 \sim \epsilon_3 \equiv \epsilon$ are small parameters. One can see that the modulational instability of the uniform solution (4) appears if $q_0 < q_* = 2a_0 + O(\epsilon^2)$. Let us consider the region near threshold where

$$q_0^2 = 4a_0^2(1 - \epsilon^2 K),\ K = O(1). \tag{6}$$

In this region, all the linear modes (m,n) can be separated into two groups: *slow* (active) modes $(0,0)$, $(0,1)$ and $(1,0)$ which grow or decay in such a way that $\mathrm{Im}\,\sigma(m,n)=0$, $\mathrm{Re}\,\sigma(m,n)=O(\epsilon)$, and *fast* (slaved) modes which oscillate rapidly: $\mathrm{Im}\,\sigma(m,n)=O(1)$ (all other modes). This fact gives the possibility to construct an asymptotic finite-dimensional model for active modes by eliminating the slaved ones.

Indeed, let us represent the solution of the problem (3) in the form:

$$\begin{aligned} a &= a_0 + \sum_{m,n} b_{mn}(t) \cos mq_0x \cos nq_0y, \\ \phi &= 2a_0^2 t + \sum_{m,n} \psi_{mn}(t) \cos mq_0x \cos nq_0y. \end{aligned} \tag{7}$$

The active variables are b_{10}, b_{01}, b_{00}, ψ_{10}, ψ_{01} and ψ_{00}. A closed finite-dimensional model can be derived under the assumption that

$$b_{10} = O(\epsilon),\ b_{01} = O(\epsilon),\ b_{00} = O(\epsilon^2),\ \psi_{10} = O(\epsilon^2),\ \psi_{01} = O(\epsilon^2),\ \psi_{00} = O(\epsilon). \tag{8}$$

In the leading order, we obtain the following expressions for the slaved variables:

$$b_{20} = \frac{b_{10}^2}{4a_0},\ b_{02} = \frac{b_{01}^2}{4a_0},\ b_{11} = \frac{b_{10}b_{01}}{a_0},\ \ldots \tag{9}$$

The variable $\psi_{00}(t)$ describing the uniform change of the phase is governed in the leading order by the equation

$$\frac{d\psi_{00}}{dt} = 4a_0 b_{00} + 3b_{10}^2 + 3b_{01}^2 \tag{10}$$

and does not affect the evolution of other variables. Thus, the actual dynamical system is five-dimensional. We shall use the leading order equations for db_{10}/dt and db_{01}/dt for elimination of the variables ψ_{10} and ψ_{01}:

$$\begin{aligned} \psi_{10} &= \frac{1}{4a_0^3}\left[\frac{db_{10}}{dt} + a_0^2(2\epsilon_2 + 4\epsilon_3)b_{10}\right], \\ \psi_{01} &= \frac{1}{4a_0^3}\left[\frac{db_{01}}{dt} + a_0^2(2\epsilon_2 + 4\epsilon_3)b_{01}\right]. \end{aligned} \tag{11}$$

Finally, using the scaling transformation

$$b_{10} = (\epsilon_1\epsilon_2)^{1/2}B_{10},\ b_{01} = (\epsilon_1\epsilon_2)^{1/2}B_{01},\ b_{00} = (\epsilon_1\epsilon_2^3)^{1/2}B_0,\ T = \epsilon_1 t \tag{12}$$

we obtain the following dynamical system:

$$\begin{aligned} \frac{d^2B_{10}}{dT^2} + 2(1+4A)\frac{dB_{10}}{dT} - [8(2K - A - 2A^2) + 32B_0 + 24B_{01}^2]B_{10} &= 0, \\ \frac{d^2B_{01}}{dT^2} + 2(1+4A)\frac{dB_{01}}{dT} - [8(2K - A - 2A^2) + 32B_0 + 24B_{10}^2]B_{01} &= 0, \\ \frac{dB_0}{dT} + 2B_0 + \frac{1}{2}\left(B_{10}\frac{dB_{10}}{dT} + B_{01}\frac{dB_{01}}{dT}\right) + \frac{1}{2}(5+4A)(B_{10}^2 + B_{01}^2) &= 0, \end{aligned} \tag{13}$$

where $A \equiv \epsilon_3/\epsilon_2$.

3 Analytical results

In this section we summarize some results concerning the dynamics of the system (13) that can be obtained without numerical computations. The system has three 3-dimensional invariant manifolds. Coming back to the full GL equation approximated by the considered dynamical system, one can interpret each of the invariant manifolds as a certain stationary solution to the GL equation, as it is specified below: M_{10}:$B_{01} = 0$ (a roll parallel to the y-axis); M_{01}:$B_{10} = 0$ (a roll parallel to the x-axis); $M_{11}^{\pm}$:$B_{10} = \pm B_{01} \equiv B_1$ (a square pattern). Earlier, it was shown that the dynamics on M_{10} and M_{01} is governed by a modification of the Lorenz system [3]. Hereafter we will concentrate on the trajectories belonging to the invariant manifold M_{11}. This manifold is locally attracting in the domain

$$3B_1^2 > 4B_0 + 2K - A - 2A^2. \tag{14}$$

On M_{11}, we perform the following change of variables:

$$\begin{gathered} B_0 = -\frac{|4A-3|(5+4A)}{32}z,\ B_1 = \frac{4A-3}{4\sqrt{2}}x, \\ \frac{dB_1}{dT} = \frac{(4A-3)(5+4A)}{4\sqrt{2}}(y-x),\ \tau = |4A-3|T \end{gathered} \tag{15}$$

which transforms the system (13) into Lorenz-like equations:

$$\begin{aligned} \frac{dx}{d\tau} &= \sigma(y-x), \\ \frac{dy}{d\tau} &= -sy + rx - xz + qx^3, \\ \frac{dz}{d\tau} &= -bz + xy. \end{aligned} \tag{16}$$

The parameters of this system are determined by two independent parameters A and K:

$$\sigma = \frac{5+4A}{|4A-3|},\ b = \frac{2}{|4A-3|},\ s = \mathrm{sign}(4A-3), r = s + \frac{16(K - A/2 - A^2)}{(5+4A)|4A-3|},\ q = \frac{3}{4\sigma}. \tag{17}$$

The most important difference between the equations (16) and the Lorenz system is the existence of trajectories tending to infinity. It is possible to find analytically the asymptotics of some trajectories of this kind describing: (i) a "fast collapse"

$$B_1 \sim \pm\frac{1}{2(\tau-\tau_1)},\ B_0 \sim -\frac{1}{8(\tau-\tau_1)^2},\ \tau_1 = const; \tag{18}$$

and (ii) a "slow collapse"

$$B_1 \sim \pm e^{(7+8A)(\tau-\tau_2)},\ B_0 \sim -\frac{3}{4}e^{2(7+8A)(\tau-\tau_2)},\ \tau_2 = const. \tag{19}$$

The "collapse" leads to violation of the assumptions (8) and is probably connected with formation of non-stationary patterns which cannot be obtained by means of the perturbation theory, e.g., spiral waves or some "imitations" of a collapse (not necessarily leading to an actual singularity).

Nevertheless, the system (13) has also some bounded attractors. The stationary point O: $B_{10} = B_{01} = B_0$ is unstable at $K > K_0 = A/2 + A^2$. The other stationary solutions are:

$$P_{10}^{\pm}: B_{10}^2 = \frac{2(K - A/2 - A^2)}{5+4A},\ B_{01} = 0,\ B_0 = -\frac{1}{2}(K - A/2 - A^2); \tag{20}$$

$$P_{01}^{\pm}: B_{10} = 0,\ B_{01}^2 = \frac{2(K - A/2 - A^2)}{5+4A},\ B_0 = -\frac{1}{2}(K - A/2 - A^2); \tag{21}$$

$$P_{11}^{\pm\pm}: B_{10}^2 = B_{01}^2 = \frac{2(K - A/2 - A^2)}{7+8A},\ B_0 = -\frac{5+4A}{7+8A}(K - A/2 - A^2). \tag{22}$$

The fixed points $P_{10}^{\pm}$ and $P_{01}^{\pm}$ are always unstable, while $P_{11}^{\pm\pm}$ are locally stable if

$$K < K_1(A) = \frac{A}{2} + A^2 + \frac{(1+2A)(1+4A)(7+8A)}{12(3+4A)}. \tag{23}$$

The frequency ω of oscillations on the stability boundary $K = K_1(A)$ where the Hopf bifurcation takes place is equal to

$$\omega = \sqrt{\frac{4(7+8A)(1+4A)}{3(3+4A)}} \tag{24}$$

4 Time-dependent states

Beyond the curve $K_1(A)$ in the plane of parameters A and K all steady solutions are unstable. The evaluation of the branching coefficient of the Hopf bifurcation shows that the bifurcation is supercritical; this ensures the existence of the stable periodic orbit with the frequency close to (24) in the strip adjacent to $K_1(A)$. The further bifurcations which lead to the formation of the bounded chaotic attractor, are global, and our description is based on numerical results. The respective diagram with lines of several principal bifurcations is presented in Figure 1. For all the found attractors the condition (14) ensuring their relevance for the whole 5-dimensional system, was checked.

In the domain $K > A/2 + A^2$ the trivial zero steady solution is a saddle point with one-dimensional unstable manifold and two-dimensional stable manifold. The former is locally tangent to the eigenvector corresponding to the positive eigenvalue λ_1, and the local dynamics on the latter is characterized by two real negative eigenvalues $\lambda_3 < \lambda_2 < 0$, with the last one being responsible for the return to the saddle along the z-axis. The eventual increase of K leads to the creation of homoclinic saddle-connections, when the components of unstable manifold get captured by the stable manifold of the saddle. Similarly to the Lorenz equations, the system (16) is invariant with respect to the simultaneous change of the sign of the variables x and y; because of this symmetry the homoclinic orbits arise in pairs (Fig. 2a). The events which follow the formation and destruction of the pair of homoclinic orbits are determined by the saddle index – the ratio of eigenvalues

$$\nu = -\frac{\lambda_2}{\lambda_1} = \frac{2}{-1 - 4A + \sqrt{1+16K}} \tag{25}$$

According to the general theory [8, 6] for $\nu < 1$ the destruction of the couple of homoclinic orbits is accompanied by the "homoclinic explosion" – creation of the Ω-set including the

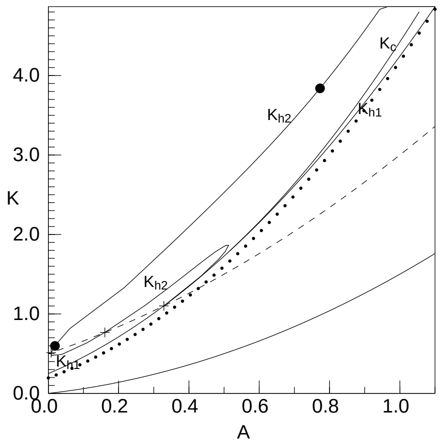

Fig. 1

Bifurcation diagram on the parameter plane

dotted line - Hopf bifurcation;
dashed line - $\nu = 1$
K_{h1} - primary homoclinic bifurcation;
K_{h2} - formation of 2-turned homoclinic loops;
K_c - chaotic set becomes attractive;
black circles - change of the homoclinic loop orientability;
crosses - homoclinic loops with $\nu = 1$

countable number of unstable periodic orbits and the continuum of recurrent trajectories. In the opposite case $\nu > 1$ contraction near the saddle prevails over expansion, and the bifurcation glues two stable mutually symmetric closed orbits into a single one. This new orbit is twice as long in the phase space as any of its parents; in this sense one may speak about the "homoclinic doubling" (note that at the bifurcation point the period is infinite).

For small A the inequality $\nu > 1$ holds; on the bifurcation line $K_{h1}(A)$(Fig.1) corresponding to the formation of the couple of 1-looped homoclinic trajectories the two stable periodic orbits which stem from the Hopf bifurcation at $K_1(A)$, merge into a 2-looped attracting limit cycle. At A=A_0=0.328726, the curve $K_{h1}(A)$ intersects with the line

$$K = A^2 + \frac{3}{2}A + \frac{1}{2}, \tag{26}$$

at which $\lambda_1 + \lambda_2$=0; to the right of A_0 one observes at $K_{h1}(A)$ the homoclinic explosion followed by the birth of the (yet unstable) chaotic set.

The unfolding of the codimension-2 bifurcation at which the formation of a couple of homoclinic orbits coincides with the vanishing of the sum of eigenvalues, was described by Lyubimov and Zaks in [9]. The two-dimensional Poincaré mapping on the secant plane transversal to the stable manifold of the saddle, was reduced to the discontinuous one-dimensional mapping, whose simplest truncation can be written as

$$\xi_{i+1} \; = \; (a|\xi_i|^{\nu} - \mu)\; \mathrm{sign}\xi_i \tag{27}$$

Here ξ parameterizes the local unstable manifold, the pre-factor a is the so-called separatrix value [11], and the parameter μ measures the decline of K from $K_{h1}(A)$: it vanishes at K=K_{h1} and is positive above this line. The two branches of (27) correspond to two symmetric components of the unstable manifold.

The ordering of bifurcations near the intersection of the curves K_{h1} and (26) depends on the sign and magnitude of the separatrix value a. In our case a is positive but small

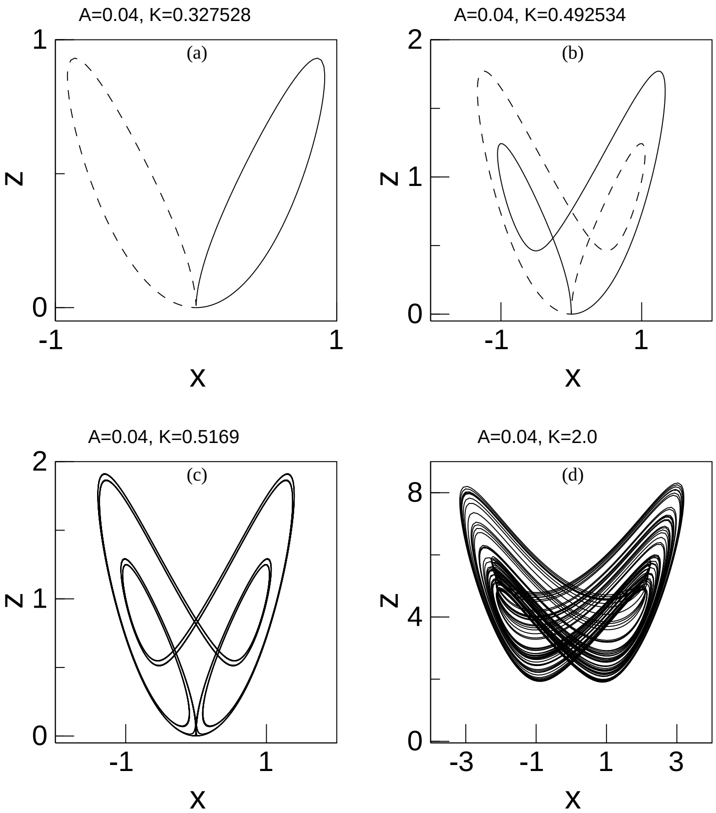

Fig. 2

(a) - primary homoclinic orbits

(b) - 2-looped homoclinic orbits

(c), (d) - chaotic attractors

$(a < 1)$; this implies that for $\nu > 1$ no further homoclinic bifurcations can happen in the vicinity of this point. On the contrary, the whole pencil of the lines of secondary multi-looped homoclinic bifurcations is issued in the direction of $\nu < 1$. The line of the saddle-node (tangent) bifurcation at which the stable and unstable closed orbits coalesce and disappear, also starts in this point and goes to the right, lying above the curve K_{h1}.

For $A > A_0$ the increase of K beyond K_{h1} leads to the domain of metastable chaos: after several erratic jerks the trajectory eventually settles onto a "regular" attractor which in this case is either of two stable symmetric periodic orbits born at $K_1(A)$. Similarly to the Lorenz case, the chaotic set acquires attractive properties when the unstable manifold of the trivial saddle point hits the stable manifold of the unstable periodic orbit (the simplest of the closed trajectories born in the homoclinic explosion); on the bifurcation diagram this event is marked by the curve K_c. The further increase of K leads to the domain of hysteresis in which the Lorenz-type chaotic attractor coexists with stable periodic orbits. Later these periodic orbits disappear via the above mentioned saddle-node bifurcation, and the chaotic set remains the only bounded attractor. For much higher values of K one observes the events reminiscent of the Lorenz equations in case of high Rayleigh numbers, with periodic windows and reverse sequences of period-doubling bifurcations, and finally a stable 2-looped periodic trajectory appears.

Most of the curves belonging to the mentioned pencil of the lines of multi-looped secondary homoclinic bifurcations are characterized by a monotonic increase of both A and K. However the line K_{h2} corresponding to the formation of the couple of 2-looped homoclinic orbits (Fig.2b) has the S-shaped form; after the turning point it goes back to the left, and intersects the line (26) at A=0.16072 thus intruding into the domain $\nu > 1$. Once again one can construct the return mapping; now it must be the mapping after *two* successive turns of the orbit. Naturally, the mapping has the familiar structure (27); however this time the value of a happens to be larger than 1. As shown by Arneodo *et al* [10] and Lyubimov and Zaks[9], in this case a cascade of homoclinic doublings should occur: for $\nu > 1$ the increase of K leads through the succession of homoclinic bifurcations at each of which the number of the loops of the attracting stable periodic orbit is doubled. The convergence rate of the sequence, as well as the scaling characteristics of the resulting chaotic attractor are entirely determined by the value of the saddle index ν. As can be seen from Fig.1, the excursion of K_{h2} into the domain $\nu > 1$ is not very deep, and the values of ν larger than 1.08 cannot be reached. As a result along the accumulation line the convergence rate $\delta(\nu)$ is relatively low: $\delta(\nu) < \delta(1.08)$=2.757688. On the other hand, the values of the scaling factor α characterizing the decrease of the distance between the 2^n-th and the 2^{n+1}-th loops of the orbit are rather high: $\alpha(\nu) > \alpha(1.08)$=9.11402; the latter circumstance implies that the chaotic attractor is very "compressed", and graphically its transversal Cantor structure can be hardly distinguished from, say, 8-looped limit cycle (cf. Fig.2c).

Further on K_{h2} the value of a decreases with the decrease of A; at the new intersection point with (26) at A=0.00798 it is already smaller than 1. Soon after the new turning point the value of a crosses zero; this marks the transition from the orientable homoclinic orbit to the non-orientable one (in the former case the invariant manifolds of the bifurcating periodic solutions are homeomorphic to a cylinder, whereas in the latter they are Möbius strips). The detailed analysis of the events accompanying this codimension-two bifurcation will be presented elsewhere; the similar phenomena were observed in the Shimizu-Morioka equations by Shilnikov [11] and Rucklidge [12]. Above the "non-orientable" segment of K_{h2} (which is limited from the right by the other similar point in which a vanishes) the secondary multi-looped homoclinic bifurcations as well as direct and reverse cascades of period-doublings are encountered. The typical shape of the attractor is presented in Fig.2d. For still higher values of K the only bounded attractor is the 2-looped periodic orbit.

If the initial conditions are chosen near the trivial equilibrium (which corresponds to the modulationally unstable uniform oscillations (4) in the initial problem (1)), the trajectory departs along one of the components of the unstable manifold of this saddle-point. For small and moderate values of K these components are eventually attracted to one of the bounded attractors. However the numerical evidence shows that for each A there is a sufficiently high value of K beyond which the components of the unstable manifold become unbounded, being attracted by the respective "collapse" solutions. Physically this means that for the very high K the collapse is nearly unavoidable, although the bounded attractors with small basins of attraction may also be reached from the other initial conditions.

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