BOSE-EINSTEIN CONDENSDATION OF EXCITONS: PROMISE AND DISAPPOINTMENT

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Abstract
The Bose-Einstein condensation of excitons has a long history with seminal contributions from Leonid V. Keldysh. Despite numerous efforts, however, a compelling experimental evidence is still missing. A brief survey of attempts to realize exciton condensation in different semiconductor systems is given. Specific problems compared with atomic Bose condensation are highlighted. More details are given on coupled quantum wells as a possible candidate in the search for exciton condensation. While here extremely long radiative lifetimes of indirect excitons can be achieved, their strong dipole-dipole repulsion leads to a genuine non-ideal behavior. Theoretical results from a dynamical T-matrix theory are presented which allow to explain blue shift and line broadening under strong excitation which have been seen in recent high-density photoluminescence experiments using a lateral trap.

18.1 A short history of exciton condensation
My first encounter with exciton physics was not reading one of the famous textbooks like the ‘Knox’ (Knox, 1963), but trying to understand the ‘Excitonic Insulator’ from a review written by Halperin and Rice (1968). Soon we realized that this branch of solid state theory was shaped and brought forward by (at that time) Soviet physicists, with cornerstone papers by Keldysh and coworkers (Keldysh and Kopaev, 1964; Keldysh and Kozlov, 1968). The basic idea was to look at a small-gap semiconductor whose energy gap could be tuned to values below the exciton binding energy. Then, the ground state must become unstable against formation of excitonic correlations. The theoretical description is very close to the famous BCS (Bardeen-Cooper-Schrieffer) theory of superconductivity, only that here the attraction between electron and hole is the standard Coulomb force, while in the BCS the dynamical screening by acoustic phonons provides the necessary attraction. More important, however, are two other facts. In the exciton case, anomalous propagators have a simple physical meaning — they are the optically driven interband polarization. Therefore, what needs sophisticated experiments using Josephson junctions in superconductors, can be
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easily done in the excitonic case via interband optics. Secondly, outside the excitonic ground state (or condensate), we find excitons as normal bound states, being thermally excited. There is a long-standing claim that the intermediate valent compound Tm-SeTe fits to the original idea of the excitonic insulator (Bucher et al., 1991; Wachter, 2005). When driving the band gap through zero by applying hydrostatic pressure, the material exhibits a phase transition with a critical temperature as high as 250 K, which is deduced from measuring thermal properties. Further specific experiments would be needed to clarify if excitons are the main players in this game – ruling out competitive mechanisms like charge density wave or lattice instability.

A new turn came in when the excitonic insulator was identified as a possible phase transition in wide-gap semiconductors, too. If optical recombination is slow enough, excited electrons and holes may reach quasi-equilibrium (with common temperature, but different chemical potentials). Although more speculative than the small-gap case with strict equilibrium, a lot of interesting physics was to be expected. However, the pressing question ‘Does it work, even in principle?’ was always lurking behind this scenario. This feeling of uncertainty was often seen already in the paper titles, as, e.g., ‘Possibility of the excitonic phase in insulators’ (which was my first scientific publication, Zimmermann, 1970).

The story with the excitonic insulator was coming to rest for some time when a new phase transition – from dilute exciton gas to the electron-hole liquid – was shown to dominate the physics in highly excited semiconductors. Again, Leonid V. Keldysh was among the pioneers of this new ‘electron-hole droplet physics’ (Keldysh, 1968), which is mainly electron-gas theory extended to two species (electrons and holes). In fact, the ground state energy minimum for multi-valley semiconductors (Si or Ge) is rather deep and lies at relatively high densities. Therefore, the excitonic character of the dilute gas around the droplets was of not much importance, and a random-phase-like approximation for the two-component electron-hole plasma did rather well. Indeed, at these high densities, strong screening prevents the formation of excitons as bound states (Mott transition). Things were expected to be different for single-valley semiconductors (GaAs, CdS). Theoretical work revealed that in this case even the high-density side of the first-order phase diagram bears excitonic signatures (Zimmermann, 1976). However, these materials have a direct band gap with dipole-allowed optical transitions, and the much shorter radiative lifetime prevents the build-up of quasi-equilibrium under high excitation. Still it was general belief that here a quite interesting sequence could be expected with rising density: Formation of excitons, their Bose-Einstein condensation (BEC), strong nonideal effects due to the underlying fermionic structure (excitonic insulator), and a high-density plasma which looses the excitonic correlations gradually (Zimmermann, 1988). When including polar optical phonons into the electron-hole plasma theory, Leonid V. Keldysh was once more paving the way (Keldysh and Silin, 1975), which gave me the first chance to come into personal contact with him, and to
learn quite a lot (which continued to be the case at all further meetings).

How to overcome the exciton lifetime problem? One way was to use wide-gap semiconductors with dipole-forbidden optical transitions. The paradigm material is here cuprous oxide which has indeed a long history on the search for exciton BEC. Following earlier claims on biexciton condensation in CuCl (Nagasawa et al., 1975), pioneering work on Cu$_2$O was done by Wolfe (Lin and Wolfe, 1993) and Mysyrowicz (Mysyrowicz et al., 1996). However, several findings were not really conclusive, for instance to read off directly the bosonic distribution function from the (phonon-assisted) photoluminescence. Other BEC claims found a different and much less spectacular explanation, as, e.g., exciton super transport being driven by phonon wind (Bulatov and Tikhodeev, 1992; Tikhodeev, 1997). Quite recently, new findings on Cu$_2$O give new hopes, as the exploration of 1s-2p transitions using infrared femtosecond spectroscopy (Kuwata-Gonokami, 2005).

Another way to achieve extremely long radiative lifetimes of excitons is using coupled quantum wells. Application of a static electric field in the growth direction allows to tilt the confinement potentials such that the lowest state has electrons and holes residing in different quantum wells. A spatially indirect exciton is formed which acquires microsecond lifetime since the overlap between electron and hole is exponentially small. Already in 1976, Lozovik made a first theoretical prediction for the excitonic insulator in such a system (Lozovik and Yudson, 1976). The first experimental hint on a possible BEC was reported quite early, too (Fuzukawa et al., 1990). But here again, spectacular findings as the nearly millimeter-sized ring emission has been considered first as BEC evidence (Butov, 2002; Snoke et al., 2002), but are now discussed in terms of a dynamic p-n junction. Still, the quite regular fragmentation of this ring emission is waiting for a conclusive explanation. Another interesting issue is to take the fluctuations (noise) of the photoluminescence as indicator of BEC (Butov, 2003). In Sections 18.3 and 18.4 we give more details and own theoretical results for this very promising system, focussing on the emission lineshape.

It is quite common knowledge that at a given density, the BEC critical temperature is larger if the Bose particles have a lighter mass (a specific example is given in eqn (18.3) below). Usually, the exciton mass is dictated by the underlying semiconductor material. However, if exciton-polaritons are formed within a microcavity, the dispersion is dominated by the cavity mode, and the relevant polariton mass can be orders of magnitude smaller than the exciton mass. However, there is always a price to be paid! Here, the system has to be pumped hard in order to get reasonable polariton densities. Furthermore, the cooling is slowed down since only the exciton part in the polariton is able to transfer energy to the lattice (Doan et al., 2005). Both effects hinder to establish quasi-equilibrium, which we consider to be one condition for classification as BEC. At least bosonic stimulation has been seen (Deng et al., 2003), and interesting features like parametric scattering under resonant pumping have been reported, too (Baumberg et al., 2000). For the latter, a description as ‘driven condensate’ would be probably right. In a genuine BEC, we hope to see a condensate whose phase coherence
Bose-Einstein condensation of excitons is rather improbable – if not impossible – since:

<table>
<thead>
<tr>
<th>Excitons are:</th>
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<th>While for atoms,</th>
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<tr>
<td>instable</td>
<td>radiative decay</td>
<td>stability is given</td>
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<td>hard to equilibrate within lifetime</td>
<td>slow cooling</td>
<td>sophisticated cooling in use</td>
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<tr>
<td>composite bosons</td>
<td>electron-hole pairs, Mott transition!</td>
<td>electron-ion plasma is far away</td>
</tr>
<tr>
<td>strongly non-ideal</td>
<td>dipolar repulsion (hinders condensation)</td>
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builds up spontaneously, and is not triggered from outside.

A more exotic version of exciton condensation is discussed for the quantum Hall effect in electron bilayer systems. At half filling, a description in terms of electrons and holes within the Landau level can be applied. Using the exciton terminology provides a new look on this system (Eisenstein and MacDonald, 2004), but it may be questioned if a classification as exciton BEC really works.

Quite a good overview on experimental attempts to find exciton BEC and the ambiguities involved in their interpretation has been given by David Snoke at the NOEKS-7 conference in 2003 (Snoke, 2003). The paper title ‘When should we say we have observed Bose condensation of excitons?’ speaks on its own. A broader survey of relevant work is compiled in a special issue of Solid State Communications (Snoke, 2005), being an offspring of a workshop held in 2004 near Pittsburgh (USA).

The actual strong interest in exciton BEC – often revitalizing old concepts – is surely triggered by the tremendous success to reach experimentally Bose-Einstein condensation of atomic systems (Cornell and Wiemann, 2002; Ketterle, 2002). In contrast, clear-cut proofs of excitonic BEC in solid state systems are still missing. The small exciton mass is an advantage, but other facts are less promising. Several of them are listed in Table 18.1 and compared with their counterparts in atomic systems. In spite of the many cons and only few pros, I am rather confident that we will see clear evidence of excitonic BEC in semiconductor systems at some time. However, before being too optimistic on a too short time scale, we should try to learn more from the hard work which was needed to reach BEC in atomic systems.

To formulate tasks for theory, we should concentrate more on the way towards exciton BEC, while the study of the condensate itself might be deferred to a later stage. A particular problem is to understand how the excitons can manage to cool down. Nonequilibrium Green’s functions and the ‘Keldysh technique’ (Keldysh, 1964) provide the proper machinery, which has been used in the present context by Haug and coworkers (Schmitt et al., 2001). Even when assuming that quasi-equilibrium has been established (as I will do in the following Sections), we face another complication: Since excitons are composite quantum particles with a sizable interaction, the formulation and solution of the many-exciton problem...
is more challenging than treating the nearly ideal atomic Bose gas. However, in doing so we will learn a lot which might be of use for other problems in interacting solid state systems.

18.2 The ideal Bose gas in a trap

Let us start with compiling results being valid for ideal (i.e. noninteracting) bosons. The general expression for the boson number in dependence on temperature \( T \) and chemical potential \( \mu \) is

\[
N(\mu, T) = N_0 + \int_0^\infty dE \rho(E) n_B(E - \mu), \quad n_B(E) = \frac{1}{\exp(\beta E) - 1},
\]

(18.1)

where the Bose distribution function \( n_B(E) \) has been introduced as usual (\( \beta = 1/k_B T \)). With rising boson number \( N \), the chemical potential moves toward the lowest available states (onset of the Boson density of states \( \rho(E) \), here taken at \( E = 0 \)). If \( \mu = 0 \) leads to a finite value of the integral, all remaining bosons have to occupy the lowest state, forming the condensate part \( N_0 \). Therefore, the shape of \( \rho(E) \) at small energies is crucial. A behavior \( \rho(E) \propto E^p \) with \( p > 0 \) gives a finite integral. Therefore, a two-dimensional system with \( \rho(E) \propto \Theta(E) \) (\( p = 0 \)) does not allow for condensation – any boson number can be accommodated in the normal phase at \( E > 0 \). This would apply for excitons in the lowest sublevel state of a quantum well. However, introducing a trap changes the situation completely. For instance, a parabolic trap profile

\[
V(R) = \frac{\alpha}{2} R^2
\]

(18.2)

leads to a density of states which rises linearly with energy, \( \rho(E) \propto E \). Therefore, BEC is possible in principle. If at a given temperature the total exciton number \( N \) exceeds the critical number

\[
N_c = (k_B T)^2 \frac{\pi^2}{6} \frac{g_S M}{\hbar^2 \alpha},
\]

(18.3)

a condensate forms, \( N_0 = N - N_c \) (\( g_S \) is the spin degeneracy).

To get numbers, we adopt for the (kinetic) exciton mass \( M \) of a typical GaAs QW a value of \( M = 0.3 m_0 \) (Siarkos et al., 2000), and take for the trap strength \( \alpha = 0.31 \mu eV/\mu m^2 \), a value which was extracted from the experimental data by Snoke (Vörös et al., 2005). At a temperature of \( T = 2 \) K, eqn (18.3) gives \( N_c = 2.5 \times 10^6 \), which seems to be not out of reach.

In a trap, only a finite number of Bose particles can be placed. What about the general theorem that a true phase transition can be realized only in the thermodynamic limit (both the system volume and particle number going to infinity)? Indeed, using the density of states as a continuous function of energy, we have implicitly made such a limiting process already in eqn (18.1). However, in the present rather simple case we can do the finite-volume problem from
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Fig. 18.1. Ideal Bose gas in a 2D trap: (a) Condensate fraction \( N_0/N \) versus temperature for \( N = 2000 \) bosons (full curve), and its thermodynamic limit (dashed). (b) Density distribution in the trap at different temperatures. The ground state wave function squared is marked by “GS”.

scratch: In the parabolic trap potential of eqn (18.2), the eigenenergies are \( \epsilon_j = jh\omega_0 \) \((j = 1, 2, \ldots)\) with \( \omega_0 = \sqrt{\alpha/M} \). Accounting properly for the degeneracy of levels, we obtain for the total exciton number

\[
N(\mu, T) = g_S \sum_{j=1}^{\infty} \frac{e^{\beta(\hbar\omega_0j - \mu)} - 1}{e^{\beta(\hbar\omega_0j - \mu)} - 1}.
\]  

The numerator resembles the density of states (linear increase with energy), but now completely discretized. The occupation of the lowest state \((j = 1)\) gives the condensate number \(N_0\). Obviously, for any finite \(N\), the chemical potential will never reach the lowest state, which a purist would call ‘absence of phase transition’. However, even for the unrealistically small number of \(N = 2000\) bosons, the dependence of the condensate number on temperature (Fig. 18.1a) resembles closely the ‘thermodynamic’ relation from eqn (18.3), here written as \(N_0 = N(1 - (T/T_{c0})^2)\), and \(k_B T_{c0} = \hbar \omega_0 \sqrt{6N/\pi^2 g_S}\). The point of maximum curvature may serve as a practical definition of the critical temperature. This can be defined a bit more accurately: Leave out in the sum over states eqn (18.4) the lowest state \((j \neq 1)\), and put \(\mu = \hbar \omega_0\). The resulting critical exciton number (or better the corresponding temperature at \(N = 2000\)) is marked by a vertical arrow in Fig. 18.1a.

With the the harmonic oscillator eigenfunctions \(\phi_j(X)\) at hand, we can calculate the spatial distribution of bosons in the trap quite easily,

\[
n(R) = g_S \sum_{j,k=0}^{\infty} \phi_j^2(X) \phi_k^2(Y) \frac{1}{e^{\beta(\hbar\omega_0(j+k+1) - \mu)} - 1}.
\]  

Integration over \(X, Y\) gives the total Boson number of eqn (18.4), as it should be. In Fig. 18.1b we display the density profile for a few different temperatures.
After reaching the condensation threshold, a strong central peak evolves which is due to the macroscopic number of bosons sitting in the ground state (marked by GS). Thus, for ideal bosons, the evolution of a sharp central peak in the trap density signals condensation. However, we can read the same figure in reciprocal space as well: Then, the plot gives the emission intensity in dependence on (in-plane) photon momentum. For this to work, we employ that the Fourier transform of the harmonic oscillator functions have the same functional form as their originals. A sharply peaked angular emission normal to the quantum well plane is to be expected, which might be used as another indication of exciton BEC. The numbers involved here are $\hbar \omega_0 = 0.3 \mu eV$ (really tiny), and a length scale of $l_0 = \sqrt{\hbar \omega_0 / \alpha} = 1 \mu m$.

However, results obtained for ideal bosons are of rather limited value for the spatially indirect excitons in coupled quantum wells, which feel a strong dipole-dipole repulsion. The trap potential is going to be flattened out, which will obscure the sharp central peak. On the other hand, the directional emission will persist. More details will be presented in the following Sections.

### 18.3 Excitons in coupled quantum wells

In a recent experiment by Snoke and coworkers (Vörös et al., 2005), a sample with two coupled GaAs quantum wells of 10 nm width each has been used, being separated by a 4 nm wide barrier. A static electric field is applied in the growth ($z$-) direction which allows to tune the indirect exciton state below the direct

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**Fig. 18.2.** Band edge diagram for the coupled quantum well system used in (Vörös et al., 2005): Two GaAs quantum wells of 10 nm width are separated by a Al$_x$Ga$_{1-x}$As barrier of 4 nm. A static electric field of $F = 36 \text{kV/cm}$ is tilting the band edges. Each confinement wavefunction is plotted with a vertical offset being its energy. From (Zimmermann, 2006).
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exciton. The band edge diagram is shown in Fig. 18.2. Pressing with a needle on the sample allowed to form a lateral trap which is guiding the indirect excitons towards the trap center. With increasing excitation, a blue shift of up to 5 meV and a broadening of the exciton emission line has been seen.

In order to model the system we have solved the Schrödinger equation for the confinement functions $u_{a}(z)$ ($a = \text{dir}, \text{ind}, h$), as displayed in Fig. 18.2. For calculating the single-exciton states, we have split off the confinement functions and introduced for the in-plane motion relative ($r$) and center-of-mass ($R$) co-ordinates, writing

$$\Psi(r_e, r_h) = u_e(z_e) \ u_h(z_h) \ \phi(r) \ \psi(R). \quad (18.6)$$

The Schrödinger equation for the 1s exciton wave function $\phi(r)$ is solved numerically (Zimmermann 2005) with the attractive electron-hole potential $-v_{eh}(r)$.

The in-plane Coulomb potentials between different carriers are given by

$$v_{ab}(r) = \int dz \ dz' \frac{e^2}{4\pi \varepsilon_0 \varepsilon_s} \ [\frac{2}{R} - \frac{2}{\sqrt{R^2 + d^2}}]. \quad (18.7)$$

The indirect exciton has a binding energy of $E_B = 3.5$ meV, not much different from the GaAs bulk value. Due to the extremely small overlap of the confinement functions, the radiative lifetime of the indirect exciton at zero momentum is quite long, $\tau_{\text{ind}} = 0.45 \mu$s. Consequently, an equilibration of the indirect exciton gas at low bath temperatures can be expected. For the sake of comparison, we give the values for the direct exciton as well: $E_B = 19.1$ meV, $\tau_{\text{dir}} = 31 \ \mu$s.

Let us now investigate the forces acting between two indirect excitons which are at a distance $R$ apart from each other. Reducing for the moment the excitons to point charges, we can easily add those four Coulomb potentials which are not already ‘eaten up’ by forming the excitons itself, with the result

$$U_{\text{dir}}(R) = \frac{e^2}{4\pi \varepsilon_0 \varepsilon_s} \left[ \frac{2}{R} - \frac{2}{\sqrt{R^2 + d^2}} \right]. \quad (18.8)$$

Here, $d$ is the effective charge separation between electron and hole, which comes out a bit larger ($d = 15.5$ nm) compared to the distance between the quantum wells (14 nm) – an effect induced by the static electric field. A strong repulsive potential is obtained (thin full curve in Fig. 18.3a). For large exciton separation, the famous dipole-dipole law

$$\lim_{R \to \infty} U_{\text{dir}}(R) = \frac{e^2}{4\pi \varepsilon_0 \varepsilon_s} \ \frac{d^2}{R^3} \quad (18.9)$$

appears (dashed curve). However, the real charge distribution of the exciton leads to a somewhat different shape, (thick full curve in Fig. 18.3a). It can be best formulated in reciprocal space as
Excitons in coupled quantum wells

Fig. 18.3. (a) Direct repulsion between two indirect excitons: Full calculation (full curve) compared with the point-charge exciton model (thin curve) and the dipole-dipole limit (dashed curve). The arrows denote the size of the exciton and its binding energy. (b) Strengths of the contact interaction: Direct, exchange and Hartree-Fock combination $U_{HF} = (5/4)U_d + U_x$. The dotted straight line is the point-charge result $U_{d0}$.

$$U_d(q) = v_{ee}(q) \chi^2(\beta q) + v_{hh}(q) \chi^2(\alpha q) - 2v_{eh}(q) \chi(\beta q) \chi(\alpha q), \quad (18.10)$$

where the mass ratios $\alpha = m_e/M, \beta = m_h/M$ ($M = m_e + m_h$) appear, and $\chi(q)$ is the Fourier transform of $\phi^2(r)$.

Now, the composite nature of excitons consisting of two fermions comes into play (Hanamura and Haug, 1977). This leads to a non-local potential due to fermionic exchange between the constituents. Forcing this into a local potential $U_d(R)$, one can construct the following Hamiltonian for many center-of-mass excitons (we closely follow Laikhtman and coworkers (Ben-Tabou de-Leon and Laikhtman, 2001)):

$$H = \int dR \sum_s \Psi_s^\dagger(R) \left[ -\frac{\hbar^2 \nabla^2}{2M} + V(R) \right] \Psi_s(R) + \frac{1}{2} \int dR dR' [U_d(R - R') + U_x(R - R')] \sum_{ss'} \Psi_s^\dagger(R) \Psi_{s'}(R') \Psi_{s'}(R) \Psi_s(R) + \int dR dR' U_x(R - R') \sum_{ss'} \left( \frac{1}{4} - \delta_{ss'} \right) \Psi_s^\dagger(R) \Psi_{s'}(R') \Psi_{s'}(R).

(18.11)$$

Here, it was essential to take into account the spin structure of excitons which are composed from spin 1/2 conduction electrons and spin 3/2 heavy hole states. The exciton spin label $s$ denotes the four exciton states, which are $s = \pm 1$ (bright).
and \( s = \pm 2 \) (dark). The one-exciton potential \( V(R) \) can model a lateral trap confinement, including disorder if present.

There is an intense discussion in the literature on to what degree it is allowed to treat excitons as bosons, as done in eqn (18.11). Note that in eqn (18.11), both effects are implemented, namely the Coulomb interaction between two excitons as well as their departure from being true bosons. However, this is done in a perturbative fashion only (linear in the Coulomb potential). An improvement has been formulated by Okumura (Okumura and Ogawa, 2001), taking into account the composite-particle effect properly. Restricting to the extreme mass ratio \((m_e/m_h \to 0)\), their result agrees nicely with a Heitler-London type treatment. Another route is followed by Combescot and coworkers (Combescot and Betheder-Maribet, 2002), who try to avoid Bose commutation rules completely. Coming up with an overcomplete set of exciton operators, however, leads to a series of contradicting results, which makes their approach questionable, if not useless.

To simplify matters we reduce in what follows the exciton-exciton (XX) potential to contact form, \( U_d(R) \Rightarrow U_d \delta(R - R') \), where the strength \( U_d \) is given by integrating the full potential over \( R \), or taking its zero-momentum value. Explicitly, we arrive at

\[
U_d = \int dR [v_{ee}(R) + v_{hh}(R) - 2v_{eh}(R)] = \lim_{q \to 0} [v_{ee}(q) + v_{hh}(q) - 2v_{eh}(q)].
\]

The point-charge result is simply

\[
U_{d0} = \frac{e^2}{\varepsilon_0 \varepsilon_s} d.
\]

The exchange strength is

\[
U_x = \sum_{k,k'} [2v_{eh}(k-k') \phi_k^* \phi_{k'} - (v_{ee}(k-k') + v_{hh}(k-k')) \phi_k^2 \phi_{k'}^2].
\]

A calculation for the CQW of Fig. 18.2 gave the values of \( U_d = 18.8 \text{ eV nm}^2 \) and \( U_x = -8.9 \text{ eV nm}^2 \).

In Fig. 18.3b, the dependence on charge separation \( d \) is plotted. In order to reach smoothly the direct exciton, we have taken here single potential wells for electron and hole, displaced by \( d \). It is interesting to see how the exchange interaction (dashed) which is dominant and repulsive at \( d = 0 \), reduces and changes sign at larger well separation. On the other hand, the direct repulsion (full) is taking over, so that the total effect is always repulsive.

We have added to Fig. 18.3b as dash-dotted curve the combination \( U_{HF} = (5/4)U_d + U_x \) which is the central quantity appearing in a Hartree-Fock decoupling (and assuming spin equilibration). The (local) shift of the bare dispersion \( \epsilon_k = \hbar^2 k^2/2M \) is given by
Dynamical T-matrix theory

\[ T = U - T \]

\[ \Sigma = T + T \]

\[ G = G_0 + \Sigma \]

**Fig. 18.4.** Diagrammatic representation of the dynamical T-matrix scheme.

The full line with arrow denotes the single exciton Green’s function \( G_k(z) \), while the dotted arrow represents the non-interacting Green’s function. The dashed line is the XX interaction \( U^\pm \) (for simplicity, we have omitted the spin degree of freedom). The two terms in the self-energy are the direct and (boson) exchange contributions.

\[ \Delta(R) = U_{HF} n(R). \quad (18.15) \]

In a smooth trap potential, the exciton density follows from

\[ n(R) = g_s \sum_k n_B(\epsilon_k + V(R) + \Delta(R) - \mu), \quad (18.16) \]

which needs to be solved selfconsistently.

### 18.4 Dynamical T-matrix theory

The Hartree-Fock decoupling of the many-exciton problem as formulated in eqn (18.15) and eqn (18.16) can easily explain the blue shift. However, while the Hartree-Fock approach (including a c-number term) is at the heart of the Bogolubov theory of BEC (Griffin, 1996), it is not able in principle to describe line broadenings. Going one step further, we outline here a dynamical T-matrix theory which we have formulated recently (Zimmermann, 2006). Note that this theory cannot be applied to the BEC state itself, but gives a reasonable insight into the route towards condensation.

In diagrammatic language, multiple XX scattering events are summed up and form the dynamical T-matrix \( T_q(z) \) (Fig. 18.4, first line). Plugging this into the one-exciton self energy \( \Sigma_k(z) \) (second line) gives via the Dyson’s equation (third line) an improved one-exciton propagator \( G_k(z) \). Selfconsistency has to be achieved by repeated iteration.

The specific spin structure of eqn (18.11) allows to split the T-matrix into a bonding/antibonding part \( T^\pm \), which obey
where the exciton pair propagator (two parallel arrows in Fig. 18.4) is given by

$$G_q(\Omega) = \sum_k \int \frac{d\omega}{\pi} \frac{d\omega'}{\pi} A_k(\omega) A_{k-q}(\omega') \frac{1 + 2n_B(\hbar\omega - \mu)}{\omega + \omega' - \Omega}. \quad (18.18)$$

The real part of eqn (18.18) has a logarithmic divergence coming from the integration over $k$. This is a well-known shortcoming of using in two dimensions a contact potential. We have chosen to cut off the integration at $\epsilon_k = 100$ meV, and postpone a proper treatment of the full $k$-dependence to further work.

For the contact interaction used, both diagrams of the self-energy can be combined into one term, with $T \equiv (9/2)T^+ + (1/2)T^-$, resulting in

$$\Sigma_k(z) = \sum_q \int \frac{d\omega}{\pi} A_{k-q}(\omega) \times$$

$$\times \left[ T_q(z + \omega) n_B(\hbar\omega - \mu) + \int \frac{d\omega'}{\pi} \frac{\text{Im} T_q(\omega' - i0) n_B(\hbar\omega' - 2\mu)}{\omega' - z - \omega} \right]. \quad (18.19)$$

This enters the exciton Green’s function resp. its spectral function

$$A_k(\omega) = \text{Im} G_k(\omega - i0) = \frac{\text{Im} \Sigma_k(\omega - i0)}{[\omega - \epsilon_k - \text{Re} \Sigma_k(\omega - i0)]^2 + [\text{Im} \Sigma_k(\omega - i0)]^2}. \quad (18.20)$$

For the exciton system, the spectrally and directionally resolved spontaneous optical emission is given by

$$I(k, \omega) = \mu^2 c v \int dR \int_t e^{i k (R - R')} \int dt e^{i \omega t} \langle \Psi_{s}(R, t) \Psi_{s}(R', 0) \rangle, \quad (s = \pm 1) \quad (18.21)$$

which shows clearly the importance of contributions $R \neq R'$ (off-diagonal long range order) for the directional characteristic (dependence on $k$). Further, eqn (18.21) can be simply expressed via the single-exciton propagator resp. the spectral function,

$$I(k, \omega) \propto \text{Im} G_k(\omega - i0) \equiv n_B(\hbar\omega - \mu) A_k(\omega). \quad (18.22)$$

For understanding the spectral shape, it is important to note that the spectral function changes sign at the chemical potential $\mu$ where the Bose distribution function $n_B(\hbar\omega - \mu)$ has a pole, resulting in a strictly positive emission. Portions below $\mu$ are due to photon emission accompanied by XX scattering. Results of the full dynamical T-matrix calculation (at zero momentum) are shown in Fig. 18.5. Nothing special happens if the chemical potential is located within the luminescence line. Obviously, the classical argument for BEC onset which we have
Formulated in Section 18.2 has to be refined here: Instead of ‘\( \mu \) is touching the energy of the lowest state’, a phase transition can only happen if the chemical potential hits the quasiparticle dispersion, \( \text{Re} \Sigma(\hbar \omega = \mu) = \mu \). The sharpening of the spectrum (Fig. 18.6a) is related to the undamping of XX scattering close to the phase transition. Technically, the imaginary part of the self energy must be zero at the chemical potential. As shown in Fig. 18.6b, the initial increase in line width due to XX scattering turns into a shrinkage, a finding which can be used to locate in a photoluminescence experiment the approach towards condensation. For the given density range (blue shift up to 2 meV) and an exciton temperature of \( T = 5 \) K, however, condensation is not achieved. The directional characteristics (not shown here) evolves into a sharp angular peak, too. Similar conclusions on the condensation signature in the angular emission have been drawn by Littlewood and coworkers (Keeling et al., 2004).

The exciton densities given in Fig. 18.6 are calculated with the standard expression using the momentum- and frequency-dependent spectral function eqn (18.20) and the Bose distribution function. It is worth noting that the T-matrix comes out appreciably smaller than the bare interaction \( (U^\pm) \), as shown by the dotted lines in Fig. 18.5. Consequently, at a given density, the blue shift of the emission is much less than the simple Hartree-Fock argument would predict. Obviously, the strong dipole-dipole repulsion and its dynamical character hinder an easy build-up of coherence. Refined calculations are needed before reliable predictions for exciton condensation in coupled quantum wells can be made.
Fig. 18.6. (a) Emission line shape of indirect excitons for different excitation levels (logarithmic plot). The lines are getting sharper as the chemical potential $\mu$ (dots) approaches the emission maximum (quasiparticle position $\Delta$). The temperature is held fixed at $T = 5$ K, and exciton densities $N_X$ are given in units of $10^{10}$ cm$^{-2}$. (b) The line width (right scale) shows a non-monotonous behavior on exciton density.

References


