Mathematische Grundlagen

Übungsblatt 1 – Lösungen

1.

2. Man vereinfache die folgenden Ausdrücke:

$$\begin{array}{ll} \text{a)} & \frac{c}{3ab} \\ \text{b)} & \frac{a^2+b^2}{(a-b)^2(a+b)} \\ \text{c)} & \frac{ac^{2n}}{b} \\ \text{d)} & a \end{array}$$

3.

$$u^4 - v^4 = (u-v)[u^3 + u^2v + uv^2 + v^3]$$

oder

$$\begin{aligned} (u^2)^2 - (v^2)^2 &= (u^2 - v^2)(u^2 + v^2) \\ &= (u-v)(u+v)(u^2+v^2) \end{aligned}$$

4.

$$\begin{aligned} \binom{n}{k} + \binom{n}{k+1} &= \frac{n!}{k!(n-k)!} + \frac{n!}{(k+1)!(n-k-1)!} \\ &= \frac{n![(k+1)+(n-k)}{(k+1)!(n-k)!} \\ &= \frac{(n+1!)}{(k+1)!(n-k)!} \end{aligned}$$

$$\begin{aligned} (a+b)(a+b)^n &= (a+b)\sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k} \\ &= \sum_{k=0}^{n}\binom{n}{k}a^{k+1}b^{n-k} + \sum_{k=0}^{n}\binom{n}{k}a^k b^{n-k+1} \end{aligned}$$

$$
\begin{aligned}
&= \sum_{k=1}^{n+1} \binom{n}{k-1} a^k b^{n-k+1} + \sum_{k=0}^{n} \binom{n}{k} a^k b^{n-k+1} \\
\Rightarrow \quad \binom{n+1}{0} &= \binom{n}{0} \quad k = 0 \\
\binom{n+1}{k} &= \binom{n}{k-1} + \binom{n}{k} \quad k = 1..n \\
\binom{n+1}{n+1} &= \binom{n}{n} \quad k = n+1
\end{aligned}
$$