Dr. B. Bunk WS 2014/15

Mathematische Grundlagen

Übungsblatt 9 – Lösungen

1. (a) $x = g(y) = \dfrac{y^2-1}{2y} = \dfrac{1}{2}\left(y - \dfrac{1}{y}\right)$

   (b) $f'(x) = 1 + \dfrac{x}{\sqrt{x^2+1}}$

   $g'(y) = \dfrac{1}{2}\left(1 + \dfrac{1}{y^2}\right)$

   (c) $x^2 + 1 = \dfrac{(y^2-1)^2 + 4y^2}{4y^2} = \dfrac{(y^2+1)^2}{4y^2}$

   $\Rightarrow \quad f'(x) = 1 + \dfrac{2xy}{y^2+1} = 1 + \dfrac{y^2-1}{y^2+1} = \dfrac{2y^2}{y^2+1} = \dfrac{1}{g'(y)}$

2. $$\begin{aligned} \frac{\partial f}{\partial r} &= \cos\varphi\, e^{r\cos\varphi} & \frac{\partial f}{\partial \varphi} &= -r\sin\varphi\, e^{r\cos\varphi} \\ \frac{\partial^2 f}{\partial r^2} &= \cos^2\varphi\, e^{r\cos\varphi} & \frac{\partial^2 f}{\partial \varphi^2} &= (-r\cos\varphi + r^2\sin^2\varphi)\, e^{r\cos\varphi} \\ \frac{\partial}{\partial\varphi}\frac{\partial f}{\partial r} &= (-\sin\varphi - r\sin\varphi\cos\varphi)\, e^{r\cos\varphi} & &= \frac{\partial}{\partial r}\frac{\partial f}{\partial \varphi} \end{aligned}$$

3. (a) $-\dfrac{1}{a}\cos at$

   (b) $2\sqrt{1+x^2}$

   (c) $-\log|1-t^2|$

   (d) $\dfrac{x^3}{3}\log x - \dfrac{x^3}{9}$

4. ♡

   (a) Integrand ist ungerade, Intervall ist symmetrisch

   (b) Integrand ist ungerade bzgl. $\phi = \pi/2$