Dr. B. Bunk WS 2014/15

Mathematische Grundlagen

Übungsblatt 10 – Lösungen

1. (a) $\dfrac{1}{(x+4)(x+5)} = \dfrac{1}{x+4} - \dfrac{1}{x+5}$

   (b) $\dfrac{2x+3}{x^3+x^2-2x} = \dfrac{2x+3}{x(x-1)(x+2)} = -\dfrac{3}{2}\,\dfrac{1}{x} + \dfrac{5}{3}\,\dfrac{1}{x-1} - \dfrac{1}{6}\,\dfrac{1}{x+2}$

   (c) $\dfrac{x+1}{x^2-6x+9} = \dfrac{x+1}{(x-3)^2} = \dfrac{1}{x-3} + \dfrac{4}{(x-3)^2}$

2. (a) Integrand divergiert an beiden Grenzen.

   (b) Integrand divergiert (nur) wie $|x-a|^{-1/2}$ an den Grenzen.

   (c) $u = \sqrt{x} \quad du = dx\,\dfrac{1}{2\sqrt{x}}$

   $\Rightarrow \quad \displaystyle\int dx ... = 2\int_0^1 du\,\frac{1}{\sqrt{1-u^2}} = 2[\arcsin u]_0^1 = \pi$

3. (a) $\cos\varphi = 1 - \dfrac{1}{2}\varphi^2 + \dfrac{1}{24}\varphi^4 + ... \Rightarrow \quad \dfrac{1}{\cos\varphi} = 1 + \dfrac{1}{2}\varphi^2 + \dfrac{5}{24}\varphi^4 + ...$

   (b) $\tan\varphi = \varphi + \dfrac{1}{3}\varphi^3 + \dfrac{2}{15}\varphi^5 + ...$

bitte wenden

4. ♡

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\begin{aligned}
x_k &= \sqrt{2}\, e^{i(2k+1)\pi/4} \quad k = 0..3 \\
&= \pm 1 \pm i \quad \text{(alle 4 Kombinationen)} \\
\Rightarrow \quad \frac{1}{x^4+4} &= \sum_k \frac{a_k}{x - x_k} \\
\text{mit} \quad a_k &= \frac{1}{4x_k^3} = \frac{x_k}{4x_k^4} = \frac{-x_k}{16} \\
\Rightarrow \quad 16 \int dx\, \frac{1}{x^4+4} &= \sum_k (-x_k) \log(x - x_k)
\end{aligned}
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Das wäre schon die Lösung, allerdings noch in komplexer Notation.
Die Rückkehr ins Reelle ist mühsam, z.B. für $-x_k = 1 \pm i$:

$$
\begin{aligned}
x + 1 \pm i &= |x+1+i|\, e^{\pm i\phi} \\
\text{mit} \quad \phi &= \arctan \frac{1}{x+1} = \frac{\pi}{2} - \arctan(x+1) \\
\Rightarrow \quad \log(x+1 \pm i) &= \log|x+1+i| \pm i\phi \\
\sum_\pm (1 \pm i) \log(x+1 \pm i) &= \sum_\pm (1 \pm i)(\log|x+1+i| \pm i\phi) \\
&= 2\log|x+1+i| - 2\phi \\
&= \log(x^2+2x+2) + 2\arctan(x+1) - \pi
\end{aligned}
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und für $-x_k = -1 \pm i$:

$$
\begin{aligned}
x - 1 \pm i &= |x-1+i|\, e^{\pm i\phi} \\
\text{mit} \quad \phi &= \arctan \frac{1}{x-1} = \frac{\pi}{2} - \arctan(x-1) \\
\Rightarrow \quad \log(x-1 \pm i) &= \log|x-1+i| \pm i\phi \\
\sum_\pm (-1 \pm i) \log(x-1 \pm i) &= \sum_\pm (-1 \pm i)(\log|x-1+i| \pm i\phi) \\
&= -2\log|x-1+i| - 2\phi \\
&= -\log(x^2-2x+2) + 2\arctan(x-1) - \pi
\end{aligned}
$$

Alles zusammen und reell:

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\begin{aligned}
\Rightarrow \quad 16 \int dx\, \frac{1}{x^4+4} &= \log(x^2+2x+2) - \log(x^2-2x+2) \\
&\quad +2\arctan(x+1) + 2\arctan(x-1)
\end{aligned}
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