Future precision physics at high energies

and

$\overline{\mathrm{MS}}$ vs. pole mass of the gauge bosons

the two–loop electroweak bosonic corrections

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❶ **Perspectives for future precision physics at TESLA**

❷ **Theoretical requirements and challenges**

❸ **State of the art in 2 $\to$ 2 processes**

❹ **The role of $\alpha(E)$ in precision physics**

❺ **Full two-loop calculations: first steps**

❻ **Outlook**

Abstract: At future linear colliders, which may reach very high luminosity, a very interesting option for precision physics is a new scan of the Z--resonance and of the W^+W^---threshold. I briefly review what accuracy possibly can be reached and what is needed to match the corresponding precision on the theory side. First steps towards a full two--loop calculation of the weak mixing angle $\sin^2\Theta^\ell_{\rm eff}$ in terms of α, G_μ and M_Z are discussed. The mass counter-terms of the W and Z bosons required for the two-loop renormalization in the on--shell scheme have been calculated recently.I briefly describe the calculation of the complete bosonic contribution in form of a double series expansion in $\sin^2\Theta_W \lesssim 0.24$ and $v = M_V^2/m_H^2 \lesssim 0.64$. Conceptual problems are discussed briefly. The two-loop results are numerically compared with the one--loop results.

❶ Perspectives for future precision physics at TESLA

Giga-Z and Mega-W

Precision Observables

Most interesting for electroweak precision physics:

new scan at Z

$10^6 \to 10^9$ Z**'s**

higher statistics:

model independent scan with 5 scan points!

plus

threshold scan of WW–threshold region

(single diagram t–channel dominated)

$10^4 \to 10^7$ W**–pairs**

mandatory large $\mathcal{P}_{e^-} (> 80\%)$

plus

either $\delta\mathcal{P}_{e^-} < 0.2 - 0.5\%$ **or** $\mathcal{P}_{e^+} (> 20\%)$

good control of beam strahlung and energy spread

Remarks:	**beam-strahlung:**	$\Gamma_Z \to \Gamma_Z + 60$**MeV**
	energy-spread:	$\sigma^0_{\mathrm{had}} \to \sigma^0_{\mathrm{had}} - 1.8\%$

Most important:

- $a_f \rightarrow \Delta\rho_f$
- $v_f/a_f \rightarrow \sin^2\Theta^f_{\rm eff}$
- $M_W \rightarrow \sin^2\Theta_W$
- $\Gamma_{\rm had} \rightarrow \alpha_S(M_Z)$
- $\Gamma_b, A^b_{\rm FB} \rightarrow \Delta\rho_b, \sin^2\Theta^b_{\rm eff}$

Observable	**LEP**	**GigaZ**
M_Z	91.1874 ± 0.0021	± 0.0021
Γ_Z	0.9×10^{-3}	0.4×10^{-3}
$\sigma^0_{\rm had} = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_{\rm had}}{\Gamma_Z^2}$	0.9×10^{-3}	0.6×10^{-3} *
$R_\ell = \frac{\Gamma_{\rm had}}{\Gamma_\ell}$	1.2×10^{-3}	0.3×10^{-3}
$\alpha_s(M_Z)$	0.1183 ± 0.0027	± 0.0009
$\Delta\rho_\ell$	$(0.55 \pm 0.10) \times 10^{-2}$	$\pm 0.05 \times 10^{-2}$
N_ν	2.984 ± 0.008	± 0.004

* **improved Bhabha required**

improving selection efficiencies for μ, τ and hadrons by factor 3 assumed

most promising:

$$\sin^2\Theta^\ell_{\rm eff} \ \text{ via } \ A_{LR}$$

$$A_{LR} = \frac{1}{\mathcal{P}}\,\frac{\sigma_L-\sigma_R}{\sigma_L+\sigma_R} = \frac{2v_\ell a_\ell}{v_\ell^2+a_\ell^2}$$

$$(1-v_e/a_\ell)\frac{1}{4} = \sin^2\Theta^\ell_{\rm eff}$$

- **ratio of total cross sections**
- **independent on final state**

$\otimes$ **need extremely good control of polarization**

$\cdots$ **Blondel scheme** $\to \Delta A_{LR} \sim 10^{-4}$

$\Rightarrow \Delta\sin^2\Theta^\ell_{\rm eff} \simeq 0.000013$ **!**

factor of 13 improvement vs. LEP/SLC

and others $\cdots$

Expected improvement factors

$\sin^2\Theta^\ell_{\rm eff}$	**13**
R_b	**5**
$\mathcal{A}_b$	**15**

M_W

- threshold scan dominated by one diagram the t–channel ν–exchange
- no fragmentation
- no color recombination etc.

$\delta M_W \sim 6$MeV possible

$\Rightarrow$ big challenge for theory !

❷ Theoretical requirements and challenges

- **Typical progress required:**

❑ threshold scan of WW–threshold region
(single diagram t–channel dominated)
$10^4 \to 10^7$ W–pairs **MegaW** $\delta M_W \sim 6$MeV possible

go from $2 \to 2$ **LEP 1** to $2 \to 4(6)$ processes **LEP 2/LC**
(plus γ radiation)
at 0.2% level or better

❑ new $2 \to 2$ scan at Z (A_{LR}, ...)
$10^6 \to 10^9$ Z's **GigaZ** $\Delta \sin^2 \Theta^\ell_{\mathrm{eff}} \simeq 0.000013$ **!**
higher statistics:
model independent scan with 5 scan points!

go to next order for $2 \to 2$ processes
(plus γ radiation)
increase precision by factor 10 relative to LEP 1

Note: complexity of calculations grows factorial
(a battle against combinatorics)
SM in renormalizable gauge without fermion doublet multiplicity
11 types of lines , 57 types of vertices !
more loops and/or more legs become a true challenge
SUSY extensions even much more complex !

In addition:

the many mass scale complexity

What complicates matters dramatically are the many different mass scales which can be very large and lead to extremely complicated analytic structure of the amplitudes and to dramatic numerical cancellation problems[a]**.**

[a] For 1–loop 2 to 2 processes M. Veltman wrote a CDC assembler routine *FORMF* (which essentially did what *FF* is doing today in a different way in standard double precision FORTRAN) at 128 digits internal precision which allowed him to guarantee 8 digits final precision in such calculations. Alternatively one has to do the cancellations analytically by appropriate series expansions, usually a tedious task.

Examples: W^+W^--production, ZH-production, $t\bar{t}$-production (while $f\bar{f}$-production is much easier for the light fermions)

Compare QCD : 3 types of lines, 4 types of vertices

only mass scales $\alpha_s \Leftrightarrow \Lambda_{\rm QCD},\; m_c,\; m_b,\; m_t$

except for top–production calculations

QCD calculations are mostly done in the massless approximation

❑ **the high precision challenge: GigaZ:**

2 $\longrightarrow$ 2 full 2–loop + leading 3–loop

Altogether: new technical challenge

going from a few 100 to several 10000 diagrams !

Example: # of diagrams at tree level WW production and decay:

signal 3 + background
$$\left\{ \begin{array}{ll} e^+e^- \longrightarrow \mu^-\bar{\nu}_\mu\tau^+\nu_\tau & 6 \\ \vdots & \vdots \\ e^+e^- \longrightarrow e^-\bar{\nu}_e e^+\nu_e & 53 \end{array} \right.$$

Other examples: tree unitary gauge, loops linear R_ξ gauge

process		# diagrams	loop level
$e^+e^- \longrightarrow 4\mathrm{f}$	$\Rightarrow$	6 to 144	tree
$e^+e^- \longrightarrow 4\mathrm{f}\gamma$	$\Rightarrow$	14 to 1008	tree
$e^+e^- \longrightarrow \mu^-\bar{\nu}_\mu u\bar{d}$	$\Rightarrow$	1907	1–loop
$e^+e^- \longrightarrow e^+e^-d\bar{d}$	$\Rightarrow$	8522	1–loop
$e^+e^- \longrightarrow e^+e^-e^+e^-$	$\Rightarrow$	21444	1–loop
$V \longrightarrow V$ ($V = W, Z$)	$\Rightarrow$	$\sim$ 50	1–loop
$W \longrightarrow W$	$\Rightarrow$	4084 (792 1pi)	2–loop
$Z \longrightarrow Z$	$\Rightarrow$	2348 (616 1pi	2–loop

❸ State of the art in 2 $\longrightarrow$ 2 processes

Complete 2 $\longrightarrow$ 2 at 1–loop calculations have been performed long time ago.

(Veltman, Passarino '79, Sirlin, Marciano, '79, ...)

Today largely automatized, standard procedures. At the 2–loop level a substantial set of leading corrections have been calculated (heavy top, heavy Higgs, QCD–corrections, QED-corrections). For no channel of $2f \longrightarrow 2f$ a complete 2–loop calculation exists! This, in spite of the fact, that such a calculation would have been very important for the interpretation of the Higgs mass bound from LEP/SLC.

Most SM calculations concern $\Delta\rho$, Δr, $\Gamma(Z \to b\bar{b})$, etc.
Typically: (incomplete)

- $O((G_\mu m_t^2)^2)$
  (van der Bij, Hoogeveen, '87, Consoli et al., '89)
  $O(G_\mu m_t^2 \alpha_s)$
  (Fleischer et al. '93, Avdeev et al. '94, Chetyrkin et al. '95)
  $O(G_\mu^2 m_t^2 M_Z^2)$
  (Degrassi, Gambino, Vicini, '96)
- $O(\alpha^2)$ **in** $\Delta\alpha$ **and for** μ**–decay**
  (van Ritbergen, Stuart, '99)
- **full 2–loop fermion contributions to** Δr
  (Freitas, Hollik, Walter, Weiglein, '02)

Most accurate electroweak precision predictions

Calculate $\sin^2\Theta_i$ **from** α, G_μ **and** M_Z **:**

$$\boxed{\sqrt{2}\,G_\mu\,M_Z^2\,\sin^2\Theta_i\,\cos^2\Theta_i = \pi\alpha\,\frac{1}{1-\Delta r_i}}$$ **Sirlin 1980**

Various definitions of $\sin^2\Theta_i$**, which coincide at tree level:**
From weak gauge boson masses, from electroweak gauge couplings and from the neutral current coupling of the charged fermions:

$$\begin{aligned}
\sin^2\Theta_W &= 1-\frac{M_W^2}{M_Z^2} \\
\sin^2\Theta_g &= e^2/g^2 \\
\sin^2\Theta_f &= \frac{1}{4|Q_f|}\left(1-\frac{v_f}{a_f}\right)\;,\quad f\neq\nu
\end{aligned}$$

$$\begin{aligned}
\Delta r_i &= \Delta r_i(\alpha,\,G_\mu,\,M_Z,m_H,\,m_{f\neq t},\,m_t) \\
\Delta r_i &= \underset{\substack{\uparrow \\ \text{universal}}}{\Delta\alpha} - f_i(\sin^2\Theta_i)\,\Delta\rho+\Delta r_{i\,\mathrm{rem}}
\end{aligned}$$

$\longrightarrow$ **predictions for** M_W, A_{LR}, A_{FB}^f, Γ_f, $\cdots$

$$\delta\Delta\alpha\to\delta M_W\;,\quad\delta\sin^2\Theta_i\;,\;\cdots$$

❹ The role of $\alpha_{\rm em}(s)$ in precision physics

Uncertainties of hadronic contributions to effective α are a problem for electroweak precision physics:

Precision physics limitations:

α , G_μ, M_Z **most precise input parameters**

partially

$\Updownarrow$

non-perturbative **precision predictions**

relationship $\sin^2\Theta_f, v_f, a_f, M_W, \Gamma_Z, \Gamma_W, \cdots$

$\alpha(M_Z), G_\mu, M_Z$ **best effective input parameters**

for VB physics (Z,W) etc.

$$\begin{array}{cccc} \frac{\delta\alpha}{\alpha} & \sim & 3.6 & \times & 10^{-9} \\ \frac{\delta G_\mu}{G_\mu} & \sim & 8.6 & \times & 10^{-6} \\ \frac{\delta M_Z}{M_Z} & \sim & 2.4 & \times & 10^{-5} \\ \frac{\delta\alpha(M_Z)}{\alpha(M_Z)} & \sim & 1.6 \div 6.8 & \times & 10^{-4} \end{array}$$

$\Rightarrow$ **now a limiting factor**

for precision physics

For perturbative QCD contributions very crucial:

precise QCD parameters $\alpha_s,\ m_c,\ m_b,\ m_t$

Status and Perspectives:

$$\frac{\delta \sin^2\theta}{\sin^2\theta} \sim \frac{\cos^2\theta}{\cos^2\theta - \sin^2\theta}\,\delta\Delta\alpha \sim 1.54\,\delta\Delta\alpha$$

$\delta\Delta\alpha$	$\delta \sin^2\theta$	Ref.	Method
0.00065	0.000232	[EJ95]	data $< 12.$ GeV
0.00016	0.000057	[DH98]	data < 1.7 GeV
0.00025	0.000089	[FJ98]	Euclidean > 2.5 GeV
0.000197	0.000070	[MOR00]	rescaled data, pQCD 2.8-3.7, 5-∞
0.000220	0.000078	[MOR00]	same but “exclusive”
0.000460	0.000164	[BP00]	data
0.000391	0.000139	[FJ00]	as EJ95 + new data
0.00007	0.000025	$\delta\sigma \lesssim 1\%$ up to J/ψ	
0.00005	0.000018	$\delta\sigma \lesssim 1\%$ up to Υ	
WA	0.000160	LEP+SLD	

❺ Full two-loop calculations: first steps

Aim: so far little feeling for size of corrections from bosonic sector.

- **QED and QCD on electroweak processes: limited number of diagrams**
- **relatively small number of diagrams involving top or physical Higgs**
- **full gauge boson sector (incl. Higgs- and Fadeev-Popov ghosts) large number of diagrams**

Steps of technical complications:
self–energies $\longrightarrow$ form–factors $\longrightarrow$ boxes

Complete calculations of observables involve all type of correction ! not feasible at present

Full two–loop renormalization program: need full set of counter terms. e.g., on-shell renormalization scheme α, M_Z and M_W as basic parameters (QED–like scheme) $\longrightarrow$ calculate gauge boson mass counter-terms (equiv. $\overline{\mathrm{MS}}$ vs. pole mass relation)

Theoretical issue: About the proper definition of masses of unstable particles

(Stuart, '91, Sirlin, '91, ..., Kniehl, Sirlin, '98)

The pole mass of the weak gauge bosons

(at two–loops)

The mass and width of a massive gauge boson V are defined via the position s_P of the pole of the full propagator (=zero of its inverse)

$$\boxed{s_P - m_V^2 - \Pi_V(s_P, m_V^2, \cdots) = 0,}$$

$\Pi_V(p^2, \cdots)$ transversal part of the one-particle irreducible self-energy (depends on all SM parameters)

- **bare amplitude in terms of bare parameters ($m_V \to m_{V,0},\ \Pi_V \to \Pi_{V,0}$)**
- **renormalized amplitude in terms of renormalized parameters, e.g., $\overline{\mathrm{MS}}$ (no index)**

Properties of the pole:

- **gauge invariant**
- **infrared finite**
- **complex in general**

Defines *pole (on–shell) mass* M and *width* Γ via

$$\boxed{s_P \equiv M^2 - \mathrm{i} M \Gamma.}$$

Renormalization

The renormalized amplitudes

$$\Pi_{V,r}(p^2, m_{V,r}^2, \cdots) = \Pi_{V,r}^{(1)}(p^2, m_{V,r}^2, \cdots) + \Pi_{V,r}^{(2)}(p^2, m_{V,r}^2, \cdots) + \cdots$$

to two-loops read (indices: 0=bare, r=renormalized)

$$\Pi_{V,r}^{(1)}(p^2, m_{V,r}^2, \cdots) = \left[\Pi_{V,0}^{(1)}(p^2, m_{V,0}^2, \cdots) + (\delta m_V^2)^{(1)} - (p^2 - m_{V,r}^2)\, \delta Z_V^{(1)} \right] \Bigg|_{\substack{m_{j,0}^2 = m_{j,r}^2 \\ e_0 = e_r}}$$

$$\Pi_{V,r}^{(2)}(p^2, m_{V,r}^2, \cdots) = \Bigg[\Pi_{V,0}^{(2)}(p^2, m_{V,0}^2, \cdots) + \left(\Pi_{V,0}^{(1)}(p^2, m_{V,0}^2, \cdots) + (\delta m_V^2)^{(1)} \right) \delta Z_V^{(1)}$$
$$+ \sum_j (\delta m_j^2)^{(1)} \frac{\partial}{\partial m_{j,0}^2} \Pi_{V,0}^{(1)} + (\delta e)^{(1)} \frac{\partial}{\partial e_0} \Pi_{V,0}^{(1)} + (\delta m_V^2)^{(2)} - (p^2 - m_{V,r}^2)\, \delta Z_V^{(2)} \Bigg] \Bigg|_{\substack{m_{j,0}^2 = m_{j,r}^2 \\ e_0 = e_r}}$$

where in the $\overline{\mathrm{MS}}$ scheme order by order the mass-counter-term $(\delta m_V^2)^{(j)}$ subtracts the ϵ–poles at $p^2 = m_{V,r}^2$ and the wave-function renormalization counter-term $\delta Z_V^{(j)}$ subtracts the ϵ–poles remaining when $p^2 \neq m_{V,r}^2$

Strictly speaking the renormalization of the ghost sector (in particular of the gauge parameter) is not discussed here, because its not needed for what follows. In case of the Z the $\gamma - Z$-mixing is an additional complication (see below).

Pole mass and $\gamma - Z$-mixing

In the neutral gauge boson sector because of $\gamma - Z$-mixing we have to consider a 2×2 matrix propagator

$$D^{-1}\left(p^2\right) = \begin{pmatrix} p^2 - \Pi_{\gamma\gamma}(p^2) & \Pi_{\gamma Z}(p^2) \\ \Pi_{Z\gamma}(p^2) & p^2 - m_Z^2 - \Pi_{ZZ}(p^2) \end{pmatrix}$$

Position of Z-pole:

$$\boxed{s_P - m_Z^2 - \Pi_{ZZ}(s_P) - \frac{\Pi^2_{\gamma Z}(s_P)}{s_P - \Pi_{\gamma\gamma}(s_P)} = 0.}$$

- **Mixing term $\Pi^2_{\gamma Z}$ starts contributing at two-loops**

- **Photon term $\Pi_{\gamma\gamma}$ only contributes beyond two-loop**

Notation for self-energies Π_V ($V = W, Z$) with

$$\Pi_W(p^2, \cdots) = \Pi_{WW}(p^2, \cdots)$$

$$\Pi_Z(p^2, \cdots) = \Pi_{ZZ}(p^2, \cdots) + \frac{\Pi^2_{\gamma Z}(p^2, \cdots)}{p^2 - \Pi_{\gamma\gamma}(p^2, \cdots)} \ .$$

Formally, same formulae apply for W and Z.

Pole mass "master formula"

By iterative solution of the pole formula to two-loops we obtain our master formula:

$$\begin{aligned} s_P \quad = \quad & m^2 \\ & + \Pi^{(1)}(m^2, m^2, \cdots) \\ & + \Pi^{(2)}(m^2, m^2, \cdots) + \Pi^{(1)}(m^2, m^2, \cdots)\Pi^{(1)\prime}(m^2, m^2, \cdots) \\ & + \cdots \end{aligned}$$

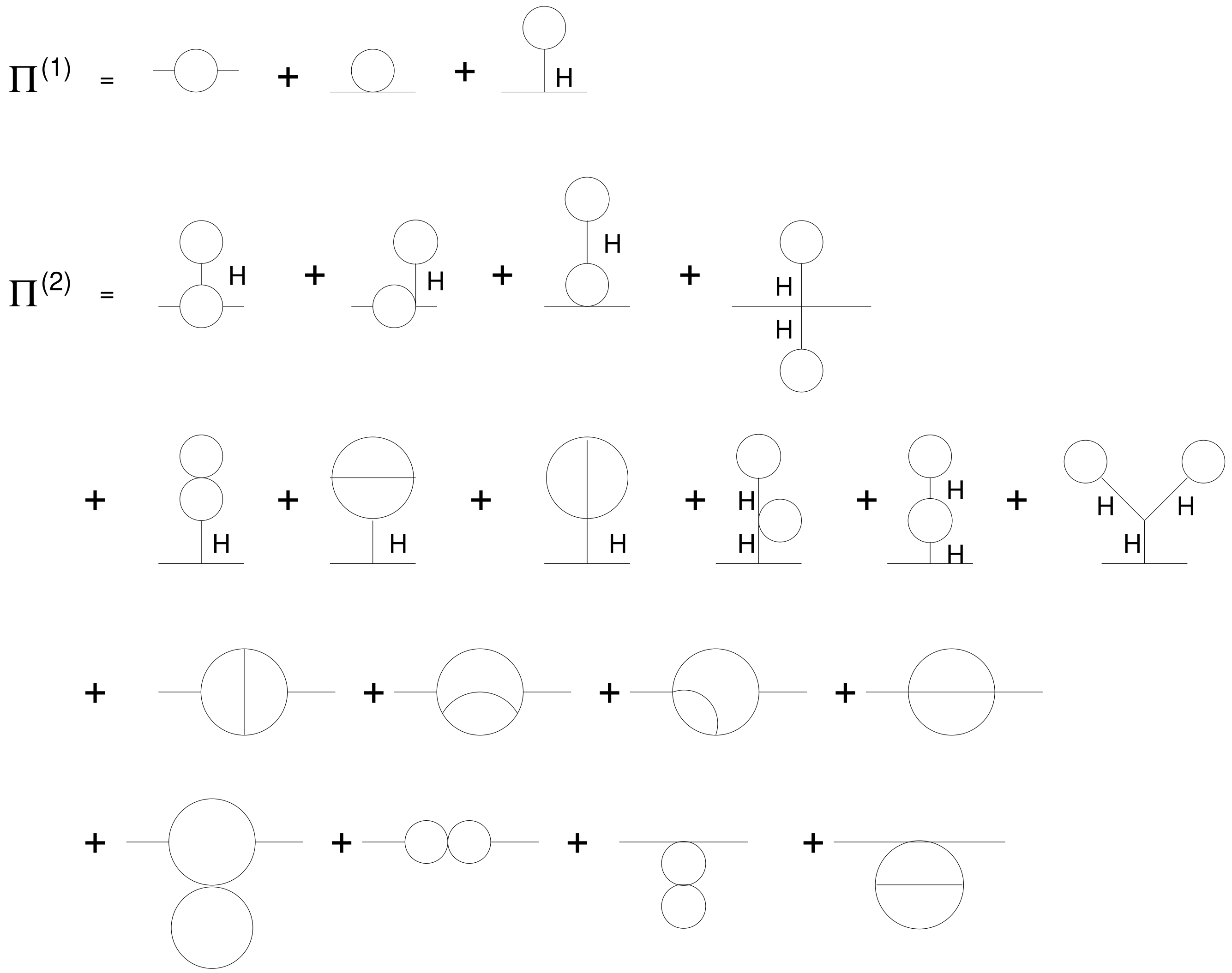
which yields the pole mass M^2 and the width Γ at this order. $\Pi^{(L)}$ is the bare ($m = m_0$) or $\overline{\mathrm{MS}}$ -renormalized (m the $\overline{\mathrm{MS}}$ -mass) L-loop contribution to Π, and the prime denotes the derivative with respect to p^2. In this way we need to evaluate propagator type diagrams and their derivatives at $p^2 = m^2$.

Note: the p^2–dependence has disappeared in this solution; it turned into a mass–dependence which cannot be disentangled from the original mass dependence of the off-shell amplitude.

Remark: the mixed $\Pi^{(1)}(m^2, m^2, \cdots)\Pi^{(1)\prime}(m^2, m^2, \cdots)$ term is crucial for getting a gauge–invariant result for the two–loop mass counter–term (see below)

Diagrams and topologies

To be computed on-shell ($p^2 = m_V^2$): $\Pi(p^2) = \Pi_1(p^2) + \Pi_2(p^2) + \cdots$



Number of diagrams	linear R_ξ gauge		non-linear R_ξ gauge	
one–loop	~ 50			
two–loop	1pi	total	1pi	total
Z	616	2348	410	1837
W	792	4084	537	2942

Evaluation of 2–loop self–energies

There exist a number of programs, which calculate the "bubble–diagrams" (analytical), obtained by low energy expansions, but also arbitrary self-energy diagrams (analytical and 1-dimensional integral-representations):

- a class of massive 2–loop–integrals (Broadhurst 90, Fleischer, Kalmykov, Kotikov 99), which depend on one scale only have been implemented in *ONSHELL2* (Fleischer, Tarasov 92, Fleischer, Kalmykov 00),
- in general exact analytic results are not known and one has to resort to series expansions at low or high energies (Broadhurst, Fleischer, Tarasov, '93, Fleischer, Tarasov,'94, Fleischer, Kalmykov, Veretin, 98), which may be combined with methods of conformal mapping and Padé–resummation,
- a combined analytical-numerical program for 2–loop self-energy functions has been developed by the W"urzburg/Leiden–Collaboration (Bauberger, Berends, Böhm, Buza, '95, Bauberger, Böhm, '95),
- for the reduction of integrals to a basis of standard-integrals there exist packages which solve the systems of recurrence-relations (Tarasov '97). Utilizing relations between integrals in different dimensions D the problem of irreducible numerators could be solved (Tarasov '96b)
- For integrals showing up in a large mass expansion the package *TLAMM* (Avdeev et al., '97) is available.
- Various expansions with respect to small parameters may by utilized (Smirnov, '90, '95, '99 ,'01)

Evaluation by expansion

Check of gauge invariance: take R_ξ gauge with independent gauge parameters ξ_W, ξ_Z and ξ_γ $\Rightarrow$ V propagator reads

$$D_{\mu\nu}(p) = \frac{i}{p^2 - m^2}\left(-g_{\mu\nu} + (1-\xi)\frac{p_\mu p_\nu}{p^2 - \xi m^2}\right)$$

Then there are several scales: $m_W,\ \sqrt{\xi_W}m_W,\ m_Z,\ \sqrt{\xi_Z}m_Z,\ m_H$

We perform expansions in 3 steps:

1. Taylor (naive) expansion in $(\xi_V - 1)$: i.e., propagators of the vector bosons and associated Higgs scalar ghosts look like

$$D^V_{\mu\nu}(p) = \frac{i}{p^2 - m_V^2}\left(-g_{\mu\nu} + (1-\xi_V)\frac{p_\mu p_\nu}{p^2 - m_V^2} - (1-\xi_V)^2\frac{m_V^2 p_\mu p_\nu}{(p^2 - m_V^2)^2} + \dots\right)$$

$$\Delta_V(p) = \frac{i}{p^2 - m_V^2}\left(1 - (1-\xi_V)\frac{m_V^2}{p^2 - m_V^2} + (1-\xi_V)^2\left(\frac{m_V^2}{p^2 - m_V^2}\right)^2 + \dots\right)$$

where $V = W, Z$.

2. Expansion in the small parameter

$$\sin^2\Theta_W = 1 - \frac{m_W^2}{m_Z^2} \lesssim 0.24$$

by which $m_W^2 = m_Z^2\,(1 - \sin^2\Theta_W)$; no–Higgs diagrams then are one–scale and can be calculated analytically with the *ONSHELL2* package.

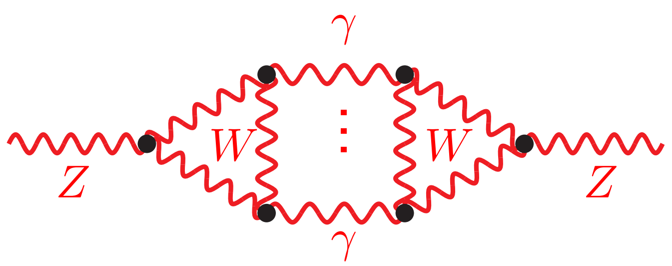
3. Diagrams with Higgs–lines are expanded for large m_H in

$$z \equiv m_Z^2/m_H^2 \lesssim 0.64$$

using the *TLAMM* package

Gauge invariance

As we know, resonant Z and W bosons decay mainly into fermion pairs. Indeed, if we switch off the fermions (as we do here) the gauge bosons are close to stable! For the purely bosonic contributions alone the imaginary part of $\Pi(p^2)$ on the mass-shell is zero at the two-loop level. This is due to the fact that in the bosonic sector we have the physical masses $m_\gamma = 0$, M_Z, M_W and M_H and by inspection of the possible two and three particle intermediate states one observes that all physical thresholds lie above the mass shells of the W and Z bosons, i.e., the self-energies of the massive gauge bosons develop an imaginary part only at $p^2 > M_V^2$ (to two loops in the SM). On kinematical grounds imaginary parts could show up from the Higgs or Faddeev-Popov ghosts, which have square masses $\xi_V M_V^2$, for small values of the gauge parameter. However, as we have verified, the two-loop on-shell self-energies are gauge independent. This implies that ghost contributions have to cancel and hence cannot contribute to the imaginary part. Thus $s_P = M_V^2$ in our case. In higher orders for the Z–propagator one gets an imaginary part as soon as $p^2 > 0$, from diagrams like



.

For the W–propagator an imaginary part is only possible for $p^2 > M_W^2$, because charge conservation requires at least one W in any physical intermediate state.

Drawback of our choice of expansion about $\xi_i = 1$:

analytic structure (ghost thresholds) lost: do not get correct imaginary part from ghost contributions !

Examples:

1.) threshold $p^2 = 4\xi_Z M_Z^2$ of
$Z \to \phi^0\phi^0$ production, ϕ^0 the neutral Higgs ghost
which is below the Z mass-shell $p^2 = M_Z^2$ when $\xi_Z < \frac{1}{4}$

2.) threshold $p^2 = \xi_W M_W^2$ of
$W^\pm \to \phi^\pm\gamma$ production, $\phi^\pm$ the charged Higgs ghosts
which is below the W mass-shell $p^2 = M_W^2$ when $\xi_W < 1$.

Thus for small values of ξ we do not get correct imaginary part diagram by diagram. However, ghost contributions must cancel on-shell. Thus by gauge invariance, which we check we know that we get the correct result. At one–loop one may check this analytically.
(Fleischer, Jegerlehner, ’81)

Cancellation is highly non-trivial: a consequence of the Slavnov-Taylor identities, which tell us how Higgs ghost, Faddeev-Popov ghosts and scalar components contained in the gauge boson fields decouple from physical amplitudes like the physical width.

For gauge parameters $\xi > 1$ the imaginary part of the W and Z self-energies in the bosonic sector up to two-loops is zero, by applying the Cutkowsky rules and inspecting all possible two and three particle intermediate states allowed by the SM Lagrangian. While for $\xi > 1$ the imaginary part is zero for each individual diagram, for small enough values of the gauge parameters a nontrivial cancellation must take place. An independent direct check of this is possible by considering the problem in the limit $\xi \to 0$, for example.

Results

- UV renormalization is exact (analytic);
  UV singularities (poles $1/\varepsilon^2$ and $1/\varepsilon$) are not affected by SSB;
  $\Rightarrow$ check against RG results within unbroken theory
  (Jones, '82, Machacek, Vaughn, '83) ✓
- confirms IR finiteness of on–shell mass for both Z and W
- gauge invariance of position of pole s_P;
  requires taking into account tadpoles ✓
- large m_H expansion breaks down at large m_H
  because of strong coupling problem (gets non-perturbative)
- relation between $\overline{\mathrm{MS}}$ and pole mass exhibits unphysical terms proportional to m_H^4, which violate Veltman's screening theorem: in observables at L–loops:

$$X(m_H) \to O((G_\mu m_H^2)^{L-1}\ \ln(m_H/M_W)^2)\ \text{ as }\ m_H \to \infty$$

  The fake terms drop in $\sin^2\Theta_W = 1 - \frac{M_W^2}{M_Z^2}$ as they should. ✓
- behavior for intermediate Higgs masses: looks O.K. down to about 130 GeV (see plots)

Form of results

$$\frac{M_V^2}{m_V^2} = 1 + \left(\frac{e^2}{16\pi^2 \sin^2\theta_W}\right) X_V^{(1)} + \left(\frac{e^2}{16\pi^2 \sin^2\theta_W}\right)^2 X_V^{(2)} ,$$

$$X_V^{(2)} = \frac{m_H^4}{m_V^4} \sum_{k=0}^{5} \sin^{2k}\theta_W \, A_k^V .$$

$$A_i^V = \sum_{j=0}^{5} A_{i,j}^V \left(\frac{m_V^2}{m_H^2}\right)^j$$

All parameters in $\overline{MS}$ scheme.

Six coefficients calculated analytically. Expansion in powers and log's (i.e., is an asymptotic expansion not a naive Taylor expansion). Expansion coefficients $A_{i,j}$ given by a small set of transcendental constants like:

$$\begin{aligned}
S_0 &= \frac{\pi}{\sqrt{3}} \sim 1.813799365..., \\
S_1 &= \frac{\pi}{\sqrt{3}} \ln 3 \sim 1.992662272..., \\
S_2 &= \frac{4}{9} \frac{\mathrm{Cl}_2\left(\frac{\pi}{3}\right)}{\sqrt{3}} \sim 0.26043413763216 2..., \\
S_3 &= \pi \mathrm{Cl}_2\left(\frac{\pi}{3}\right) \sim 3.188533097...
\end{aligned}$$

$\overline{\mathrm{MS}}$ mass in terms of on–shell mass

Inverse of "master formula":

express all $\overline{\mathrm{MS}}$ parameters in terms of on-shell ones:

$$\begin{aligned} m_V^2 &= M_V^2 - \hat{\Pi}_V^{(1)} - \left\{ \Pi_V^{(2)} + \Pi_V^{(1)} \Pi_V^{(1)\prime} \right\}_{MS} \\ &\quad - \sum_j (\Delta m_j^2)^{(1)} \frac{\partial}{\partial m_j^2} \hat{\Pi}_V^{(1)} - (\Delta e)^{(1)} \frac{\partial}{\partial e} \hat{\Pi}_V^{(1)} \Big|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}} , \end{aligned}$$

sum runs over all species of particles $j = Z,\ W,\ H$

$$(\Delta m_j^2)^{(1)} = -\mathrm{Re}\hat{\Pi}_j^{(1)} \Big|_{m_j^2 = M_j^2,\, e = e_{\mathrm{OS}}} \equiv -M_V^2 \frac{e_{\mathrm{OS}}^2}{16\pi^2 \sin^2\theta_W} X_V^{(1)} \Big|_{m_j^2 = M_j^2}$$

stands for the self-energy of the jth particle at $p^2 = m_j^2$ in the $\overline{\mathrm{MS}}$ scheme and parameters replaced by the on-shell ones. Note that in the above relation we had to perform a change from the $\overline{\mathrm{MS}}$ to the on-shell scheme also for the electric charge

$$e(\mu^2) = e_{\mathrm{OS}} \left[1 + \frac{e_{\mathrm{OS}}^2}{16\pi^2} \left(\frac{7}{2} \ln\left(\frac{M_W^2}{\mu^2} \right) - \frac{1}{3} \right) \right]$$

with $e_{\mathrm{OS}}^2/4\pi = \alpha \sim 1/137$. Accordingly, since $\hat{\Pi}^{(1)}$ depends on e by an overall factor e^2 only,

$$(\Delta e)^{(1)} \frac{\partial}{\partial e} \hat{\Pi}_V^{(1)} = \frac{e^2}{16\pi^2} \left[7 \ln\left(\frac{M_W^2}{\mu^2} \right) - \frac{2}{3} \right] \hat{\Pi}_V^{(1)} \,.$$

On-shell scheme mass counter-terms

Identifying $m_V^2 = m_{V,0}^2 = M_V^2 + \delta M_V^2$ in inverse MF $\Rightarrow$ on-shell gauge-boson mass counter-terms δM_V^2 :

$$\begin{aligned}
\delta M_V^2 &= -\mathrm{Re}\Bigg[\check{\Pi}_{V,0}^{(1)} + \check{\Pi}_{V,0}^{(2)} + \check{\Pi}_{V,0}^{(1)}\check{\Pi}_{V,0}^{(1)\,\prime} \\
&\quad + \sum_j (\delta M_j^2)^{(1)} \frac{\partial}{\partial m_{j,0}^2}\check{\Pi}_{V,0}^{(1)} + (\delta e)^{(1)}\frac{\partial}{\partial e_0}\check{\Pi}_{V,0}^{(1)}\Bigg]\Bigg|_{\substack{m_{j,0}^2 = M_j^2 \\ e_0 = e_{\mathrm{OS}}}} \\
&= \left(Z_{\overline{\mathrm{MS}}}\cdot Z_{\mathrm{OS}} - 1\right) M_V^2
\end{aligned}$$

in terms of the original bare on-shell amplitudes

$$\check{\Pi}_{V,0}^{(i)} = \Pi_{V,0}^{(i)}(p^2, m_{V,0}^2, \cdots)\Big|_{p^2=M_V^2, m_{j,0}^2=M_j^2, e_0=e_{\mathrm{OS}}}$$

and the bare on-shell counter-terms δM_j^2 and δe.

The second equality gives δM_V^2 in terms of the singular factor $Z_{\overline{\mathrm{MS}}} = m_{V,0}^2/m_V^2(\mu)$ and the finite factor $Z_{\mathrm{OS}} = m_V^2/M_V^2$. These will be needed in two-loop calculations of observables in the on-shell scheme.

Explicit expressions in (Jegerlehner, Kalmykov, Veretin, '01)

⑥ Outlook

Precise test of the limitations of the SM require a LC like the ECFA/DESY TESLA (TEV Superconducting Linear Accelerator) machine

Such accelerators can be built now !

- **Physics:**
  - Higgs properties, Higgs couplings
  - SUSY spectrum, SUSY Higgs, scenarios
  - exploring the extra dimensions
  - many other possibilities
- **Theory:**

Extraordinary challenge: must learn to handle precise and efficient computations of 10000's of diagrams.

- computer algebra; automatization of calculations
- numerical methods; multi precision calculations
- computing resources; massive parallel computers

ECFA/DESY LC working group: "The Loop Verein"

similar American initiative:

"N.N." (LoopFest, May 9 -10, BNL)

Aim: developing new tools and techniques for calculating precise rates for Standard Model and supersymmetric processes that match the expected experimental precision.

- **Motivation**

  field which got a lot of momentum from precision physics at LEP/SLC

$$\Downarrow$$

we must keep momentum and improve calculations to meet TESLA

requirements of HIGHER ENERGY **and** HIGHER LUMINOSITY

Coordinate activities, identify problems and their solution.

Time scale: a long term project.

(http://www-zeuthen.desy.de/theory/TLV/TheLoopVerein.html).