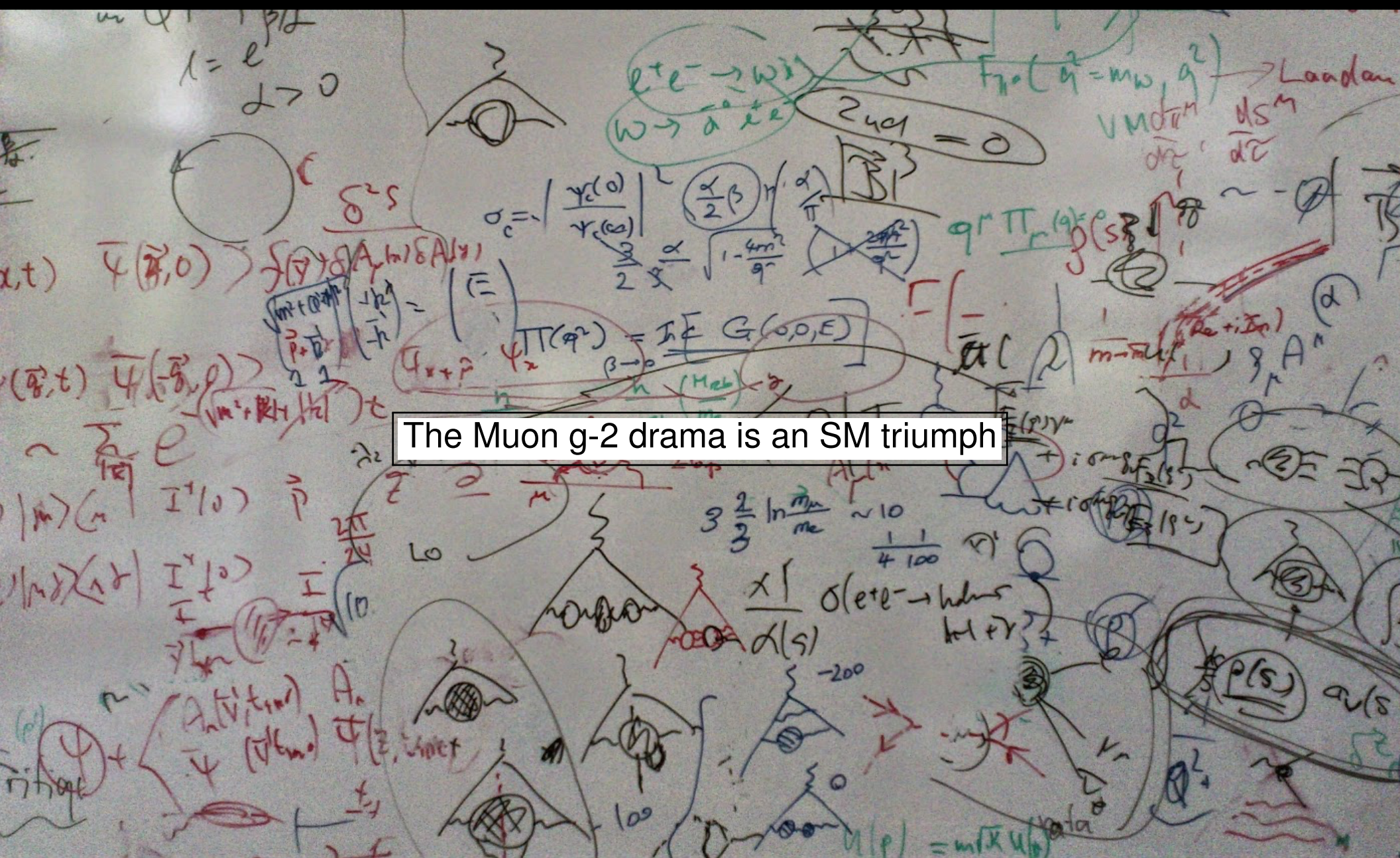




The Muon g-2 drama is an SM triumph

THE CLOSER YOU LOOK THE MORE THERE IS TO SEE — THE SM TO THE DEEPEST

Lepton $g - 2$ Moments: what they tell us

Fred Jegerlehner, DESY Zeuthen/Humboldt University Berlin

Matter To The Deepest “Conference 2025” ,
Katowice/Poland, September 15 - 19

Abstract:

The magnetic moments of leptons (g_ℓ -factors) and the associated anomalies of the muon and electron play a decisive role in high-precision verification of the Standard Model. These quantities are not only among the most precisely measured in particle physics, but can also be predicted with high accuracy. At the same time, they offer promising insights into physics beyond the Standard Model. The key relationship for testing possible new heavy states is

$$\frac{\delta a_\ell}{a_\ell} \propto \frac{m_\ell^2}{M^2} \quad (M \gg m_\ell),$$

where m_ℓ is the mass of the lepton and M is the mass scale of new physics. This equation shows that the fractional change in a_ℓ is proportional to the square of the lepton mass over the square of the new physics scale, making the muon about $(m_\mu/m_e)^2 \sim 4 \times 10^4$ times more sensitive to Beyond Standard Model (BSM) physics than the electron. For the muon, theory and experiment now agree to 9 decimal places, so that $\delta a_\mu \sim \frac{\alpha}{\pi} \frac{m_\mu^2}{M_{\text{NP}}^2}$ can effectively probe intermediate mass scales beyond M_t up to about 1 TeV (essentially ruled out directly by LHC).

Outline of Talk:

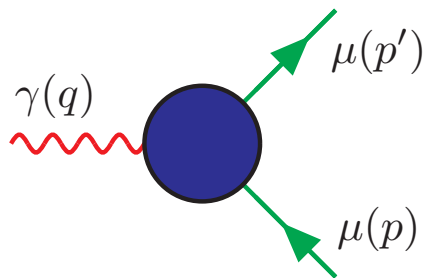
- ❖ Introduction
- ❖ Theory Status
- ❖ Hadronic Stuff: the Limitation to Theory
- ❖ Hadronic Vacuum Polarization (HVP) – Data & Status
- ❖ Ab initio Method: HVP via Lattice QCD
- ❖ What is the Problem?
- ❖ Muon $g-2$ Status and the SM
- ❖ Electron $g-2$ Status and its Future
- ❖ Conclusion

Introduction

Particle with spin $\vec{s} \Rightarrow$ magnetic moment $\vec{\mu}$ (internal current circulating)

$$\vec{\mu} = g_\mu \frac{e\hbar}{2m_\mu c} \vec{s} ; \quad g_\mu = 2 (1 + a_\mu)$$

Dirac: $g_\mu = 2$, $a_\mu = \frac{\alpha}{2\pi} + \dots$ muon anomaly



Electromagnetic Lepton Vertex

$$= (-ie) \bar{u}(p') \left[\gamma^\mu F_1(q^2) + i \frac{\sigma^{\mu\nu} q_\nu}{2m_\mu} F_2(q^2) \right] u(p)$$

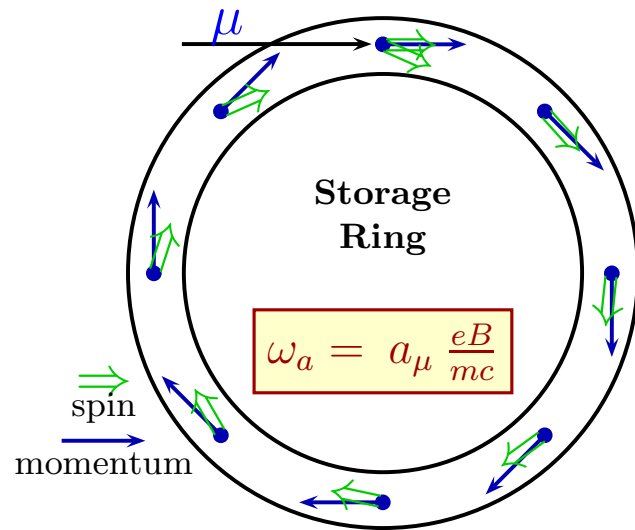
$$F_1(0) = 1 ; \quad F_2(0) = a_\mu$$

the simplest object
you can think of
in the static limit

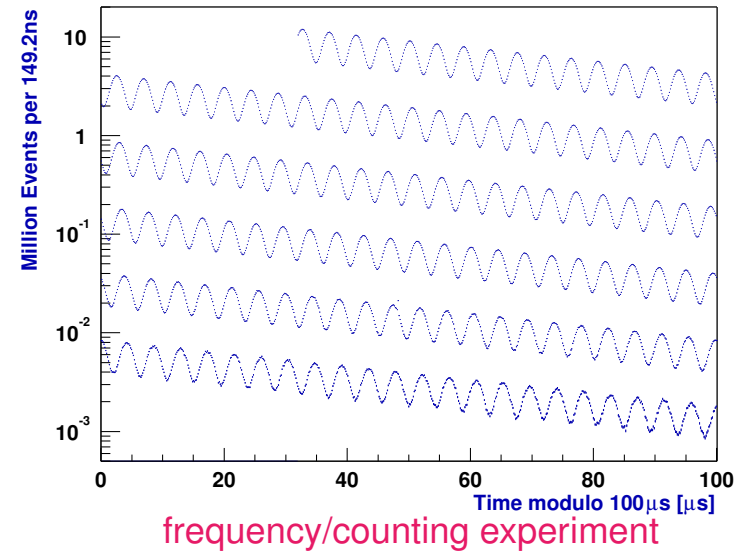
a_μ responsible for the Larmor (spin) precession $\Rightarrow$ need polarized muons orbiting
Shoot protons on target producing pions which decay by P violating weak process

$$\pi^+ \rightarrow \mu^+ \nu_\mu ; \quad \mu^+ \rightarrow e^+ \nu_e \bar{\nu}_\mu$$

Larmor precession $\vec{\omega}$ of beam of spin particles in a homogeneous magnetic field $\vec{B}$



actual precession $\times 2 \sim 12'/\text{circle}$



Magic Energy: $\vec{\omega}$ is directly proportional to $\vec{B}$ at magic energy $\sim 3.1 \text{ GeV}$

$$\vec{\omega}_a = \frac{e}{m} \left[a_\mu \vec{B} - \left(a_\mu - \frac{1}{\gamma^2 - 1} \right) \vec{\beta} \times \vec{E} \right]_{\text{at "magic } \gamma"}^{E \sim 3.1 \text{ GeV}} \simeq \frac{e}{m} [a_\mu \vec{B}]$$

CERN, BNL and FNAL g-2 experiments

Stern, Gerlach 22: $g_e = 2$; Kusch, Foley 48: $g_e = 2 (1.00119 \pm 0.00005)$

Crucial: 3.1 GeV muons life-time in lab frame $\gamma\tau_\mu$ 29 times longer!

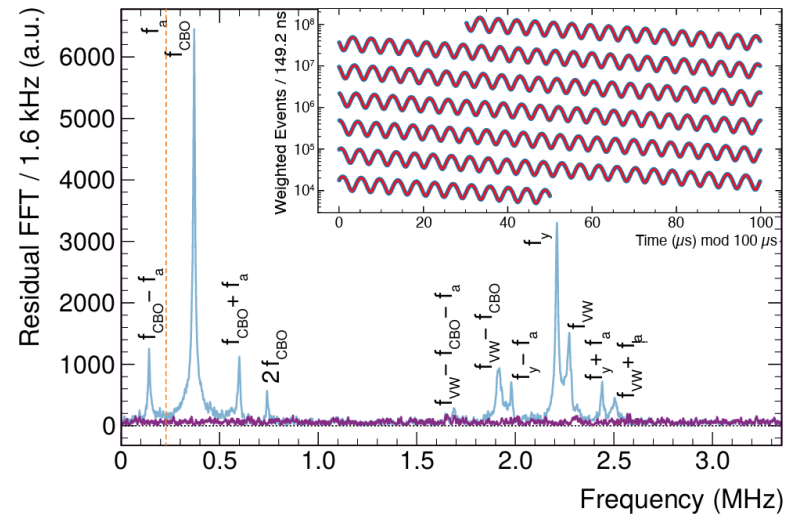
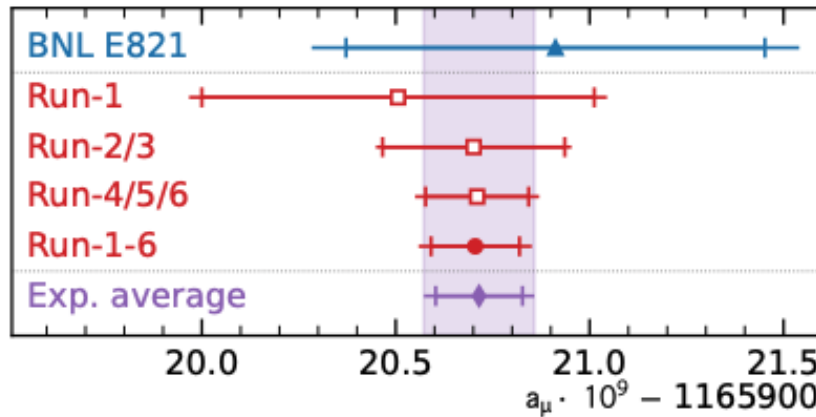
$$a_\mu^{\text{exp}} = (11\,659\,207.15 \pm 0.80 \pm 0.95)[1.24] \times 10^{-10} \text{ FNAL/BNL}$$

Precision: (124 ppb) today vs. (540 ppb) at BNL

The exciting new Fermilab Muon $g - 2$ measurement applying sophisticated improved technology brilliantly confirmed the 2004 final BNL result and triggers a new serious attack on the SM.

$$a_\mu[\text{BNL}] = 11659209.1(5.4)(3.3)[6.3] \Rightarrow a_\mu[\text{FNAL}] = 11659207.05(1.14)(0.95)[1.48]$$

in units 10^{-10} ; FNAL error down by factor 4! now: comparable syst and stat errors



Left: BNL vs. FNAL21/23/25. Right: $N(t) = N_0 e^{-t/\gamma\tau_\mu} \times \{1 + A \cos[\omega_a t - \varphi_0]\}$ modulo well understood corrections N_0 normalization, A P-violation asymmetry

To come – novel complementary experiment at J-PARC: $\vec{E} = 0$ cavity in place of magic γ determined by $\left(a_\mu - \frac{1}{\gamma^2 - 1}\right) = 0$ at CERN, BNL and Fermilab

□ ultra relativistic (CERN, BNL, Fermilab) vs. ultra cold (J-PARC) muons

⇒ very different systematics

a_μ measured via a ratio of frequencies (measurement of a_μ and B)

$$B = \frac{\hbar\omega_p}{2\mu_p}, \quad \omega_a = \frac{ea_\mu}{m_\mu c} B, \quad \mu_\mu = \left(1 + a_\mu\right) \frac{e\hbar}{2m_\mu c} \Leftrightarrow \mu_\mu = \left(1 + a_\mu\right) \frac{\hbar}{2} \frac{\omega_a}{a_\mu B} = \left(\frac{1}{a_\mu} + 1\right) \frac{\omega_a}{\omega_p} \mu_p \Leftrightarrow$$

$$a_\mu^{\text{exp}} = (g_\mu/2 - 1) = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p - \omega_a/\tilde{\omega}_p} = \frac{\mathcal{R}}{\lambda - \mathcal{R}}$$

in terms of 3 frequency measurables

- $\tilde{\omega}_p = (e/m_\mu)\langle B \rangle$ free proton NMR frequency
- $\mathcal{R} = \omega_a/\tilde{\omega}_p$ Larmor precession from BNL/FNAL
- $\lambda = \omega_L/\tilde{\omega}_p = \mu_\mu/\mu_p$ from hyperfine splitting of muonium

$$\mu'_p/\mu_B = 1.5209931551(62) \times 10^{-3}; m_\mu/m_e = 206.7682827(46) \text{ (Codata 2024)}$$

$$\lambda = 3.183345137(85) \text{ (Codata 2008); } \mathcal{R}' = 0.00370730088(36)(29) \text{ (FNAL)}$$

Experiment essentially limited at 120 ppb by μ_μ/μ_p from CODATA

□ Theory Status

all of SM counts

$$a_{\mu}^{\text{SM}} = F_2(0) = a_{\mu}^{(\text{QED} + \text{EW} + \text{HVP}_{\text{LO}} + \text{HVP}_{\text{NLO}} + \text{HVP}_{\text{NNLO}} + \text{HLbL}_{\text{LO}} + \text{HLbL}_{\text{NLO}})}$$

$$= 0.00116591810 \pm 0.00000000043 \quad \text{White Paper 2020 (WP20)}$$

based on $\text{HVP}_{\text{LO}} = 693.1(4) \times 10^{-10}$ and $\text{HLbL}_{\text{LO}} = 90(17) \times 10^{-10}$

My 2007 value: $a_{\mu}^{\text{SM}} = 0.00116591793(68)$ in arXiv:hep-ph/0703125. Value up $81.0-79.3=1.7$ in 10^{-10} , error down 30%

SM theory incl. τ -decay spectra:

Davier et al., arXiv:0906.5443 incl. τ -decay spectra got $a_{\mu}^{\text{HVP-LO}}[ee + \tau] = 705.3 \pm 4.5 \times 10^{-10}$

Masjuan, Miranda & Roig, arXiv:2310.14102: $a_{\mu}^{\text{HVP-LO}}[\tau] = 704.1^{+4.1}_{-4.0} \times 10^{-10}$

vs $\Leftrightarrow a_{\mu}^{\text{HVP-LO}}[\text{LQCD : BMW, Mainz}] = 707.5 \pm 5.5 \times 10^{-10}$

If “BSM = missing HVP” need $a_{\mu}^{\text{HVP-LO}} = 718.2 \times 10^{-10}$

Also remember:

Belle 2008: τ data-driven result

$$a_{\mu}^{\pi\pi}[2m_{\pi}, 1.8 \text{ GeV}] = (523.5 \pm 3.9) \times 10^{-10} (\tau : \text{Belle})$$

compared to e^+e^- data-driven result

$$a_{\mu}^{\pi\pi}[2m_{\pi}, 1.8 \text{ GeV}] = (504.6 \pm 10.1.1) \times 10^{-10} (e^+e^- : \text{CMD2, SND})$$

difference 18.9×10^{-10} , if added to alphaQED result

$$a_{\mu}^{\text{HVP-LO}}[e^+e^-] = (694.79 \pm 4.18) \times 10^{-10} \text{ we get } 713.7 \times 10^{-10}$$

CMD-3 2023: e^+e^- for the range 0.327 to 1.2 GeV, relative to White paper

$$a_{\mu}^{\text{had;LO}}(2\pi, \text{CMD3}) = (526.0 \pm 4.2) \times 10^{-10}, \text{ compared to } (506.0 \pm 3.4) \times 10^{-10}|_{\text{WP}}$$

difference 20.0×10^{-10} , if added to

$$a_{\mu}^{\text{HVP-LO}}[e^+e^-] = (693.1 \pm 4.0) \times 10^{-10}|_{\text{WP}} \text{ we get } (713.1 \pm 4.7) \times 10^{-10}$$

τ data driven results seem to support recent LQCD results!

High CMD-3 value in comparison to SND20 result (both e^+e^- -scan at Novosibirsk facility) is another puzzle!

News in Theory:

HVP_{LO} and HLbL_{LO} calculations in **lattice QCD**, impact by BMW 2019 and later validations

Major shifts due to lattice QCD results:

$$\text{HVP}_{\text{LO}} = 693.1(4) \Rightarrow 713.2(6.1) \text{ and } \text{HLbL}_{\text{LO}} = 90(17) \Rightarrow 112.6(9.6)$$

New puzzle: data driven dispersion relation approach (DRA) versus ab initio lattice QCD approach (BMW,..., LQCD)

5 Numbers to establish the "g-2 Test" as $\delta a_\mu(\text{NewPhysics}) \equiv a_\mu^{\text{exp}} - a_\mu^{\text{SM}}$

$\omega_a, \omega_p, \lambda$ from experiment and $a_\mu^{\text{HVP}_{\text{LO}}}, a_\mu^{\text{HLbL}_{\text{LO}}}$ from theory, other SM contributions well under control!

Theory as now in units $\times 10^{-11}$:

	$a_\mu[\text{SM}]$	precision	$\delta a_\mu^{\text{SM-exp}}$	S.D.	comment
MB17	116591783[35]	(300 ppb)	-306 ± 72	4.3σ	DRA vs BNL
WP20	116591810[43]	(369 ppb)	-279 ± 76	3.7σ	DRA vs BNL
GM25	dito	dito	-262 ± 45	5.9σ	DRA vs FNAL 25
GM21	116592019[38]	(325 ppb)	-40 ± 44	0.9σ	BMW vs FNAL 21
WP25	116592033[62]	(530 ppb)	-38 ± 63	0.6σ	LQCD&DRA vs WA

$$\begin{aligned}
 a_\mu^{\text{the}} &= 0.00 \text{ QED } \mathbf{1165} \text{ +LOHVP } \mathbf{92} \text{ +EW+HLbL+HOHVP } \mathbf{033} & (62) \\
 a_\mu^{\text{exp}} &= 0.00 \text{ 1165 } \text{ 92 } \text{ 072 } & (12)
 \end{aligned}$$

agrees to 9 decimal places! Does SM prediction match 10 digits precision?

$$\delta a_\mu = a_\mu^{\text{exp}} - a_\mu^{\text{SM}} = 38 \pm 63 \times 10^{-11} = \Delta a_\mu^{\text{BSM}}???$$

Note: EW contributes 12.9σ ; HLbL 9.6σ and HOHVP -8.2σ

For details I refer to [my book 2017](#) and references therein. The QED prediction of a_μ is given by (see [Aoyama et al. 12](#), [Laporta 17](#), [Steinhauser et al. 17](#), [Volkov 19/24](#))

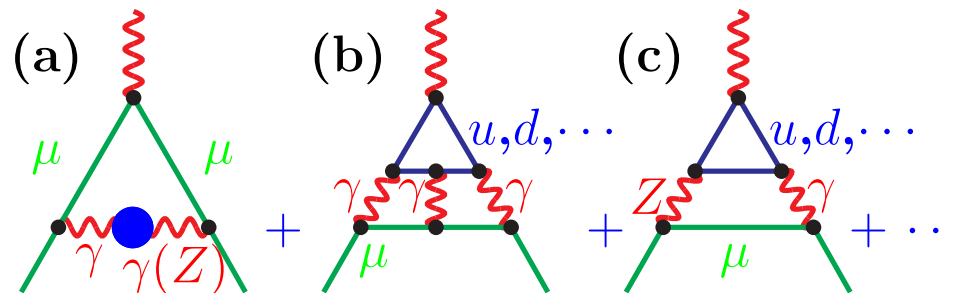
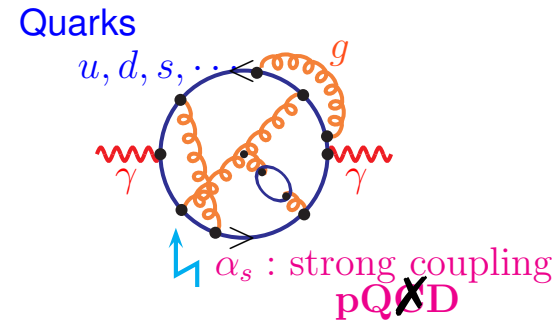
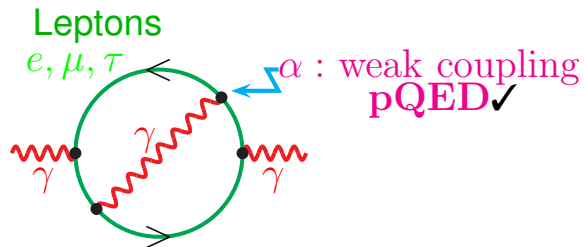
$$a_\mu^{\text{QED}} = \frac{\alpha}{2\pi} + 0.765\,857\,423(16) \left(\frac{\alpha}{\pi}\right)^2 + 24.050\,509\,82(28) \left(\frac{\alpha}{\pi}\right)^3 + 130.8734(60) \left(\frac{\alpha}{\pi}\right)^4 \\ + 750.010(872) \left(\frac{\alpha}{\pi}\right)^5 = 116584718.8(2) \times 10^{-11}$$

For the weak contributions we have (see [Czarnecki, Marciano, Stöckinger et al. ...](#))

$$a_\mu^{\text{EW}} = \underset{\text{LO}}{194.79(1) \times 10^{-11}} - \underset{\text{NLO}}{40.38(36) \times 10^{-11}} = 154.4(4) \times 10^{-11}$$

□ Hadronic Stuff: the Limitation to Theory

General problem in electroweak precision physics:
 contributions from hadrons (quark loops) at low energy scales



- (a) Hadronic vacuum polarization $O(\alpha^2), O(\alpha^3)$
 - (b) Hadronic light-by-light scattering $O(\alpha^3)$
 - (c) Hadronic effects in 2-loop EWRC $O(\alpha G_F m_\mu^2)$
- Light quark loops
 ↓
 Hadronic “blobs”

Evaluation of non-perturbative effects:

- data in conjunction with Dispersion Relations (DR),
- low energy effective modeling, RLA, HLS, ENJL
- lattice QCD

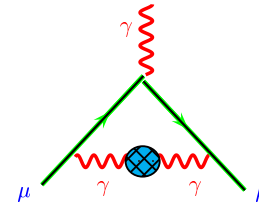
- (a) HVP via dispersion integral over $e^+e^- \rightarrow \text{hadrons}$ -data
(1 independent amplitude to be determined by one specific data set),
HLS global fit, lattice QCD
- (b) HLbL via Resonance Lagrangian Approach (RLA) (CHPT extended by VDM
in accord with chiral structure of QCD) [Knecht, Nyffeler], $\gamma\gamma \rightarrow \text{hadrons}$ -data driven dispersive
approach [Colangelo et al.] (28/19 independent amplitudes to be determined by as many
independent data sets), lattice QCD [Blum et al., Wittig, Meyer et al., ...]
- (c) quark and lepton triangle diagrams: $VVV = 0$ by Furry $\Rightarrow$ only VVA
(of $f\bar{f}Z$ -vertex) contributes $\Rightarrow$ ABJ anomaly is perturbative
and non-perturbative simultaneously i.e. leading effects calculable
(anomaly cancellation) [de Rafael, Knecht, Perrottet, Melnikov, Vainshtein]

Hadronic Vacuum Polarization (HVP) – Data & Status

Leading **non-perturbative** hadronic contributions a_μ^{had} can be obtained in terms of

$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \rightarrow \gamma^* \rightarrow \text{hadrons}) / \frac{4\pi\alpha^2}{3s}$ data via **Dispersion Relation (DR)**:

$$a_\mu^{\text{had}} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int_{4m_\pi^2}^{E_{\text{cut}}^2} ds \frac{R_\gamma^{\text{data}}(s) \hat{K}(s)}{s^2} + \int_{E_{\text{cut}}^2}^{\infty} ds \frac{R_\gamma^{\text{pQCD}}(s) \hat{K}(s)}{s^2} \right)$$

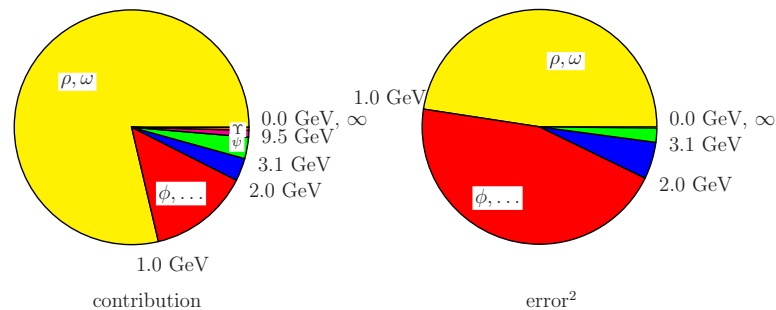
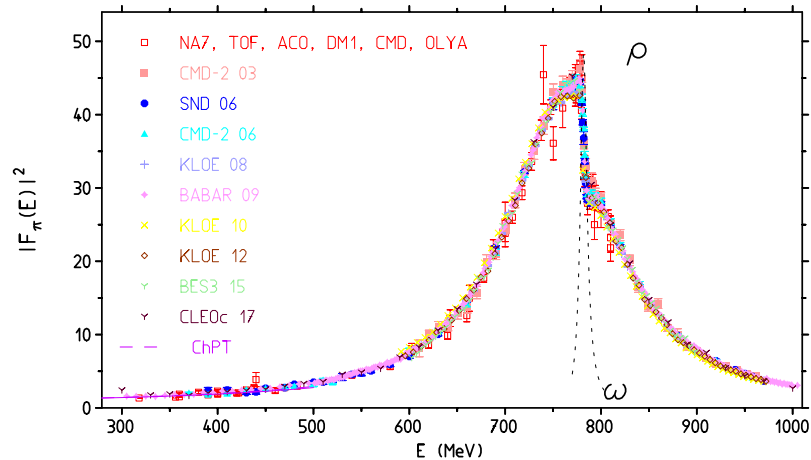


- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\phi^2$

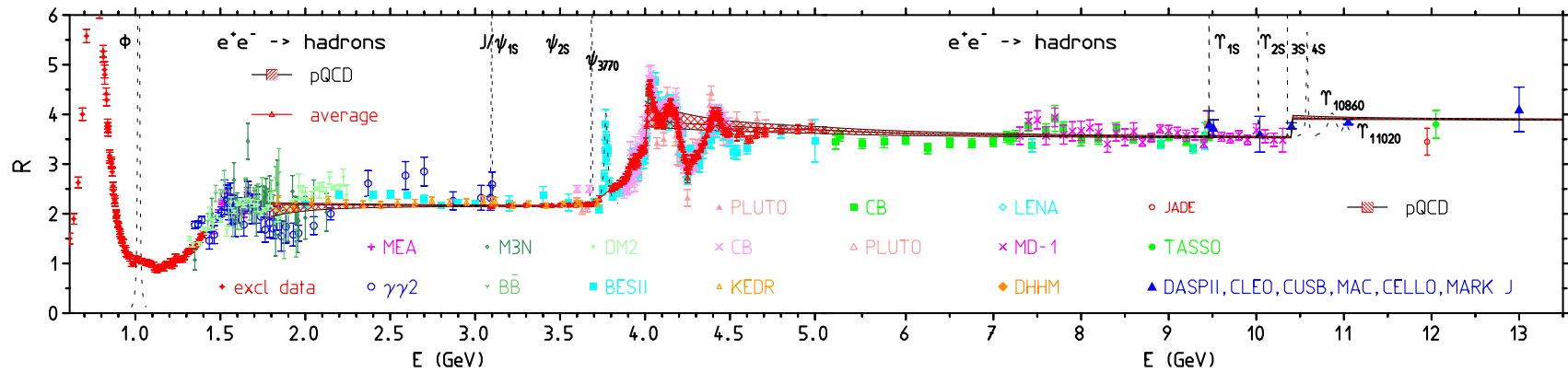
Data: **NSK, KLOE, BaBar, BES3, CLEOc, [CMD-3?]**

$$a_\mu^{\text{had}(1)} = (697.17 \pm 4.18) 10^{-10}$$

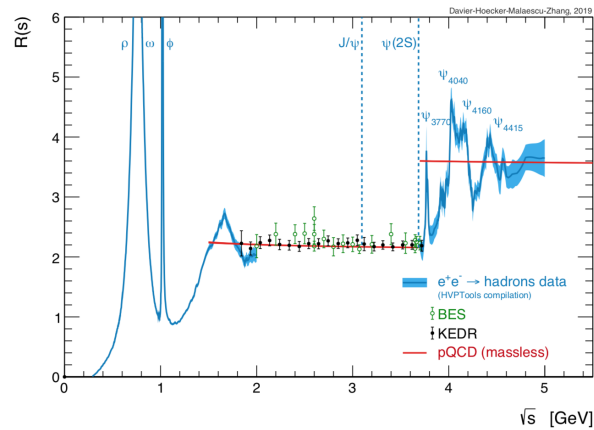
e^+e^- –data based



Higher energies:

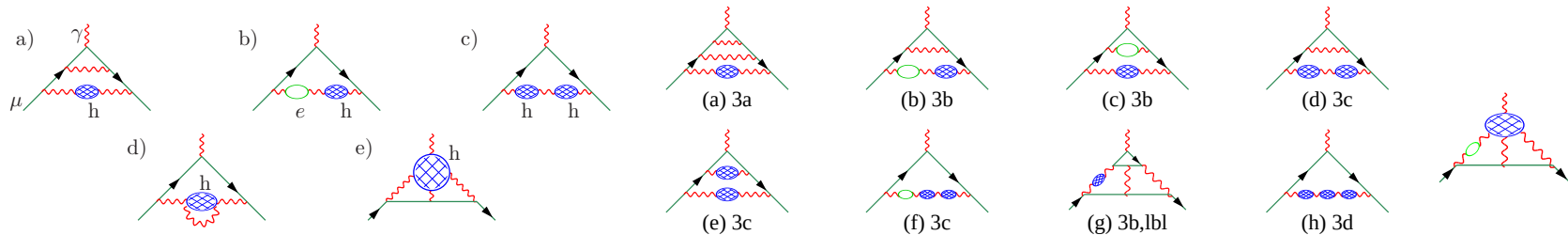


The compilation of $R(s)$ -data utilized.



$R(s)$ compilation by Davier et al.

NLO and NNLO HVP Effects:



Hadronic higher order contributions: involving LO, NLO and NNLO vacuum polarization, and light-by-light scattering insertions.

Barbieri, Remiddi 74, Krause 97, Kurz et al 14, my 2023 update below

Summary of the hadronic and weak contributions (based on dispersive approach)

$a_{\mu}^{\text{had}(1)}$	=	$(697.17 \pm 4.18) 10^{-10}$	(LO)
$a_{\mu}^{\text{had}(2)}$	=	$(-10.89 \pm 0.067) 10^{-10}$	(NLO)
$a_{\mu}^{\text{had}(3)}$	=	$(1.242 \pm 0.010) 10^{-10}$	(NNLO)
$a_{\mu}^{\text{had,LbL}}$	=	$(10.34 \pm 2.88) \times 10^{-10}$	(HLbL)
a_{μ}^{weak}	=	$(15.4 \pm 0.4) 10^{-10}$	(LO+NLO) .

LO HVP and LO HLbL to be reset by LQCD results!

● uncertainties of the subleading NLO and NNLO and EW below 1σ (DRA save)

□ Ab initio Method: HVP via Lattice QCD

Primary object for HVP in LQCD: e.m. quark current correlator in Euclidean configuration space

$$\langle J_\mu(\vec{x}, t) J_\nu(\vec{0}, 0) \rangle, \quad J_\mu = \frac{2}{3} \bar{u} \gamma_\mu u - \frac{1}{3} \bar{d} \gamma_\mu d - \frac{1}{3} \bar{s} \gamma_\mu s + \dots$$

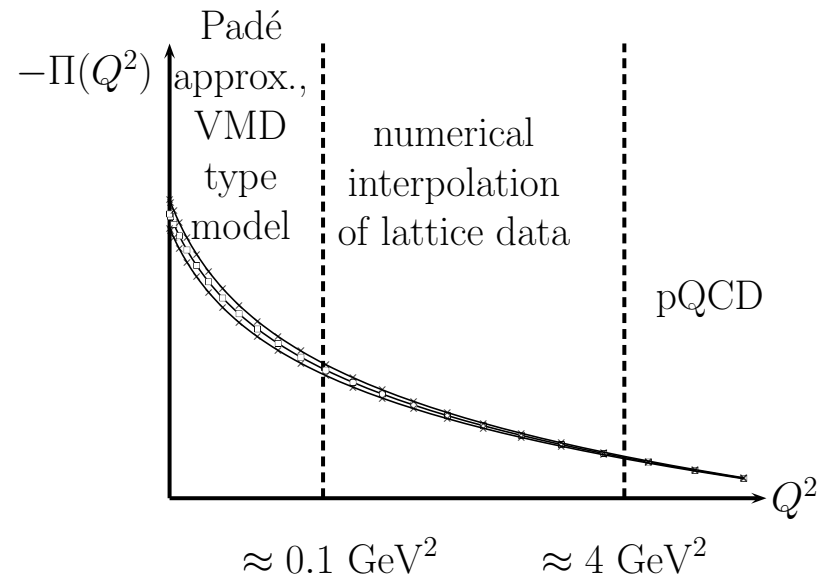
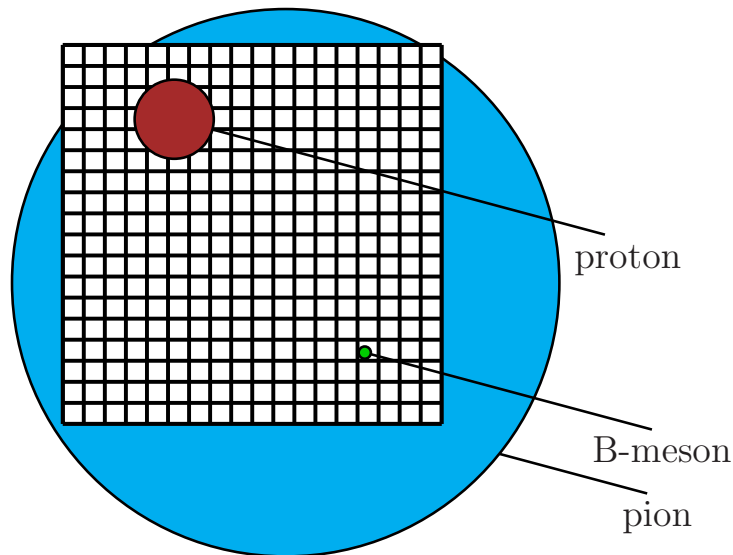
A Fourier transform yields the bare vacuum polarization function $\Pi(Q^2)$

$$\Pi_{\mu\nu}(Q) = \int d^4x e^{i Q x} \langle J_\mu(x) J_\nu(0) \rangle = (Q_\mu Q_\nu - \delta_{\mu\nu} Q^2) \Pi(Q^2)$$

needed to calculate

$$a_\mu^{\text{HVP}} = 4\alpha^2 \int_0^\infty dQ^2 f(Q^2) \{ \Pi(Q^2) - \Pi(0) \} \quad \text{with } f(Q^2) \text{ a known kernel function}$$

Limitations here: finite box $V = L^4$, lattice spacing a ; extrapolations needed



Lattice QCD (LQCD) now a competitive method! From first principles straight forward approach

- LQCD lattice in finite box: momenta are quantized $Q_{\min} = 2\pi/L$
where L is the lattice box length. $Q_{\min} \rightarrow 0 \Leftrightarrow L \rightarrow \infty$ infinite volume limit
- $Q_{\min} = 2\pi/L$ with $m_{\pi}aL \gtrsim 4$ for $m_{\pi} \sim 200$ MeV, such that $Q_{\min} \sim 314$ MeV

- ❖ lattice data: $Q^2 > (2\pi/L)^2$
- ❖ extrapolate to $Q^2 = 0$ via Padé's or complete via HVP data
- ❖ required accuracy: need LQCD data down to $Q_{\min}^2 \approx 0.1 \text{ GeV}^2$

- about 44% of the low x contribution to a_{μ}^{had} is not covered by data yet

Data from BMW, Mainz/CLS-24, RBC/UKQCD combined and averaged by WP25 group yield:

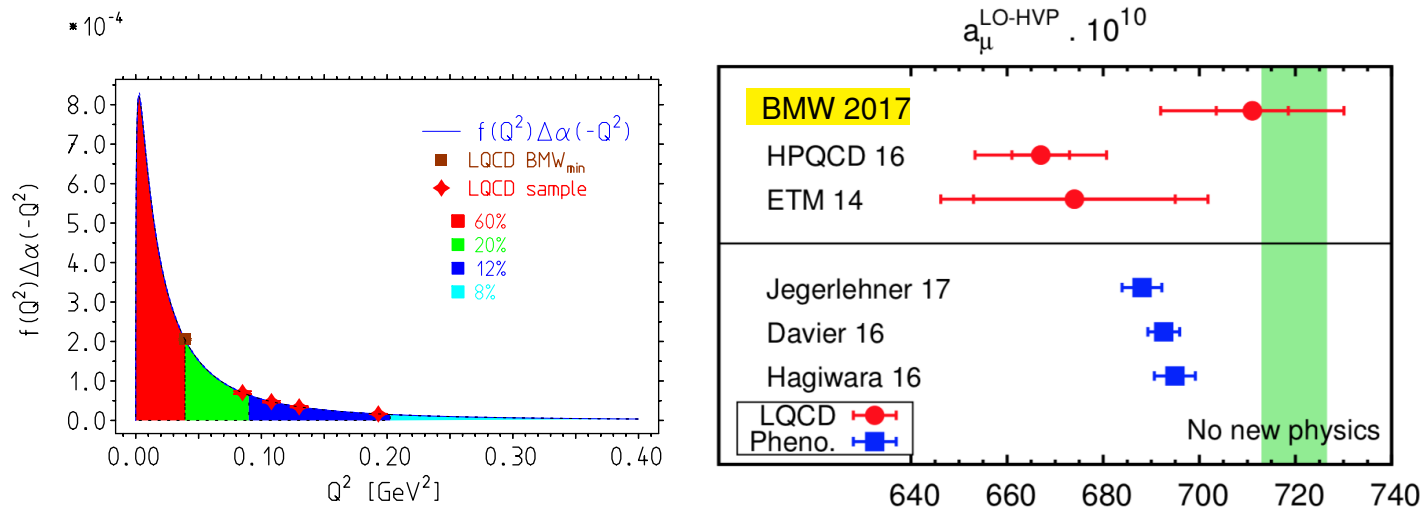
$$a_{\mu}^{\text{SD}}(\text{iso}) = 69.06(22) \times 10^{-10}, \quad a_{\mu}^{\text{W}}(\text{iso}) = 236.16(42) \times 10^{-10}, \quad a_{\mu}^{\text{LD}}(\text{iso}) = 407.9(5.0) \times 10^{-10}.$$

9.6% 33.1% 57.2%

$$a_{\mu}^{\text{HVP, LO}}(\text{iso}) \Big|_{\text{Avg.1}}^{\text{lat}} = 713.1(5.0) \times 10^{-10} \text{ including IB corrections } a_{\mu}^{\text{HVP, LO}}(\text{cor}) \Big|_{\text{Avg.1}}^{\text{lat}} = 713.2(6.1) \times 10^{-10}$$

Individual flavor contributions from light (u, d) amount to about 90%, strange about 8% and charm about 2%.

As off 2017: LQCD results for the leading order a_μ^{HVP} , in units 10^{-10} [Brookhaven, Zeuthen, Mainz, Edinburgh, ... groups]: a first hint for a larger HVP value



Left: Percent contributions from ranges. Right: Plot from Budapest-Marseille-Wuppertal 17

Most recent WP25: a large HVP result now established

Collaboration	$a_\mu^{\text{HVP, LO}}$	$a_\mu^{\text{HVP, LO}} _{\text{corr}}$
BMW-20	707.5(5.5)	707.2(5.5)
Mainz/CLS-24	724.5(7.1)	724.5(7.1)
RBC/UKQCD-24	734.5(13.0)	729.4(13.0)

$$a_\mu^{\text{HVP, LO}}|_{\text{LQCD WP25}} = 713.2(6.1) \times 10^{-10}$$

□ What is the Problem?

Note: in spite of the fact of a heroic effort in calculating and modeling QED correction to hadronic processes (see Jadach, Ward, Placzek, Was, Czyz, German et al., Piccinini et al.,...) some puzzles remain:

● $e^+e^- \rightarrow \text{hadrons}$ experiments measure the photon propagator, i.e., a mix of QCD and QED which is difficult to disentangle (requires removing external QED corrections, radiation from hadrons not well understood [sQED]). Lattice QCD “measures” the hadronic blob directly (external QED corrections switched off)

$$\begin{array}{ccc}
 \text{O} \text{---} \text{wavy line} \text{---} \text{blob} \text{---} \text{wavy line} \text{---} \text{O} & \leftrightarrow & \text{blob with external lines} \\
 \langle A(x) A(0) \rangle & & \langle j(x) j(0) \rangle \\
 \text{photon propagator} & & \text{current correlator}
 \end{array}$$

Also:

● τ data in many respects are much simpler than e^+e^- ones, because the charged $\rho^\pm$ does not mix with other states in the dominant low energy range below about 1.05 GeV (up to above the ϕ , below ρ')!

In contrast understanding NC data requires modeling: CHPT + spin 1 resonances (VMD) $\Rightarrow$ Resonance Lagrangian Approach e.g. HLS (massive Yang-Mills) $\Rightarrow$ dynamical widths, dynamical mixing of $\gamma, \rho^0, \omega, \phi$

□ Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0 - \gamma$ mixing], i.e., effect concerning relation photon propagator and one-particle irreducible blob, and is included in NC $e^+e^- \rightarrow \pi^+\pi^-$ data, absent in CC $\tau \rightarrow \nu_\tau \pi\pi$ data!

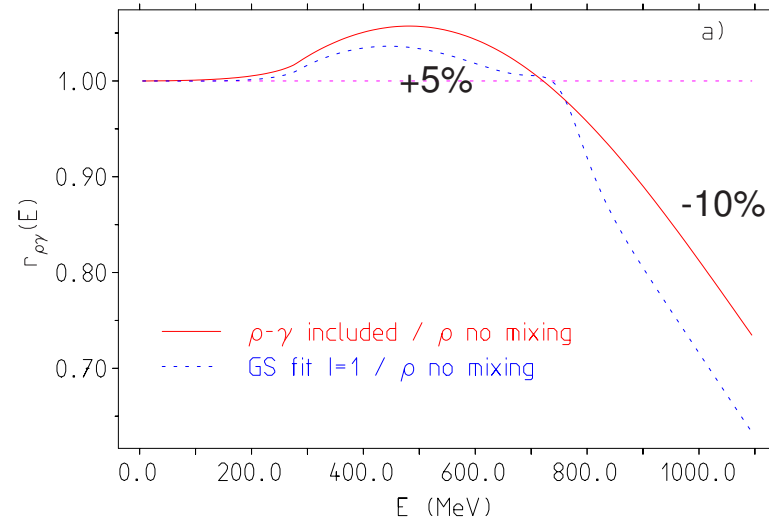
As a QED effect must be disentangled from e^+e^- data!

- Accounting the $\rho - \gamma$ interference resolves τ (charged channel) vs. e^+e^- (neutral channel) puzzle,
F.J.& R. Szafron, M. Benayoun et al. 2011

$\rho - \gamma$ interference
(absent in charged channel)
often mimicked by large shifts
in M_ρ and Γ_ρ

$$-i \Pi_{\gamma\rho}^{\mu\nu}(\pi)(q) = \text{diagram 1} + \text{diagram 2}$$

$v_0(s) = r_{\rho\gamma}(s) R_{\text{IB}}(s) v_-(s)$



- not τ spectra require to be corrected for missing $\rho - \gamma$ mixing!
rather this QED effect has to be eliminated from the e^+e^- data. Indeed absent in LQCD data!
- for getting had blob in e^+e^- the $\gamma - \rho^0$ mixing has to be removed!
- for the $l=1$ part of $a_\mu^{\text{had}}[\pi\pi]$ results in (JS 2011)

$$\delta a_\mu^{\text{had}}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10},$$

as a correction applied for the range $[0.63, 0.96]$ GeV.

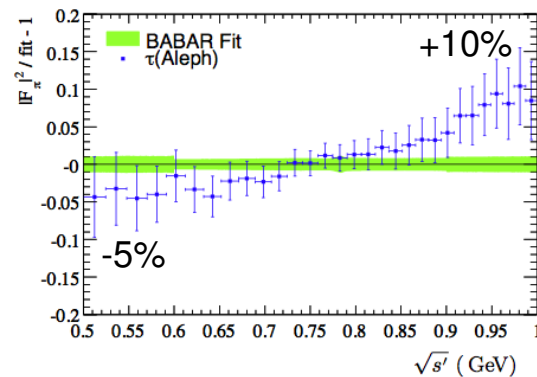
To be added to standard e^+e^- based $a_\mu^{\text{HVP, LO}}$: (in JS11 we subtracted it wrongly from the τ data)

$$a_\mu^{\text{HVP, LO}}|_{\text{cor}} = a_\mu^{\text{HVP, LO}} + \delta a_\mu^{\text{had}}[\rho\gamma] \simeq (702.3 \pm 4.2) \times 10^{-10}$$

Fits well with Davier et al. arXiv:0906.5443 incl. τ -decay spectra $a_\mu^{\text{had}}[ee + \tau] = (705.3 \pm 4.5) \times 10^{-10}$

At least a **part of the missing shift** with respect to LQCD data.

Best “proof” for our $\gamma - \rho^0$ mixing profile (Davier et al. 2009):



□ The issue of photon radiation by hadrons like pions remains (sQED vs RLA ?).

Advantage lattice QCD!

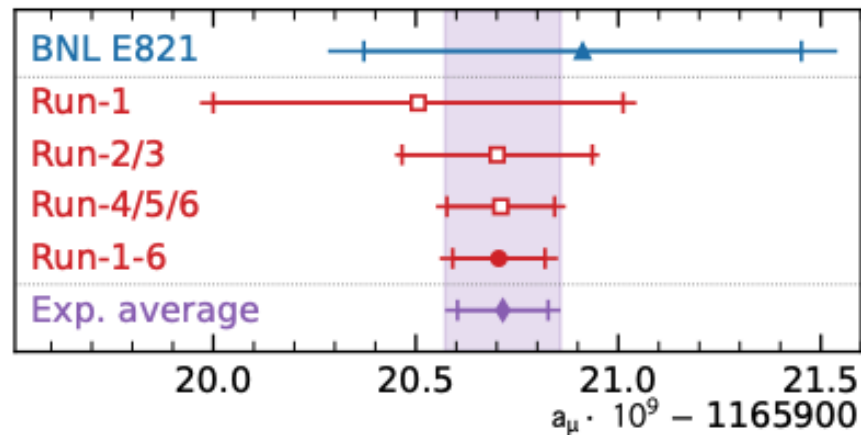
Muon $g-2$ Status and the SM

The exciting new Fermilab Muon $g-2$ measurement applying sophisticated improved technology brilliantly confirmed the 2004 final BNL result and triggers a new serious attack on the SM.

$$a_\mu[\text{BNL}] = 11659209.1(5.4)(3.3)[6.3] \Rightarrow a_\mu[\text{FNAL}] = 11659207.05(1.14)(0.95)[1.48]$$

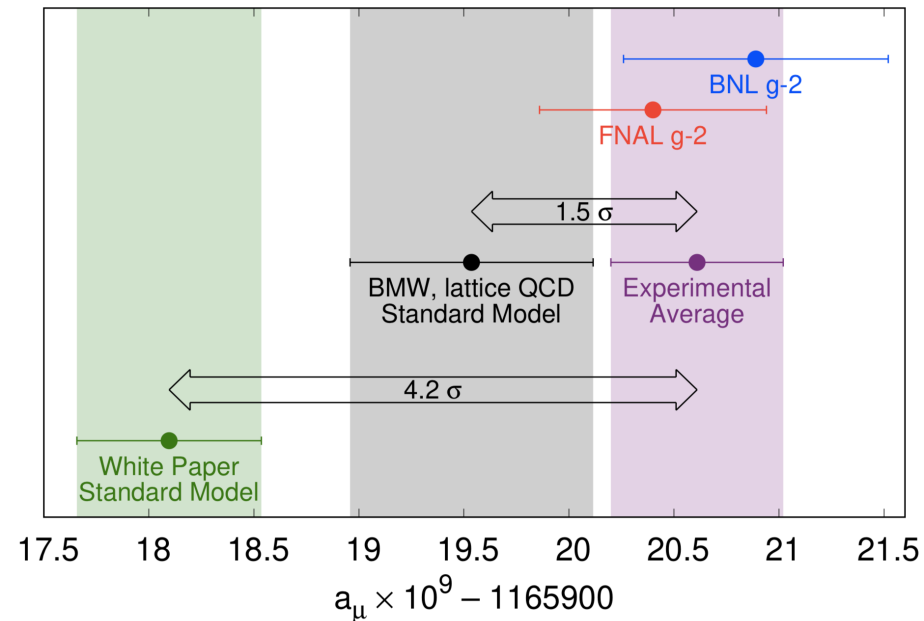
$$a_\mu[\text{FNAL/BNL}] = 11659207.15(0.80)(0.95)[1.24] \text{ (124 ppb) vs. (540 ppb) at BNL}$$

in units 10^{-10} ; FNAL error down by factor 4! now: comparable syst and stat errors



BNL vs. FNAL21/23/25

Compare 2021 status after FNAL Run-1: There's something in the air.



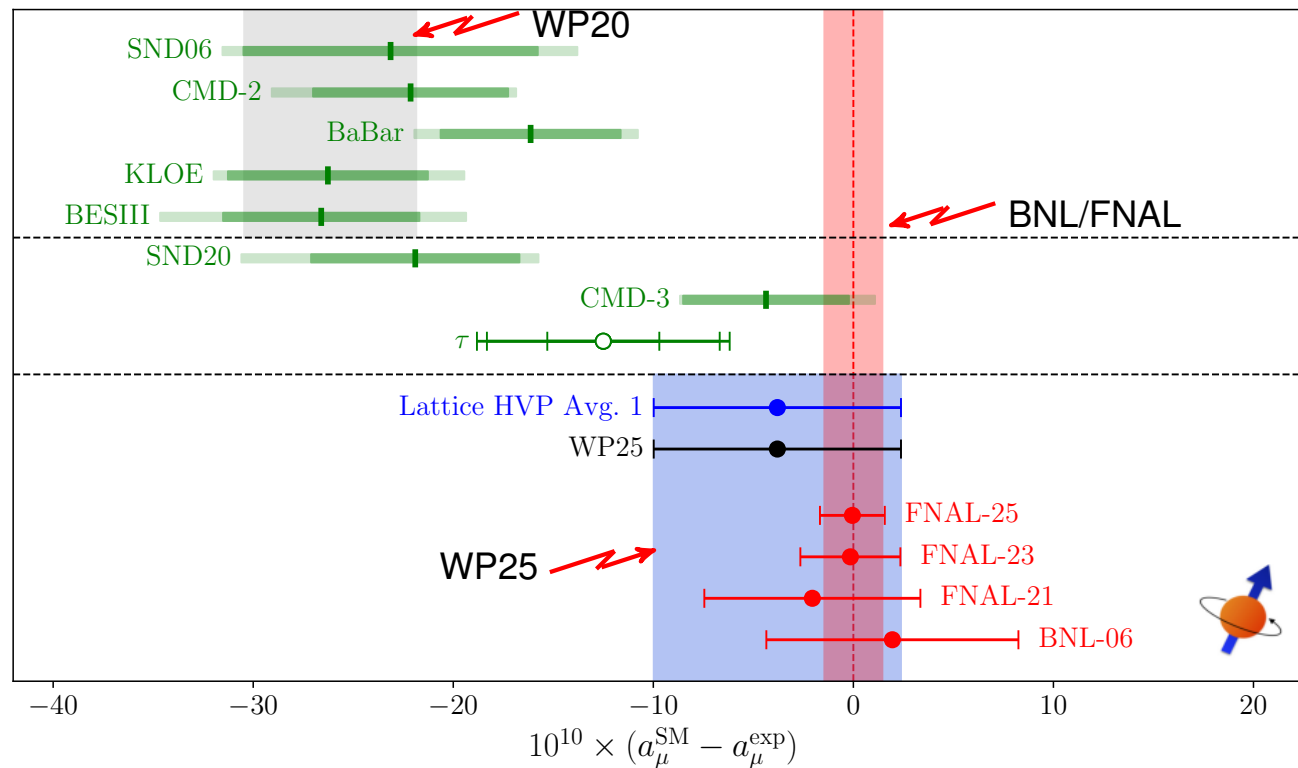
Muon g-2 plot incl. BMW lattice QCD after FNAL Run-1

WP20 Gap $\Delta a_\mu^{\text{BSM}}$ is 1.6 times EW contribution! Big effect missing?

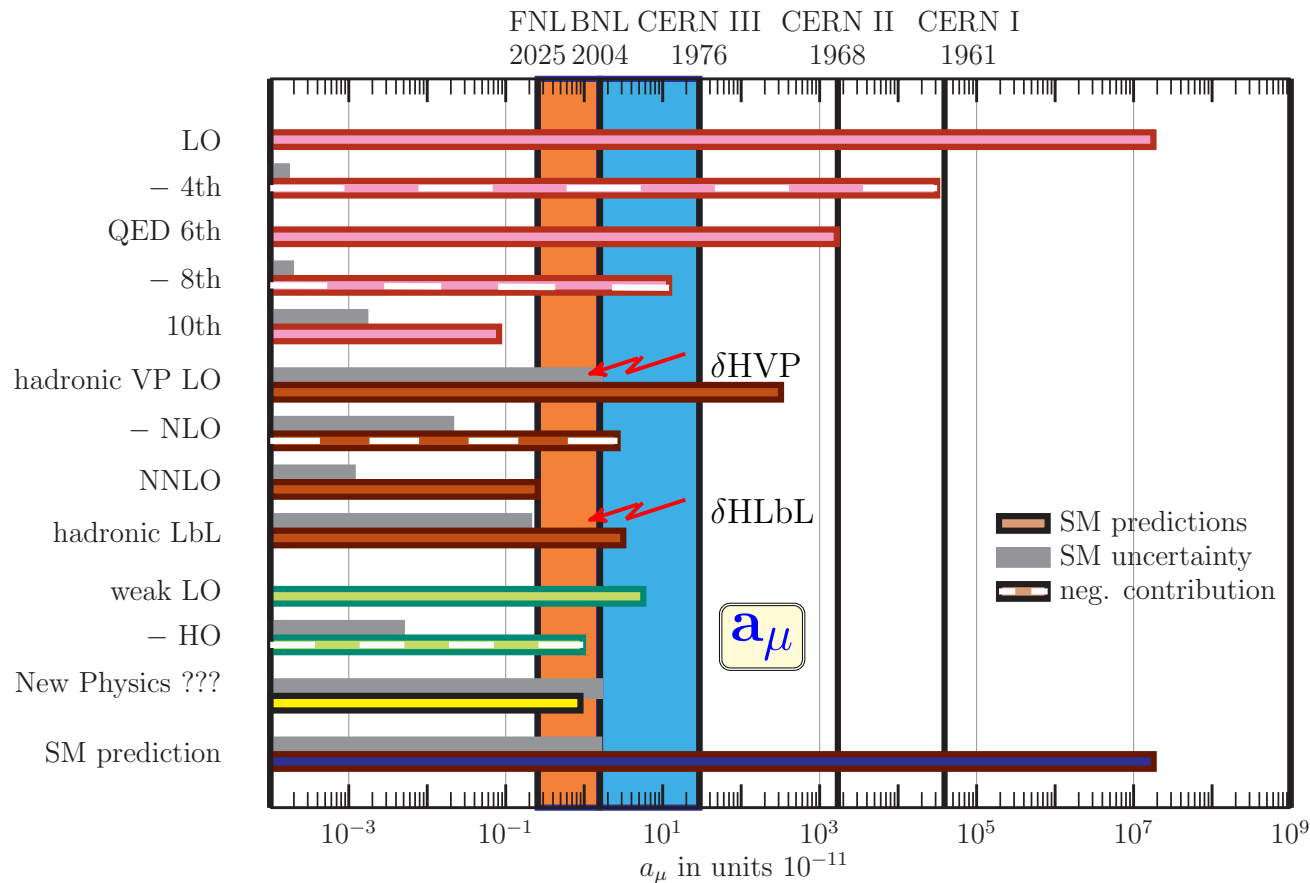
Still: theory leading uncertainty: HVP (and HLbL); DR+Data vs lattice QCD

Note: $\delta a_\mu^{\text{HVP-LO}}[\text{LQCD} - \text{DR}] = 14.4(6.8) \times 10^{-10}$ vs $a_\mu^{\text{EW}} = 15.44 \pm 0.04 \times 10^{-10}$

Now some might be shocked: a 5.6σ missing BSM turned into a “validates the SM up to the **0.37 ppm** level”, as highlighted in Fodor, Davier et al. 2024 [BMW/DMZ-24]. Status of a_μ in fall 2025:



While the consensus HVP(/HLbL) result from WP20 indicates a missing effect of 5.6σ between theory and experiment, the HVP(/HLbL) results from lattice QCD lead to an agreement with the SM at the level of 0.6σ ; graph from WP25. Puzzles regarding the true value of $a_\mu^{\text{HVP, LO}}$ remain unsolved: KLOE vs. BaBar, CMD3 vs. SND20, NC e^+e^- data vs. CC τ data, the data-driven dispersive approach, and LQCD calculations each face their own challenges.



Present muon $g - 2$ experiments testing various contributions. New Physics ? = deviation $(a_\mu^{\text{exp}} - a_\mu^{\text{the}})/a_\mu^{\text{exp}}$. Limiting theory precision: hadronic vacuum polarization (**HVP**) and hadronic light-by-light (**HLbL**)

*** digging deeper and deeper ***

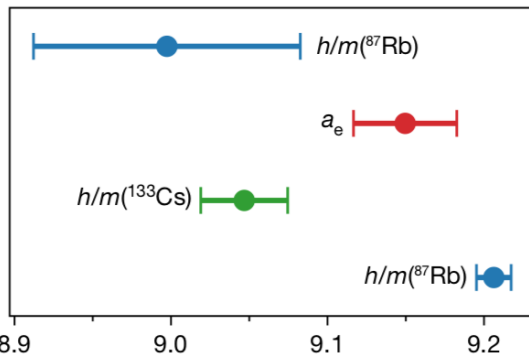
Electron g-2 Status and its Future

2018 $\alpha^{-1}(\text{Cs18}) = 137.035999046(27) \Rightarrow a_e^{\text{exp}} - a_e^{\text{the}} = (-84 \pm 36) \times 10^{-14} \Rightarrow -2.3 \sigma$

Oops! δa_e sign differs from δa_μ ! Challenge for BSM theory!

2020 $\alpha^{-1}(\text{Rb20}) = 137.035999206(11) \Rightarrow a_e^{\text{exp}} - a_e^{\text{the}} = (51 \pm 30) \times 10^{-14} \Rightarrow +1.7 \sigma$

α Rb20 vs Cs18 differ by **5.4 σ** ! Note α is basic SM input parameter!

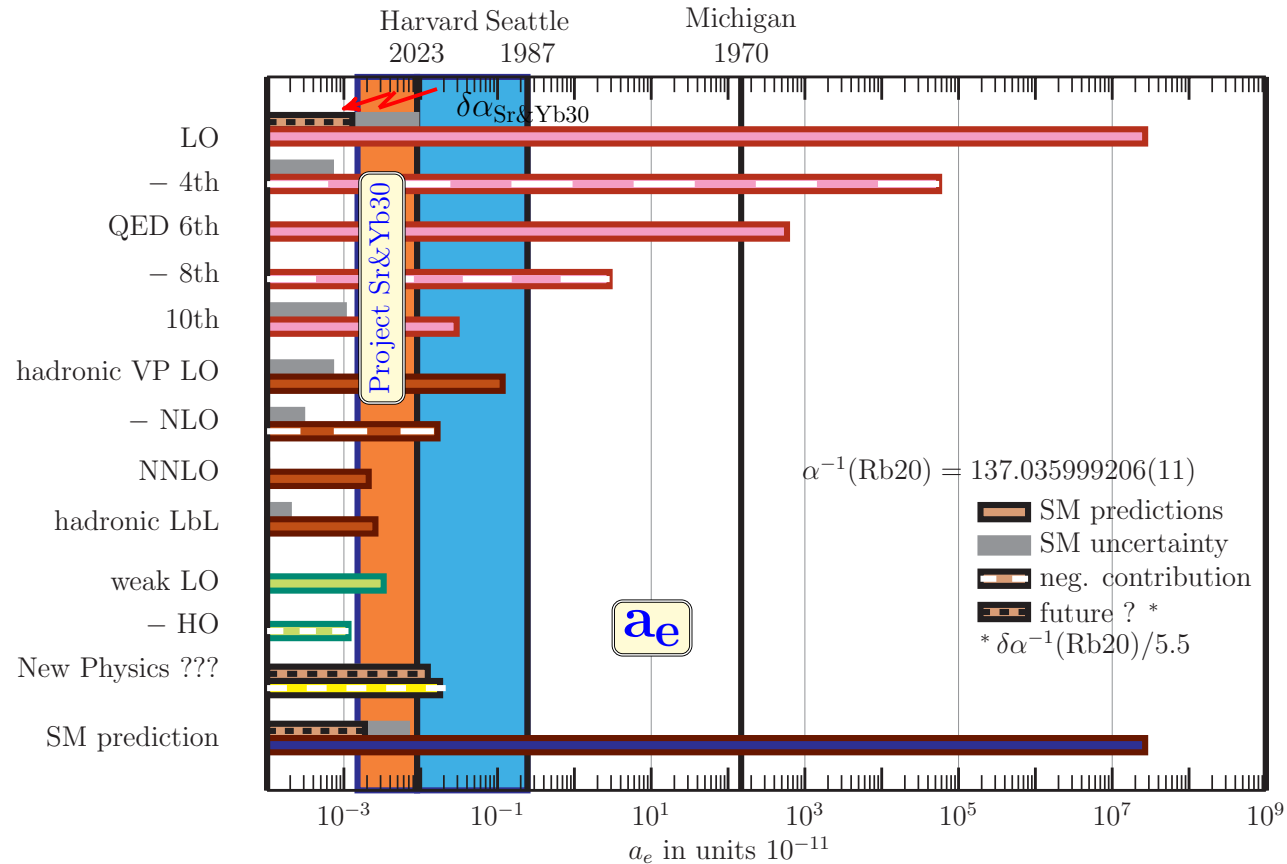


Electron anomaly $a_e - 0.00115965218050$:
from -2.3σ in 2018 to $+1.7 \sigma$ in 2020, and $+2.3 \sigma$ 2024.

Largely a QED test, highly sensitive to $\alpha_{\text{e.m.}}$.

Updates:

- ❖ 2022 update from $a_e^{\text{exp}} = 0.00115965218073(28)$ to $a_e^{\text{exp}} = 0.00115965218059(13)$ Gabrielse et al.
- ❖ 2024 $a_e^{\text{LO-HVP}}$ rescaled by $a_\mu^{\text{LO-HVP}}$ lattice vs. dispersive result, i.e.,
 $a_e^{\text{LO-HVP}} = 1.871(11) \times 10^{-12}$ to $a_e^{\text{LO-HVP}} = 1.923(09) \times 10^{-12}$
- ❖ 2024 change in universal 5-loop coefficient to $A_1^{(10)} = 5.873(128)$ Volkov, Aoyama et al.
 $a_e^{\text{the}} = 0.001159.65218023(9) \Rightarrow a_e^{\text{exp}} - a_e^{\text{the}} = (36 \pm 16) \times 10^{-14} \Rightarrow +2.3 \sigma$



Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of $\alpha(\text{Rb20})$ from atomic interferometry. “New Physics” ? = deviation $(a_e^{\text{exp}} - a_e^{\text{the}})/a_e^{\text{exp}}$, i.e., essentially absent presently. The blue band illustrates the improvement by the Harvard/Northwestern U. experiment. The orange band shows the possible progress by the Sr&Yb30 atomic interferometry project AION.

In fair agreement with the SM!

Conclusions

- One of the most prominent signs of BSM physics has gone: advantage SM, as by LHC searches!
- Does not mean hunting for BSM physics becomes less a main goal for research in particle physics
- What does the a_μ and a_e tell us?
 - ❖ High precision physics may be more challenging than expected, both a_μ^{exp} and a_μ^{the}
 - ❖ Need alternative experiment on a_μ^{exp} : so far all experiment CERN, BNL, FNAL use magic γ . New to come: J-PARC ultra cold muons in $\vec{E} = 0$ field
 - ❖ MuonE (elastic μe -scattering) determination of HVP [Abbiendi, Venanzoni et al.] at CERN (measures directly what LQCD calculates)
 - ❖ New α determination as an input for a_e AION project (with Strontium, Yttrium ions)
 - ❖ Need understand inconsistencies in data required for DRA to HVP: KLOE vs. BaBar, SND20 vs CMD-3, $e^+e^- \rightarrow \pi^+\pi^-$ vs $\tau^- \rightarrow \pi^0\pi^-\nu_\tau$ data (NC vs CC); for HLbL need more $\gamma\gamma \rightarrow \text{hadrons}$ data (better precision of known channels and new channels not experimentally accessed so far)
 - ❖ Never trust consensus papers! High precision physics is extremely important but extremely challenging.
- Beyond BSM science fiction: Scrutinizing precise SM prediction and progress in determining SM parameters like M_t and M_H (requiring high precision Higgs/Top-quark pair factory) a big challenge, important also for better understanding early cosmology. But that's another story!

Outlook: note that we have a remarkable agreement of the results:

Correcting $e + e^- \rightarrow \pi^+ \pi^-$ for $\rho^0 - \gamma$ mixing JS 2011/2024:

Davier et al., arXiv:0906.5443 incl. τ -decay :

Masjuan, Miranda & Roig, arXiv:2310.14102:

LQCD, Budapest, Marseille, Wuppertal 2020:

$$a_\mu^{\text{HVP-LO}}[ee] = 702.3 \pm 4.2 \times 10^{-10}$$

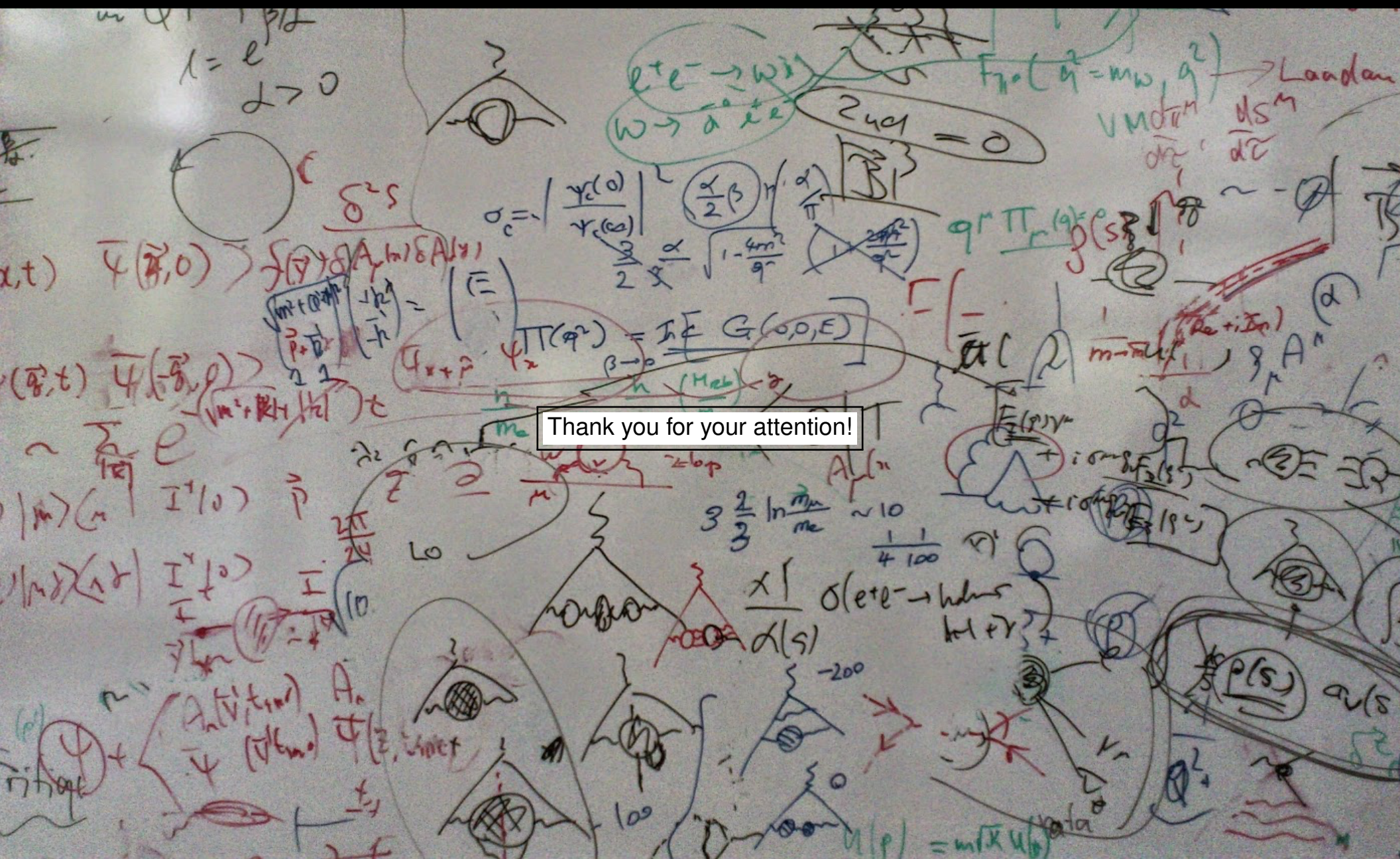
$$a_\mu^{\text{HVP-LO}}[ee + \tau] = 705.3 \pm 4.5 \times 10^{-10}$$

$$a_\mu^{\text{HVP-LO}}[\tau] = 704.1 \pm 4.1 \times 10^{-10}$$

$$a_\mu^{\text{HVP-LO}}[\text{LQCD : BMW}] = 707.5 \pm 5.5 \times 10^{-10}$$

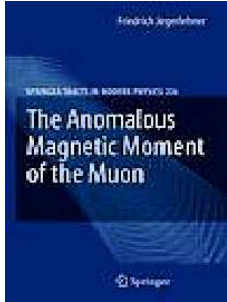
Since Maiz/CLS and RB/UKQCD obtained somewhat larger lattice results (relative to BMW), the White Paper 2025 combined the lattice results to the larger value $a_\mu^{\text{HVP-LO}} = 713.2 \pm 6.2 \times 10^{-10}$.

If progress in lattice results would consolidate the slightly smaller BMW-value, the puzzle about the gap between the dispersive approach and lattice results would be resolved when the $e^+ e^-$ data would be corrected for $\gamma - \rho^0$ interference. Then, the difference between the theoretical prediction and the experimental measurement at a 0.9σ level would still mean agreement with the SM.



Backup Slides

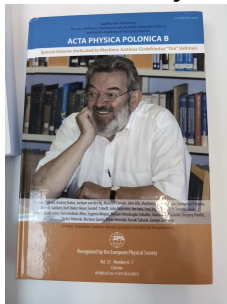
For further reading:



Jegerlehner F., The Anomalous Magnetic Moment of the Muon. Springer Tracts in Modern Physics, Vol 274 (2017). Springer, Cham (693 pages on one number to 8 digits)

Book update comment: Lepton_Moments2024 on my WEB page <https://people.physics.hu-berlin.de/~fjeger/>

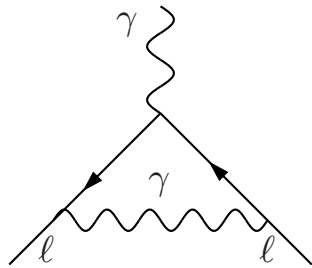
In Acta Physica Polonica B, special Volume dedicated to “Tini” Veltman:



F. Jegerlehner, The Standard Model of Particle Physics as a Conspiracy Theory and the Possible Role of the Higgs Boson in the Evolution of the Early Universe, Acta Phys. Polon. B **52**, no.6-7, 575-605 (2021)

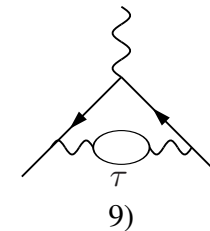
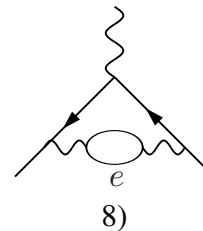
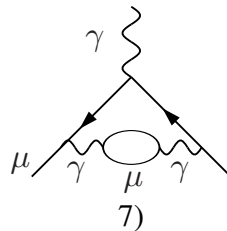
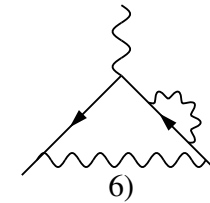
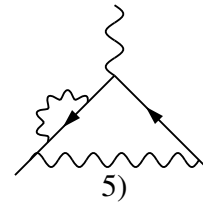
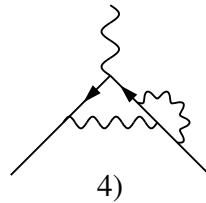
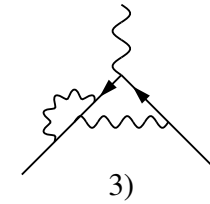
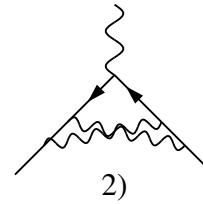
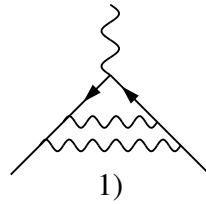
On a_μ^{QED}

Universal contributions: a_μ internal muons loops only



$$a_{\ell}^{(2)} \text{ universal} = \frac{1}{2} \left(\frac{\alpha}{\pi} \right)$$

Schwinger 1948

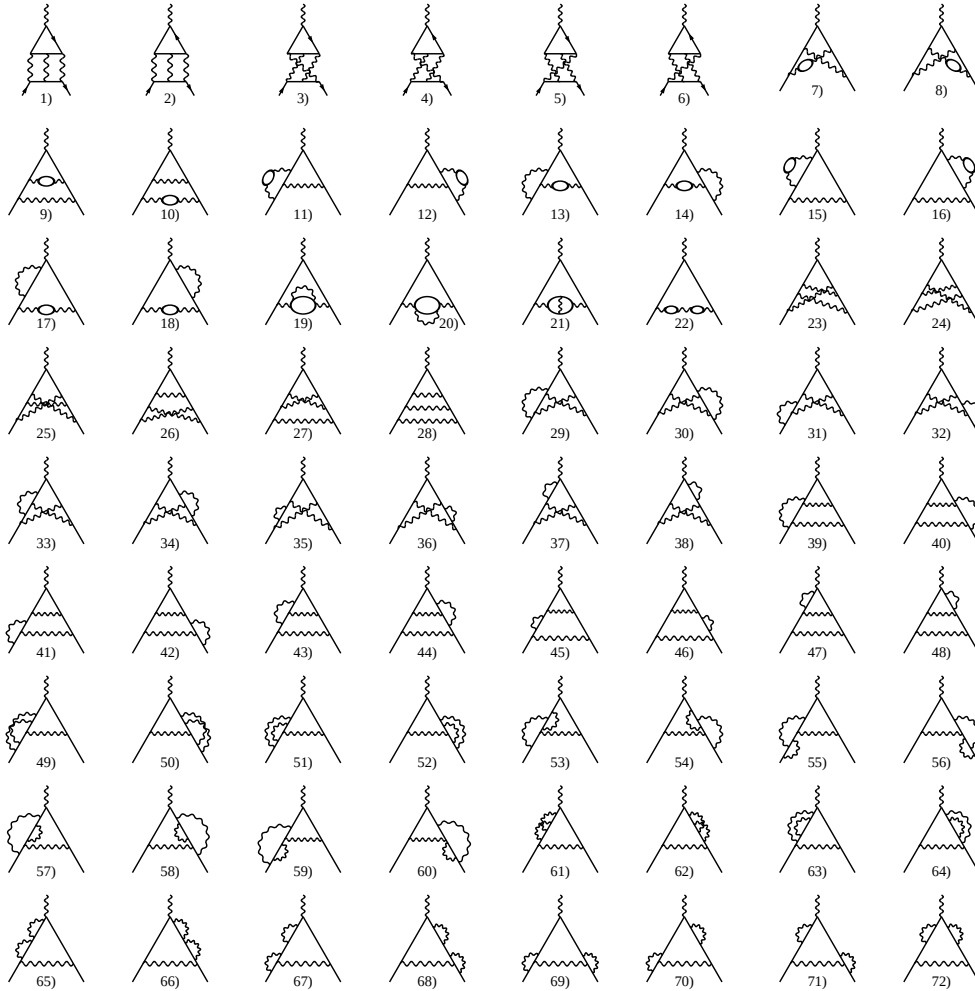


$$a_{\ell}^{(4)} \text{ universal} [1 - 7] = \left[\frac{197}{144} + \frac{\pi^2}{12} - \frac{\pi^2}{2} \ln 2 + \frac{3}{4} \zeta(3) \right] \left(\frac{\alpha}{\pi} \right)^2$$

Peterman 57, Sommerfield 57

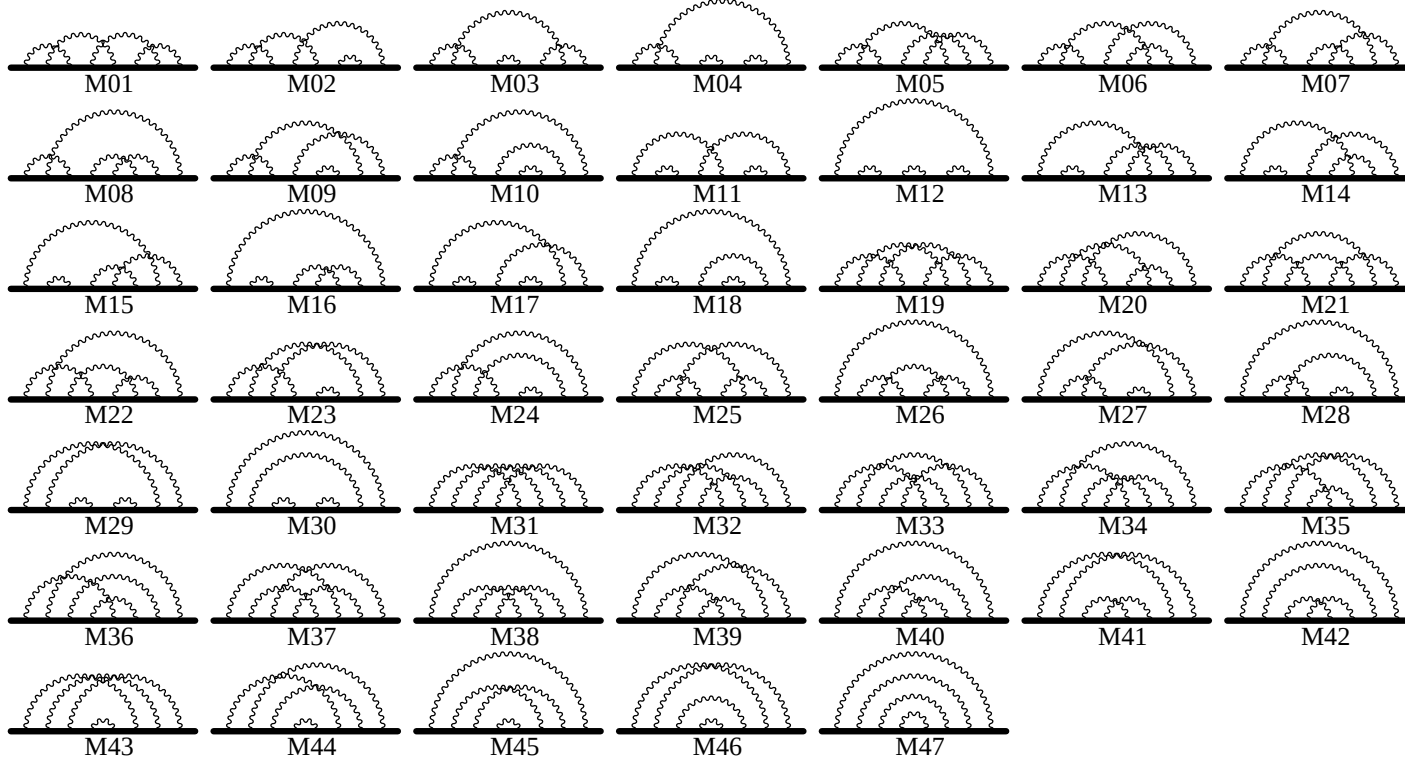
Universal 3-loop contribution:

(Remiddi et al., Remiddi, Laporta 1996 [27 years for 72 diagrams])



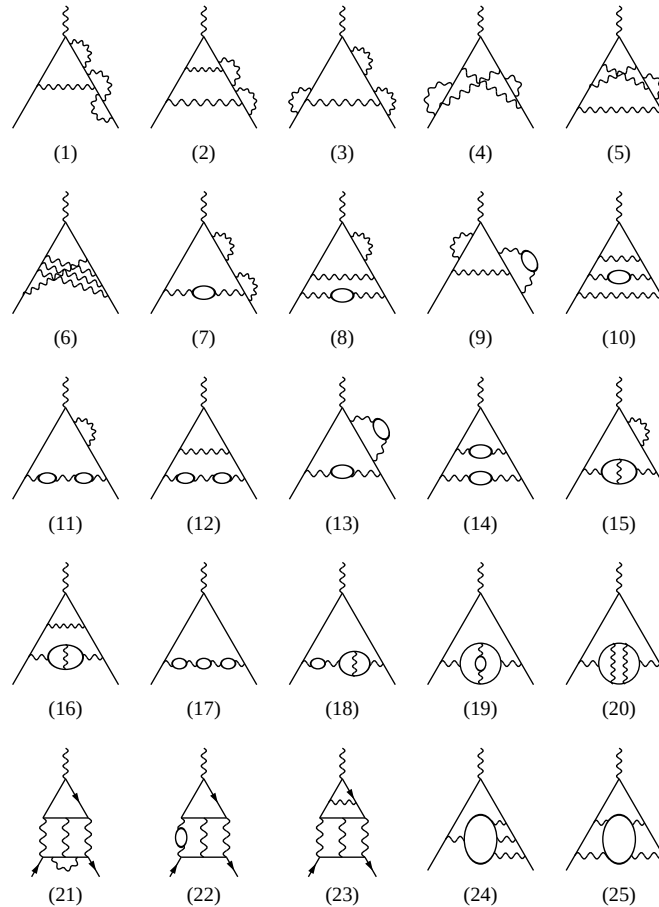
$$\begin{aligned}
 a_{\ell \text{ uni}}^{(6)} = & \left[\frac{28259}{5184} + \frac{17101}{810} \pi^2 \right. \\
 & - \frac{298}{9} \pi^2 \ln 2 + \frac{139}{18} \zeta(3) \\
 & + \frac{100}{3} \left\{ \text{Li}_4\left(\frac{1}{2}\right) \right. \\
 & \left. + \frac{1}{24} \ln^4 2 - \frac{1}{24} \pi^2 \ln^2 2 \right\} \\
 & - \frac{239}{2160} \pi^4 + \frac{83}{72} \pi^2 \zeta(3) \\
 & \left. - \frac{215}{24} \zeta(5) \right] \left(\frac{\alpha}{\pi} \right)^3
 \end{aligned}$$

30 years of heroic efforts: demanding 4-loop contribution:
(Kinoshita et al., Aoyama, Hayakawa, Kinoshita and Nio 2007)



4-loop Group V diagrams. 47 self-energy-like diagrams of $M_{01} - M_{47}$ represent 518 vertex diagrams [by inserting the external photon vertex on the virtual muon lines in all possible ways].

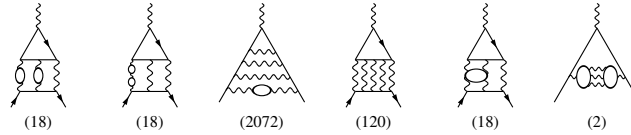
Laporta's quasi exact universal 4-loop result



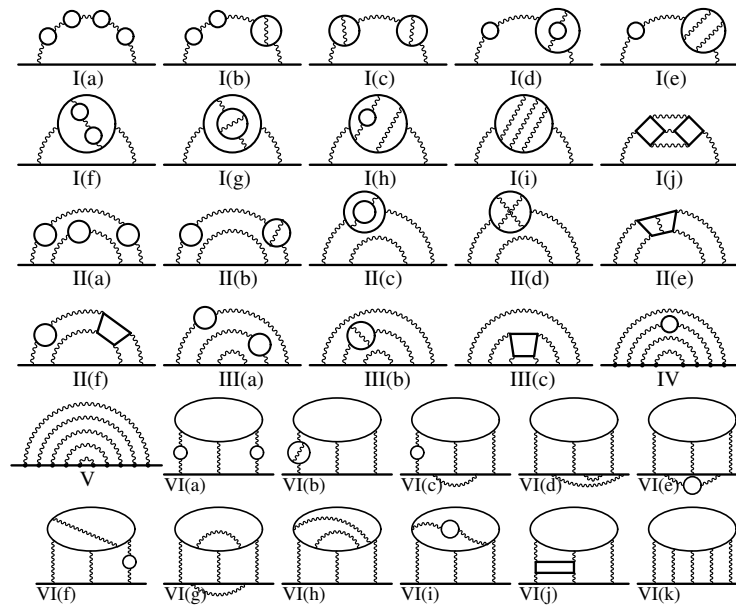
Sample diagrams of the **25** gauge-invariant subsets of the **891** universal $O(\alpha^4)$ contributions to a_e .

Contribution to $A_1^{(8)}$ of the 25 gauge-invariant sets
amounts to know a_e^{QED} and $a_\mu^{\text{QED universal}}$ to $O(\alpha^4)$ exactly
confirming Aoyama, Kinoshita & Nio numerical results

1	- 1.971075616835818943645699655337264406980
2	- 0.142487379799872157235945291684857370994
3	- 0.621921063535072522104091223479317643540
4	1.086698394475818687601961404690600972373
5	- 1.040542410012582012539438620994249955094
6	0.512462047967986870479954030909194465565
7	0.690448347591261501528101600354802517732
8	- 0.056336090170533315910959439910250595939
9	0.409217028479188586590553833614638435425
10	0.374357934811899949081953855414943578759
11	- 0.091305840068696773426479566945788826481
12	0.017853686549808578110691748056565649168
13	- 0.034179376078562729210191880996726218580
14	0.006504148381814640990365761897425802288
15	- 0.572471862194781916152750849945181037311
16	0.151989599685819639625280516106513042070
17	0.000876865858889990697913748939713726165
18	0.015325282902013380844497471345160318673
19	0.011130913987517388830956500920570148123
20	0.049513202559526235110472234651204851710
21	- 1.138822876459974505563154431181111707424
22	0.598842072031421820464649513201747727836
23	0.822284485811034346719894048799598422606
24	- 0.872657392077131517978401982381415610384
25	- 0.117949868787420797062780493486346339829



Some typical tenth order contributions to a_ℓ including fermion loops. In brackets the number of diagrams of the given type. Alone the universal $A_1^{(10)}$ term has contributions from 12 672 diagrams.



Typical self-energy-like diagrams representing 32 gauge-invariant subsets contributing to the tenth-order lepton $g - 2$. Solid lines represent lepton lines propagating in a weak magnetic field.

Kinoshita et al., Aoyama et al., Volkov

HLbL Resonance Lagrangian vs Dispersive Approaches

Compare

e.g. $a_{\mu}^{\text{HLbL}}[\text{RLA}] = 103(29) \times 10^{-11}$ (RLA, my book)
vs $a_{\mu}^{\text{HLbL}}[\text{DRA}] = 92(19) \times 10^{-11}$ (DRA WP)
vs $a_{\mu}^{\text{HLbL}}[\text{LQC}] = 107(14) \times 10^{-11}$ (LQCD Mainz)

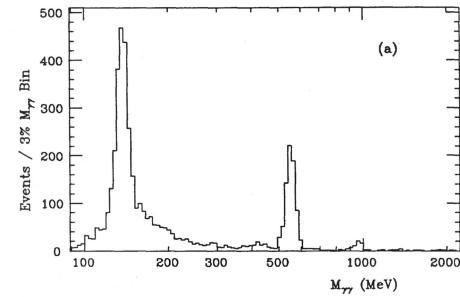
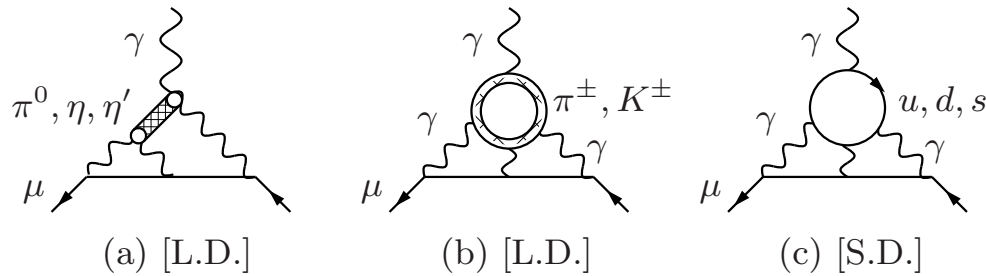
□ besides lattice, the two approaches are not very different after all

□ general data-driven DRA (Colangelo et al. 2015) reveals 19 essentially independent amplitudes (form-factors) which requires as many different channels to be measured (Bjinen et al. estimates 32)

□ only very few of the FFs are experimentally known, some single tag $\gamma\gamma^* \rightarrow \pi^0, \pi^+\pi^-, \pi^0\pi^0, \eta, f_1, \dots$ All have been used by the RLA estimates as well.

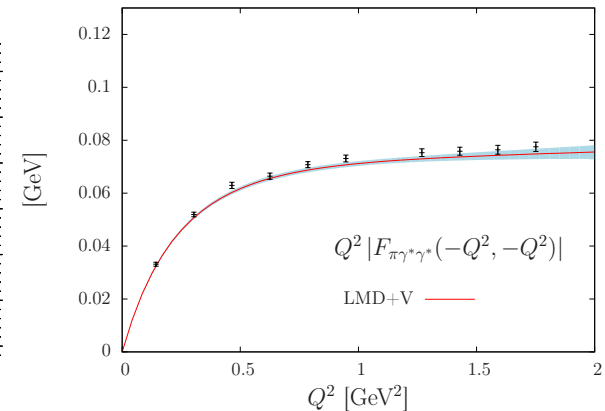
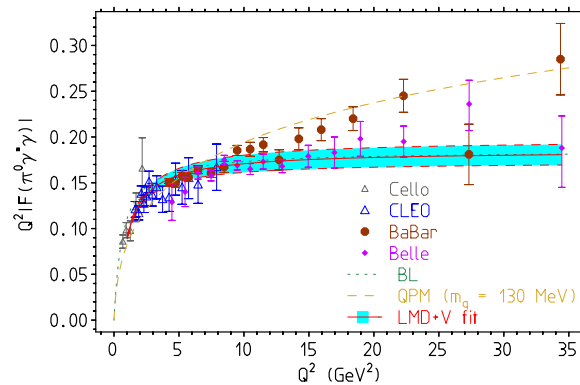
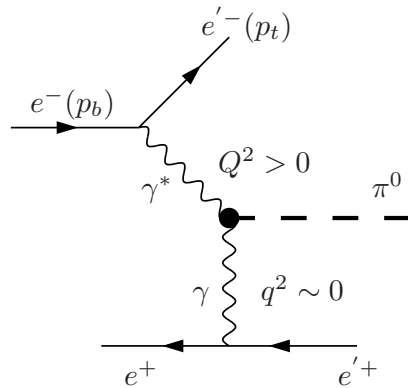
□ on experimental side $\gamma\gamma \rightarrow \text{hadrons}$ is very difficult. Is subprocess of $e^+e^- \rightarrow e^+e^- \text{hadrons}$

□ fortunately dominated by single particle exchanges!



L: HLbL classes. R: HLbL facts, pronounced non-perturbative physics.

Gérardin, Meyer, Nyffeler 16



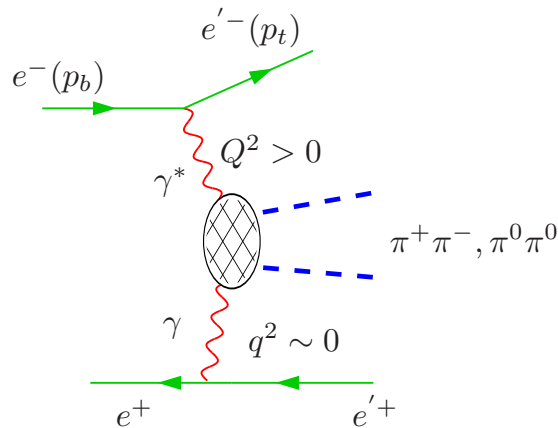
CELLO and CLEO measurement of the π^0 form factor $\mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2, -Q^2, 0)$ at high space-like Q^2 . **Outdated by BaBar?**

Belle **confirms with theory expectations!** First measurements of $\mathcal{F}_{\pi^0 \gamma^* \gamma}(m_\pi^2; -Q^2, -Q^2)$ in lattice QCD

More recent LQCD results by RBC Collab.

Tree level: signal diagrams obscured by substantial background

$e^+e^- \rightarrow e^+e^-\pi^0$	10 diagrams;	to extract $e^+e^- \rightarrow \gamma^*\gamma\pi^0 \rightarrow e^+e^-\pi^0$
$e^+e^- \rightarrow e^+e^-\pi^+\pi^-$	146 diagrams;	to extract $e^+e^- \rightarrow \gamma^*\gamma\pi^+\pi^- \rightarrow e^+e^-\pi^+\pi^-$
$e^+e^- \rightarrow e^+e^-\pi^0\pi^0$	60 diagrams;	to extract $e^+e^- \rightarrow \gamma^*\gamma\pi^0\pi^0 \rightarrow e^+e^-\pi^0\pi^0$

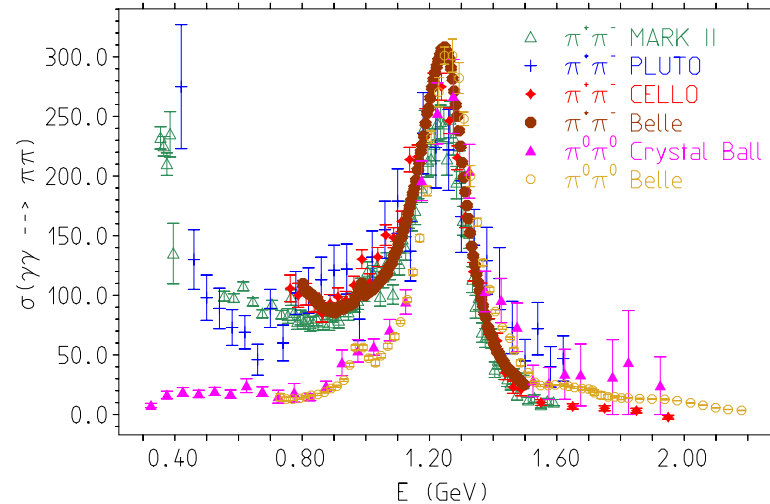


$$\frac{d\sigma_{\gamma^*\gamma}}{d|\cos\theta^*|} =$$

$$\frac{d^3\sigma_{ee}}{dWd|\cos\theta^*|dQ^2} \frac{f}{2 \frac{d^2L_{\gamma^*\gamma}}{dWdQ^2} (1+\delta)(\varepsilon/\varepsilon')\varepsilon'}.$$

The factors δ , ε , and f correspond to the radiative correction and ε' to the tagging efficiency.

Extracting $\gamma^*\gamma \rightarrow \pi\pi$ from $e^+e^- \rightarrow e^+e^-\gamma^*\gamma \rightarrow e^+e^-\pi\pi$ in a single tag experiment like Belle. Denoted by θ^* is the production angle of one of the π 's

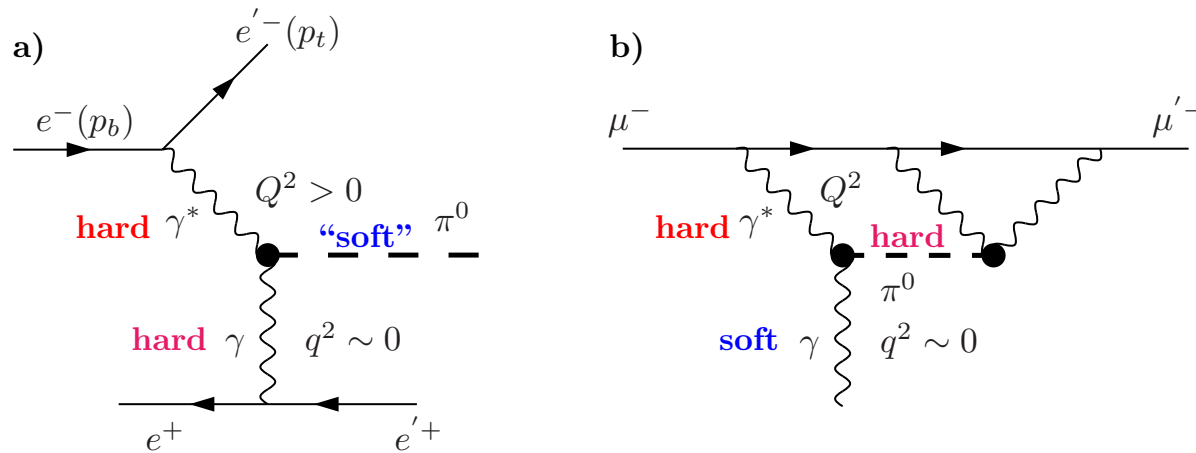


Di-pion production in $\gamma\gamma$ fusion. At low energy, we have direct $\pi^+\pi^-$ production and by strong rescattering $\pi^+\pi^- \rightarrow \pi^0\pi^0$, however, with a very much suppressed rate. Above about 1 GeV sQED is invalid, resolved $q\bar{q}$ couplings are seen. The $\pi^0\pi^0$ cross-section in this figure is enhanced by the isospin symmetry factor 2, by which it is reduced in reality.

□ The DRA to HLbL is a great achievement providing a clear general setup to implement properly the available data. Unfortunately, the experimental database is not yet satisfactory and its improvement is a very challenging task for the future.

□ In the end, LQCD must be the solution, but again HLbL is much more demanding than HVP, although a 10 % accuracy would be sufficient here.

A kinematics issue for HLbL:



- a) Measurement of the π^0 form factor $\mathcal{F}_{\pi^0\gamma^*\gamma}(m_\pi^2, -Q^2, 0)$ at high space-like Q^2 ,
 b) needed at external vertex is $\mathcal{F}_{\pi^0\gamma^*\gamma}(-Q^2, -Q^2, 0)$ as $q_\gamma^\mu \approx 0$.

● in any case analytic continuation required, from real to virtual pion!

□ actually $\pi^0\gamma^*\gamma^*$ (double tag) available from LQCD Mainz and RBC

A Comparison of WP25, WP20 and my Book:

Contribution	WP25	WP20	myBook17
HVP LO (lattice)	7132(61)	7116(184)	–
HVP LO (e^+e^-)	6947.9(41.8)	6931(40)	6894.6(32.5)
HVP LO ($e^+e^- + \tau$)	7045.0(62.0)	–	–
HVP NLO (e^+e^-)	–99.6(1.3)	–98.3(7)	–99.27(0.67)
HVP NNLO (e^+e^-)	12.4(1)	12.4(1)	12.24(0.10)
HLbL (pheno)	103.3(8.8)	92(19)	103.4(28.8)
HLbL NLO (pheno)	2.6(6)	2(1)	3(2)
HLbL (lattice)	122.5(9.0)	82(35)	–
HLbL (pheno + lattice)	112.6(9.6)	90(17)	–
QED	116 584 718.8(2)	116 584 718.931(104)	116 584 718.86(03)
EW	154.4(4)	153.6(1.0)	153.6(1.1)
HVP (LO + NLO + NNLO)	7045(61)	6845(40)	6807.6(31.9)
HLbL (pheno + lattice + NLO)	115.5(9.9)	92(18)	–
Total SM Value	116 592 033(62)	116 591 810(43)	116 591 783(35)

Comparison WP25, WP20 and my book (in units of 10^{-11}). “HLbL (lattice)” is WP20 + charm-loop. “HVP (LO + NLO + NNLO)” combines from HVP LO (lattice) [WP25] and HVP LO (e^+e^-) [WP20].

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