Lepton Magnetic Moments, the 2024 Update

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A key quantity for the high-precision test of the SM is the anomalous magnetic moment of the muon a_μ, one of the most precisely measured quantities in particle physics, which also can be predicted with comparably high accuracy and at the same time represents a susceptible window to physics beyond the SM. After the improved measurement $a_\mu^{\rm exp} \equiv (g_\mu - 2)/2 = 116\,592\,091(54) \times 10^{-11} (0.54 \text{ ppm})$ [1] (CERN 1977 had achieved a 7 ppm accuracy) at Brookhaven National Laboratory (BNL) in the years 1999 to 2004 a $4.3\ \sigma$ deviation between experiment and theory [5] fueled the hope that the SM was in trouble, and the deviation pointed to possible new physics. At that time, the minimal supersymmetric expansion of the SM (MSSM) was the most promising way to close the gap between the experiment and the SM prediction. However, the search for new particles at the LHC in 2012 already showed that no convincing new physics scenario could produce such an effect. Recently, the muon $g-2$ experiment at the Fermi National Accelerator Laboratory (FNAL) was able to further improve the accuracy with the result $a_\mu^{\rm exp} = 116\,592\,055(24) \times 10^{-11}$ in full agreement with the BNL result. The world average $a_\mu^{\rm exp} = 116\,592\,059(22) \times 10^{-11}$ (0.19 ppm) [2, 3] confronted with the White Paper result [4], appeared to increase the discrepancy to about 5.2 σ.

These results relied on the data-driven dispersive calculations of non-perturbative hadronic vacuum polarization (HVP) given by $a_\mu^{\mathrm{LO-HVP}} = 694.79(0.78)(4.11)[4.18] \times 10^{-10}$ with `alphaQEDc23` package. However, progress in the ab initio lattice QCD calculations [6] has yielded a much larger HVP $a_\mu^{\mathrm{LO-HVP}} = 714.1(2.2)(2.5)[3.3] \times 10^{-10}$, which, together with other SM contributions [4] gives the result $a_\mu^{\mathrm{the}} = 116\,592\,019(38) \times 10^{-11}$, which differs with $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{the}} = 40(44) \times 10^{-11}$ from the experiment, i.e., only 0.9 standard deviations. This result validates the standard model up to the 0.37 ppm level, as highlighted in [6]. The various contributions and their uncertainties of the SM predictions are shown in Fig. 1.

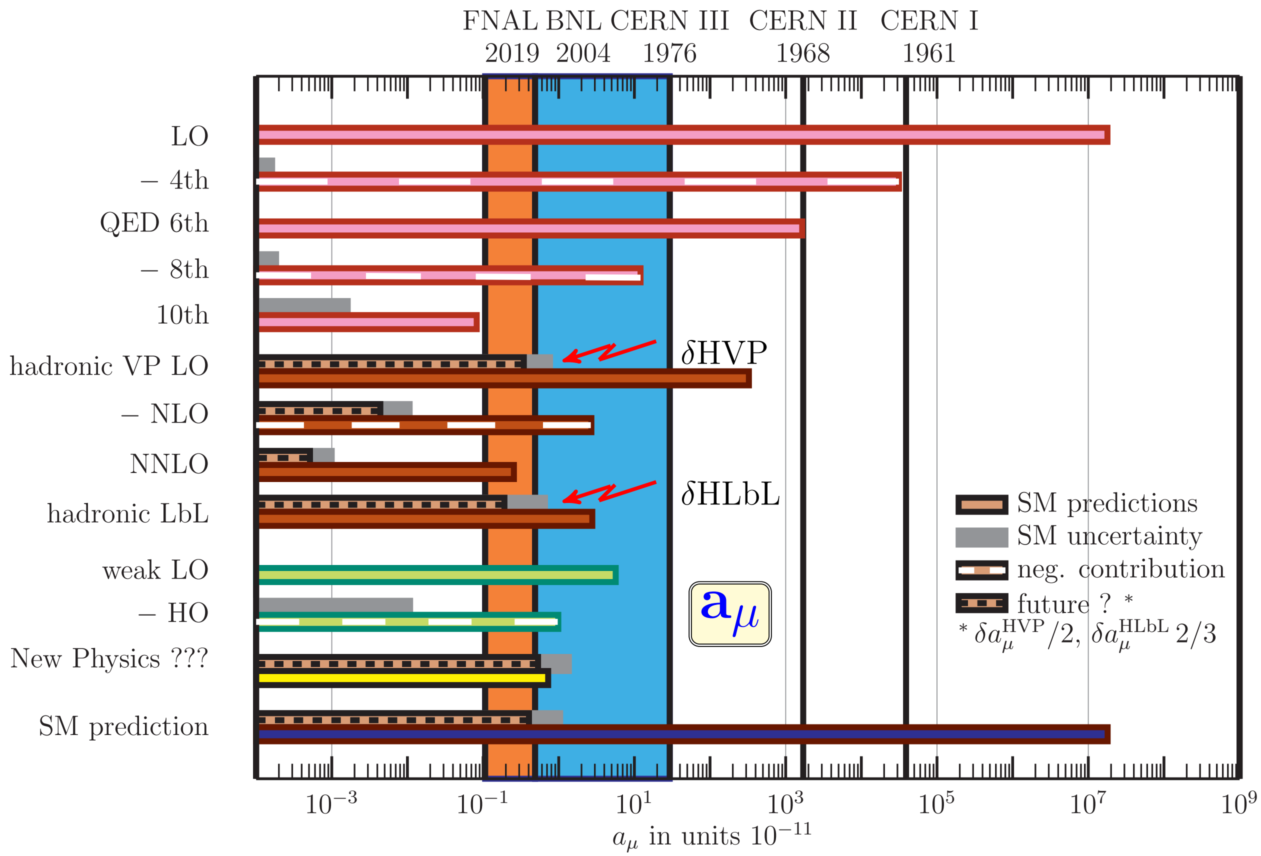

Figure 1: Past and future muon $g-2$ experiments testing various contributions. New Physics ? = deviation $(a_\mu^{\rm exp} - a_\mu^{\rm the})/a_\mu^{\rm exp}$. Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

For the anomalous magnetic moment of the electron $a_e = (g_e - 2)/2$, the agreement between theory and experiment provides a similarly impressive test of the SM prediction. On the experimental side, we have the remarkably accurate recent measurement by the Gabrielse team $g_e/2 = 1.001\,159\,652\,180\,59\,(13)(0.13\ \mathrm{ppt})$ [8]. The accurate prediction of a_e heavily depends on an accurate electromagnetic fine structure constant. Here the substantial progress in atomic interferometry with ^{87}Rubidium atoms with the result $\alpha^{-1}(\mathrm{Rb20}) = 137.035999206(11)(81\ \mathrm{ppt})$ [7] has provided a significant step towards the improved prediction $a_e^{\mathrm{the}} = 1.15965218059(9)$[1]. Confronted with the experimental value, we obtain $a_e^{\mathrm{exp}} - a_e^{\mathrm{the}}\ 0.18(16) \times 10^{-11}$, 1.1 standard deviations and an accuracy of 8.1×10^{-11}. Update Dec 2024: a correction of the tenth-order QED contribution to a_e [9, 10] lowers the value of a_e by 5.4×10^{-14} such that $a_e^{\mathrm{the}} = 1.15965218024(9)$ and $a_e^{\mathrm{exp}} - a_e^{\mathrm{the}}\ 0.35(16) \times 10^{-11}$ a 2.2 standard deviations. Accidentally, the lattice QCD shift of $+5.2 \times 10^{-14}$ of the hadronic contribution and the correction of the tenth-order QED contribution cancel each other. In future

[1] a_e^{LO-HVP} rescaled by a_μ^{LO-HVP} lattice vs. dispersive result, i.e., $a_e^{LO-HVP} = 1.871(11) \times 10^{-12}$ to $a_e^{LO-HVP} = 1.923(09) \times 10^{-12}$.

atomic interferometry experiments (the AION project based on Strontium and Yttrium) are expected to reduce the uncertainty of the electromagnetic fine structure constant such that an a_e prediction with an accuracy of 2×10^{-14} would be possible. The various contributions and their uncertainties of the SM predictions are shown in Fig. 2.

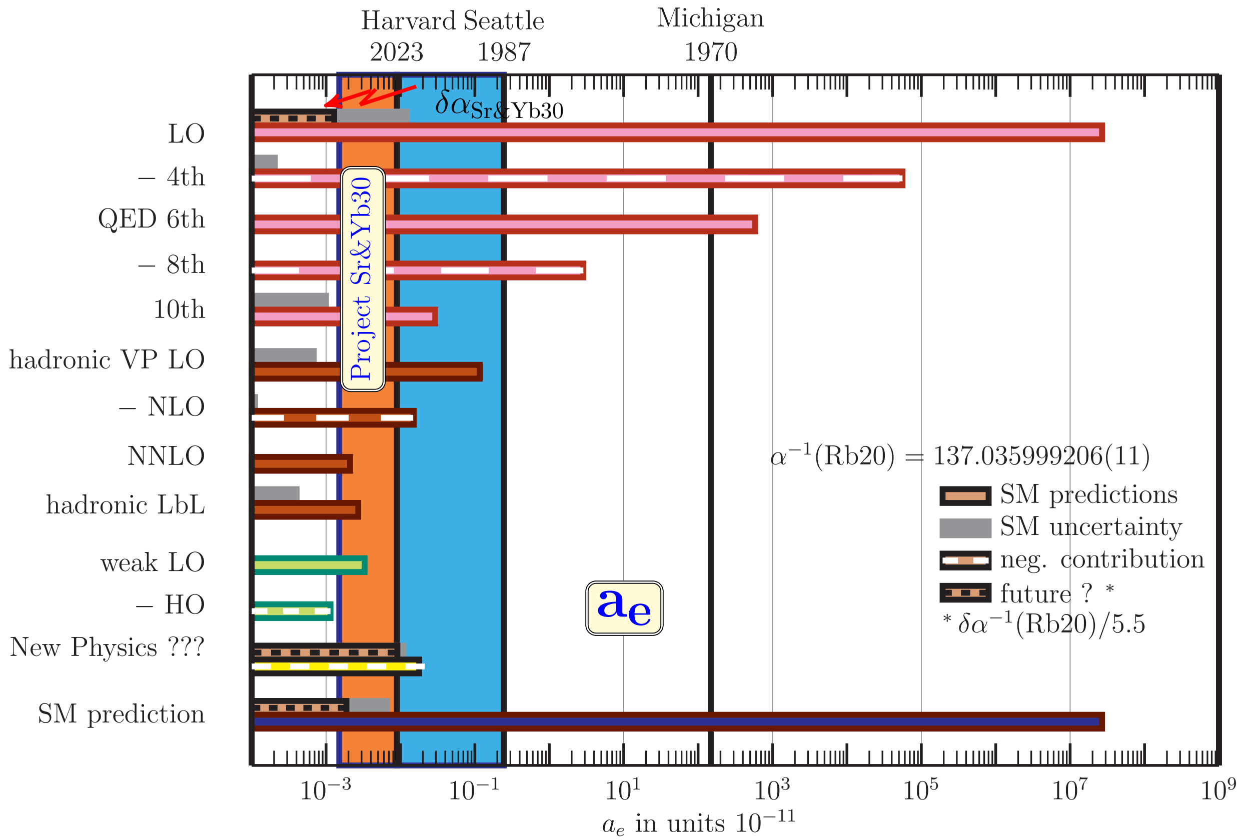

Figure 2: Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of α(Rb20) from atomic interferometry. "New Physics" ? = deviation $(a_e^{\mathrm{exp}} - a_e^{\mathrm{the}})/a_e^{\mathrm{exp}}$, i.e., esentially absent presently. The blue band illustrates the improvement by the Harvard experiment. The orange band shows the possible progress by the Sr&Yb30 atomic interferometry project.

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