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PRECISION TESTS OF ELECTROWEAK-INTERACTION PARAMETERS+

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Abstract: We review caculations of parameter renormalization effects which will be measured with high precision at LEP and SLC. Uncertainties of theoretical predictions, experimental precision errors and the size of effects from unknown particles are discussed.

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1. Future Precision Measurements of Vector Boson Parameters

The discovery of the intermediate vector bosons $W^{\pm}$ and Z at the CERN $p\bar{p}$ collider [1] opened a new chapter in the history of experimental and theoretical studies of electroweak processes. For the first time it was possible to establish the existence of new particles within the standard model [2] at precisely predicted values of their masses:

$$M_W = \left(\frac{\pi \alpha}{\sqrt{2} G_\mu \sin^2\theta_W} \frac{1}{(1-\Delta r)} \right)^{1/2} ; \; M_Z = \frac{M_Z}{\cos\theta_W} . \qquad (1.1)$$

Here $\sin^2\theta_W = 1 - M_W^2/M_Z^2 = \sin^2\theta_o \frac{1-\Delta g}{1-\Delta r}$ is the weak mixing angle calculated from the corresponding parameter $\sin^2\theta_o$ determined from low energy neutral current data. Careful studies of the experimental data [3,4] revealed already the importance of the radiative corrections Δr and Δg which will be discussed in the next section. Future precision experiments at SLC and LEP will admit to test the standard model on its quantum level with an accuary providing a "window to new physics". Apart from the masses and widths of the vectorbosons a number of asymmetries will be measured at the Z-peak, like the forward-backward asymmetry $A_{FB}^{f\bar{f}}$ (for unpolarized beams), the left-right asymmetry A_{LR} and the polarized forward-backward asymmetry A_{PolFB}^{ff} (both for longitudinally polarized beams). At the tree level (at the Z-peak) they are simple functions of $\sin^2\theta_W$:

$$A_{FB}^{f\bar{f}} = \frac{3}{4} A_e A_f , \; A_{LR} = A_e , \; A_{PolFB}^{f\bar{f}} = \frac{3}{4} A_f \qquad (1.2)$$

$$A_f = -2 \frac{v_f \, a_f}{v_f^2 + a_f^2}$$

where v_f is the vector coupling and a_f the axialvector coupling of the neutral current for a particular fermion flavor f.

The accuracies to which different quantities will be determined in future experiments have been estimated by several working groups [5,6]. Some of the estimates are listed in Tab. 1.

In view of the high precision which will be reached by experiments at LEP, we have to take into account radiative corrections and to worry about uncertainties of theoretical predictions. We may distinghuish the

Table 1: Experimental uncertainties of different measurements and corresponding uncertainties in $\sin^2\theta_W$ calculated by means of Eqs. (1.1, 1.2)

Measurement		Precision in quantity measured	$\sin^2\theta_W$
νN scattering			0.006
νe scattering	(CHARM II)		0.005
M_W, M_Z	(ACOL 20 pb^{-1})	$\delta(M_W/M_Z) \simeq 0.002$	0.004
M_Z	(LEP I 200 pb^{-1}	$\delta M_Z \simeq 20$ MeV	0.0005
$A_{FB}^{\mu^+\mu^-}$	at Z-peak)	$\delta A_{FB}^{\mu^+\mu^-} \simeq 0.003$	0.0015
A_{LR}	(LEP I 40 pb^{-1} with	$\delta A_{LR} = 0.003$	0.0004
$A_{PolFB}^{\mu^+\mu^-}$	$\langle P_L \rangle = 0.5$, $\Delta P_L/P_L = 0.01$)	$\delta A_{PolFB}^{\mu^+\mu^-} = 0.005$	0.0006
M_W	(LEP II 500 pb^{-1})	$\delta M_W = 100$ MeV	0.0006

following categories of uncertainties:

a) the limited accuracy of input parameters,
b) ambiguities due to strong interaction effects,
c) errors due to unknown higher order effects (two-loop, ...),
d) unknown parameters like the Higgs mass m_H, the top-quark mass m_t and the lowest order value ρ_{00} of the ρ-parameter which is a measure for the weak isospin of the Higgs-field(s),
e) the possible existence of new physics, like additional fermions, supersymmetric partners, extra gauge bosons (Z',...), non-standard interaction terms or composite structures.

With the precision value for M_Z from SLC/LEP I, the best set of input

parameters for theoretical calculations will be α, G_μ and M_Z. Precision measurements of any further quantity will provide tests of the higher order (one-loop) effects predicted by the standard model [7] and yield bounds to the unknown parameters like m_H, m_t and ρ_{oo}. Stringent limits on new physics are to be expected.

What can be learned on hitherto unobserved effects could be obscured by "theoretical uncertainties" due to hadronic ambiguities and due to unknown higher order effects. Estimates of these ambiguities will be discussed below.

2. Radiative Corrections and Parameter Shifts

The low energy parameters of the electroweak standard model (SM) [2] are determined in four fermion processes which are described in terms of fermion currents. The latter derive from the $SU(2)_L$ (V-A) currents $J_{\mu i}$ and the $U(1)_Y$ current J_Y in the combinations which couple to the physical gauge bosons (A^μ, Z^μ and $W^{\pm\mu}$)

$$\begin{aligned} j_{\mu em} &= J_{3\mu} + \frac{1}{2} J_{Y\mu} \\ J_{\mu Z} &= J_{3\mu} - \sin^2\theta_W \, j_{\mu em} \\ J^{\pm}_{\mu} &= J_{1\mu} \pm i J_{2\mu} \, . \end{aligned} \tag{2.1}$$

The charged current (CC) has the form

$$J^{+}_{\mu} = \bar{\nu}_\ell \gamma_\mu (1 - \gamma_5) \ell + \bar{q}_u \gamma_\mu (1 - \gamma_5) U_{KM} \, q_d \tag{2.1a}$$

exhibiting quark flavor changing, through the unitary Kobayashi-Maskawa matrix U_{KM}, the neutral current (NC) is strictly flavor conserving

$$J_{\mu Z} = \sum_f \bar{\psi}_f \, \gamma_\mu (a_f + b_f \gamma_5) \psi_f \tag{2.1b}$$

the sum extending over the individual fermion flavors (and color). In our convention the vector- and axialvector neutral current coefficients are

given by

$$(a_1,b_1) = (-Q_1 \sin^2\theta_W + \frac{1}{4}, -\frac{1}{4})$$
$$(a_2,b_2) = (-Q_2 \sin^2\theta_W - \frac{1}{4}, \frac{1}{4}), \qquad (2.1c)$$

respectively, for the upper (1) and lower (2) components of the weak isodoublets. The interaction part of the matter field Lagrangian thus takes the form

$$L_{int} = \frac{g}{\sqrt{2}} (J^+_\mu W^{\mu-} + h.c.) + \frac{g}{\cos\theta_W} J_{\mu Z} Z^\mu + e j_{\mu em} A^\mu \qquad (2.2)$$

proposed by Glashow [2] long time ago. Its low energy effective form (momentum transfer $|q^2| \ll M_W^2, M_Z^2$) is the Fermi-type Lagrangian

$$L^{(o)}_{eff,int} = -\frac{4}{\sqrt{2}} (G_c J^+_\mu J^{\mu-} + G_n J_{\mu Z} J^\mu_Z) + e j_{\mu em} A^\mu \qquad (2.3)$$

which is the basis for all studies of low energy four fermion processes. In (2.2) $\cos\theta_W$ is subject to the weak-electromagnetic unification condition

$$e = g \sin\theta_W \qquad (2.4)$$

and the Fermi couplings in (2.3) are determined by

$$\sqrt{2}\, G_c = \frac{g^2}{4M_W^2} = \frac{1}{v^2} \qquad (2.5a)$$

in case of the charged current and by

$$\sqrt{2}\, G_n = \frac{g^2 + g'^2}{4M_Z^2} = \rho \frac{1}{v^2} \qquad (2.5b)$$

in case of the neutral current, where the neutral-to-charged-current coupling ratio

$$\rho = \frac{G_n}{G_c} = \frac{M_W^2}{M_Z^2 \cos^2\theta_W} \qquad (2.5c)$$

defines the ρ-parameter. Within the standard model $\rho = 1 + O(\alpha)$ is calculable function.

The parameters of the standard model have been determined first from low energy fermion processes.[+] They are given by the fine structure constant $\alpha = e_o^2/4\pi = 1/137.035963(15)$ (from low momentum transfer Coulomb scattering) the Fermi constant $G_\mu = G_{co} = \frac{1}{\sqrt{2}v_o^2} = 1.16637(2) \times 10^{-5}\ \mathrm{GeV}^{-2}$ (from the muon decay rate) and the weak mixing parameter

$$\sin^2\theta_o = \frac{1}{4}(1-\xi). \tag{2.6a}$$

We assume $\sin^2\theta_o$ to be defined through low momentum transfer neutrino-electron scattering where the cross section ratio

$$\sigma(\bar{\nu}_\mu e)/\sigma(\nu_\mu e) = (\xi^2 - \xi + 1)/(\xi^2 + \xi + 1) \tag{2.6b}$$

is independent of the neutral current coupling $G_{NC} = G_{no}$. The latter is defined by $G_{NC} = (8\pi\sigma(\nu_\mu e)/m_e E_\nu [(1+\xi)^2 + \frac{1}{3}(1-\xi)^2])^{1/2}$ (E_ν the incident neutrino energy). Strong interaction effects in this parametrization only enter through virtual hadronic processes. For phenomenological purposes it is wise to keep the coupling ratio $\rho_o = G_{NC}/G_\mu$ as a free parameter. ρ_o has been determined experimentally to be close to the lowest order standard model value $\rho_{oo} = 1$. Present $(\overset{(-)}{\nu}_\mu e)$-scattering data are not very precise due to low statistics. A reasonably well-defined value for $\sin^2\theta_o$ can be obtained only if data from semileptonic neutral current reactions are used. Values obtained for $\sin^2\theta_o$ form different reactions are process-dependent. Therefore a process independent definition of $\sin^2\theta_W$ is needed. A convenient convention, proposed by Sirlin [8], is the definition in terms of the physical vectorboson masses (assuming $\rho = 1$)

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2}. \tag{2.7}$$

[+] In the following we put an index zero to indicate low energy parameters.

By including process - dependent radiative corrections $\sin^2\theta_W$ can be computed from $\sin^2\theta_o$ measured in a particular process [4]. As we shall discuss below, radiative corrections depend on as yet unknown parameters, namely, the t-quark mass m_t and the Higgs mass m_H. As standard values we shall assume

$$\rho_{oo} = 1, \quad m_t = 35 \text{ GeV} \quad \text{and} \quad m_H = 100 \text{ GeV}. \tag{2.8}$$

Recent updates [4] of the neutral current data yield from ν_μe-scattering

$$(1) \qquad \sin^2\theta_W = 0.2271 \pm 0.0143$$

$$(2) \qquad \sin^2\theta_W = 0.2272 \pm 0.0148, \quad \rho = 1.0014 \pm 0.0557$$

or, including the data from semileptonic reactions,

$$(1) \qquad \sin^2\theta_W = 0.2281 \pm 0.0042 \tag{2.9}$$

$$(2) \qquad \sin^2\theta_W = 0.2284 \pm 0.0069, \quad \rho = 1.0004 \pm 0.0080$$

for one- (1) and two- (2) parameter fits, respectively. The one-parameter fits assume $\rho_{oo} = 1$.

In the standard model the value of $\sin^2\theta_o$ defined by (2.6) is related to $\sin^2\theta_W$ by

$$\sin^2\theta_o/\sin^2\theta_W = 0.9948 \tag{2.10a}$$

assuming (2.8) and $M_Z = 92$ GeV [9,10]. For the same parameter values

$$\rho_o = 1.0051 \tag{2.10b}$$

is predicted. Formal expressions for the corrections involved in these numbers will be given later.

One important conclusion can be drawn from the experimental bounds on ρ. Namely, only Higgs doublets are tolerated by the NC-data and this

implies $\rho_{oo} = 1$. We therefore adapt (2.9) as the best estimate for $\sin^2\theta_W$ obtained from low energy NC-data.

The crucial consequences of the fact that masses are generated by the Higgs mechanism are

1) the mass coupling relations, which admit to predict the vectorboson masses in terms of the low energy parameters: to lowest order we have

$$M^2_{W_o} = \frac{\alpha\pi}{\sqrt{2}\, G_\mu} \frac{1}{\sin^2\theta_o}\,; \quad M^2_{Z_o} = \frac{M^2_{W_o}}{\rho_o \cos^2\theta_o} \tag{2.11}$$

or, using (2.9) and (2.10), we obtain

$$M_{W_o} = 78.26 \pm^{0.73}_{0.71} \text{ GeV} \quad M_{Z_o} = 88.78 \pm^{0.59}_{0.57} \text{ GeV}. \tag{2.12}$$

A useful parameter in this context is

$$A_o = \left(\frac{\alpha\pi}{\sqrt{2}\, G_\mu}\right)^{1/2} = 37.2805(3) \text{ GeV}. \tag{2.13}$$

The physical vectorboson masses differ from these values by calculable radiative corrections, which we will discuss below;

2) the interpretation of G_μ as being directly determined by the vacuum expectation value v of the Higgs field. In fact, G_μ does not depend on the gauge couplings and the measurement of G_μ establishes the existence of the Higgs condensate

$$v = (\sqrt{2}\, G_\mu)^{-1/2} = 246.221(3) \text{ GeV}. \tag{2.14}$$

This interpretation provides an explanation for an exceptional property of G_μ. In contrast to the running gauge couplings, G_μ is not renormalized (not running) up to a scale set by v.

We are now going to consider the higher order corrections which lead to predictions like (2.10) and which are necessary for a calculation of the physical vectorboson masses in terms of the low energy parameters. The low energy effective masses (2.11) represent the lowest order prediction.

The physical masses are determined formally by the real parts of the positions of the poles of the propagators. A simple calculatory scheme for including higher order effects is obtained as follows: the mass-coupling relations like (2.11) are exact for the bare (unrenormalized, dimensionally regularized) parameters in the Lagrangian. The bare parameters are related to the physical (experimental) ones by subtractions of appropriate counterterms (bare parameter shifts). For the masses we have

$$M_{Vb}^2 = M_V^2 + \delta M_V^2 = M_V^2\left(1 + \frac{\delta M_V^2}{M_V^2}\right);\ (V = Z,W) \tag{2.15}$$

for the fine structure constant

$$\delta\alpha_b = \alpha + \delta\alpha = \alpha\left(1 + \frac{\delta\alpha}{\alpha}\right) \tag{2.16i}$$

and for the μ-decay Fermi constant

$$G_{\mu b} = G_\mu + \delta G_\mu = G_\mu\left(1 + \frac{\delta G_\mu}{G_\mu}\right)\ . \tag{2.16ii}$$

Similarly, from $\nu_\mu e$ scattering we may define for the weak mixing angle

$$\sin^2\theta_b = \sin^2\theta_o + \delta\sin^2\theta_o = \sin^2\theta_o\left(1 + \frac{\delta\sin^2\theta_o}{\sin^2\theta_o}\right) \tag{2.16iii}$$

and for the ρ-parameter

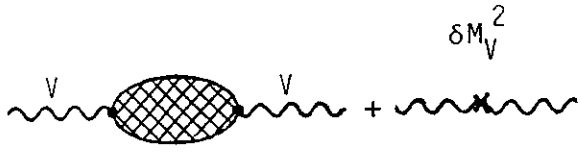
$$\rho_{ob} = \frac{G_{NCb}}{G_{\mu b}} = \rho_o\left(1 + \frac{\delta\rho_o}{\rho_o}\right) \tag{2.16iv}$$

The latter is unity within the standard model with $\rho_{oo} = 1$. The counterterms are higher order terms in which we expand in a formal power series to the desired order of perturbation theory. For example, starting from the bare analog of (2.11), we obtain for the vector boson masses

$$M_W^2 = \frac{\pi\alpha}{\sqrt{2}\,G_\mu}\frac{1}{\sin^2\theta_o}\left(1 + \frac{\delta\alpha}{\alpha} - \frac{\delta G_\mu}{G_\mu} - \frac{\delta\sin^2\theta_o}{\sin^2\theta_o} - \frac{\delta M_W^2}{M_W^2}\right)$$

$$M_Z^2 = \frac{M_W^2}{\rho_o\cos^2\theta_o}\left(1 - \frac{\delta\rho_o}{\rho_o} + \mathrm{tg}^2\theta_o\,\frac{\delta\sin^2\theta_o}{\sin^2\theta_o} - \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta M_W^2}{M_W^2}\right) \tag{2.17}$$

at one-loop order. These are the formulae for the mass shifts. The counterterms can be calculated in perturbation theory. The mass counterterms are determined by the transversal vector boson selfenergy functions



$$= -i\, g^{\mu\nu}\, \pi^{VV}(q^2) + \text{longitudinal part} \qquad (2.18a)$$

by the condition that the renormalized selfenergy vanishes on the mass shell. This yields

$$\frac{\delta M_V^2}{M_V^2} = \mathrm{Re}\,\frac{\pi^{VV}(M_V^2)}{M_V^2} = -A^{VV}(M_V^2)\ ; \quad (V = Z,W) \qquad (2.18b)$$

for the mass counterterms. The other counterterms are determined according to the definition of α, G_μ and $\sin^2\theta_o$ (Eq. (2.6)) from the electromagnetic form factor of the electron, the μ-decay rate and the $\overset{(-)}{\nu}_\mu e$ scattering cross sections, respectively, by the conditions that (up to known electromagnetic corrections (e.m.)) the Born approximations at zero momentum transfer are exact.

In Fig. 1, the renormalization conditions are shown diagramatically. The various selfenergies, form factors and box diagrams may be decomposed into invariant amplitudes. The precise definitions and explicit expression are given in the Appendix.

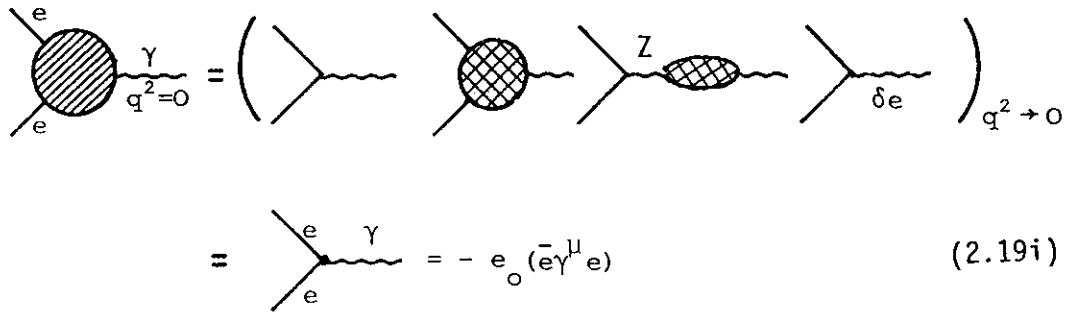
For the charged currents (CC) only left-handed amplitudes $A_L(1-\gamma_5)$ are possible owing to the fixed chirality of the neutrinos. In case of the neutral currents (NC) radiative effects are included in the two amplitudes A_1 and A_2 at the electron vertex such that

$$\begin{aligned} G_{no}(a + b\gamma_5) &= G_{no}(a_o + A_1 + (b_o + A_2)\gamma_5) \\ &= G_{no}(1 + A_L)(\sin^2\theta_o(1 + A_\kappa) - \tfrac{1}{4}(1 - \gamma_5)) \\ &\doteq G_{co}\rho_o(\kappa\sin^2\theta_o - \tfrac{1}{4}(1 - \gamma_5)) \end{aligned} \qquad (2.20)$$

where $A_L = \dfrac{A_2}{b_o}$, $A_\kappa = \dfrac{b_o A_1 - a_o A_2}{b_o \sin^2\theta_o}$ and hence

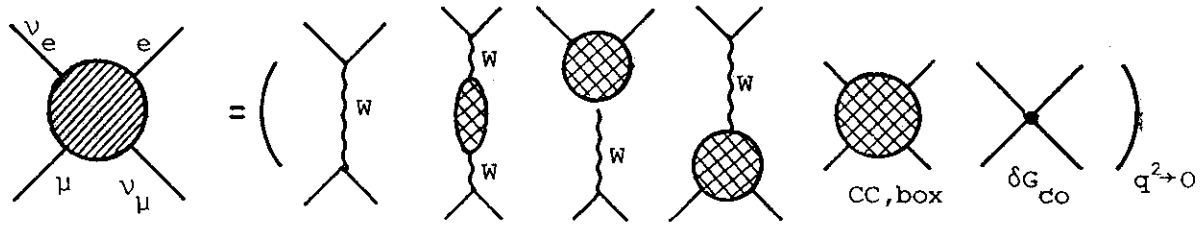
$$\rho_o = (1 + A_L), \quad \kappa = (1 + A_\kappa). \qquad (2.21)$$

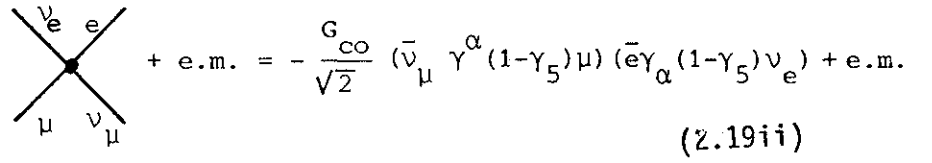
i)



$$= - e_o (\bar{e}\gamma^\mu e) \qquad (2.19i)$$

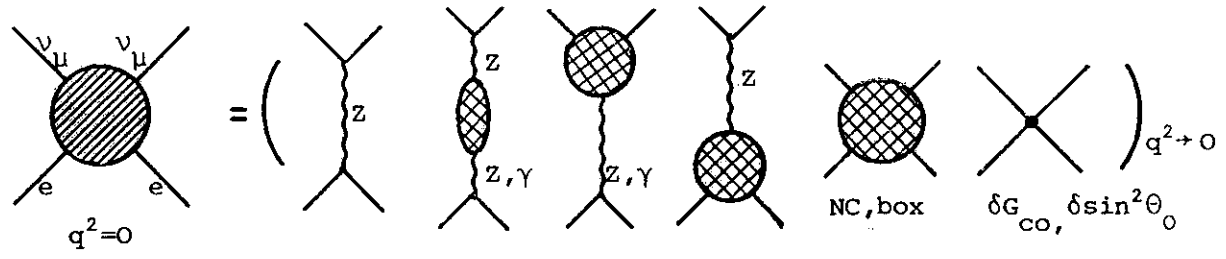
ii)

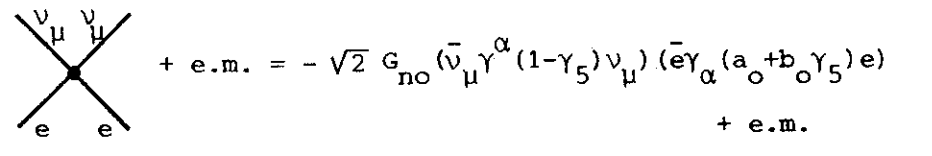




(2.19ii)

iii)





(2.19iii)

Fig. 1: Renormalization conditions defining the low energy parameters α, G_μ and $\sin^2\theta_o$.

The conditions (2.19) then read

$$\sqrt{2}\,G_\mu = \frac{1}{v_o^2} = \frac{1}{v_o^2}\frac{M_W^2}{M_W^2-q^2}\left(1 + 2\frac{\delta v_o^{-1}}{v_o^{-1}} + \left[\frac{-M_W^2}{M_W^2-q^2}A^{WW}(q^2) + A_L^{W\mu\nu_\mu}(q^2) + A_L^{We\nu_e}(q^2) + \frac{M_W^2-q^2}{M_W^2}A_{LCC}^{box}(q^2)\right]\right)\Big|_{q^2\to 0}$$

$$\sqrt{2}\,G_{no} = \frac{1}{v_o^2}\frac{M_Z^2}{M_Z^2-q^2}\left(1 + 2\frac{\delta v_o^{-1}}{v_o^{-1}} + \left[\frac{-M_Z^2}{M_Z^2-q^2}A^{ZZ}(q^2) + A_L^{Zee}(q^2) + A_L^{Z\nu\nu}(q^2) + \frac{M_Z^2-q^2}{M_Z^2}A_{L\,NC}^{box}(q^2)\right]\right)\Big|_{q^2\to 0}$$

$$\sin^2\theta_o = \sin^2\theta_o\left(1 + \frac{\delta\sin^2\theta_o}{\sin^2\theta_o} + \left[-2M_{W_o}^2\frac{A^{\gamma Z}(q^2)-A^{\gamma Z}(0)}{q^2} + 2M_{W_o}^2\left\{\frac{M_Z^2-q^2}{M_Z^2}A_1^{\gamma\nu\nu}(q^2) - A^{\gamma Z}(0)\right\}\frac{1}{q^2} + A_\kappa^{Zee}(q^2) + \frac{M_Z^2-q^2}{M_Z^2}A_{\kappa\,NC}^{box}(q^2)\right]\right)\Big|_{q^2\to 0}\cdot \qquad (2.22)$$

Accordingly, the counterterms are given by

$$\frac{1}{2}\frac{\delta\alpha}{\alpha} = \frac{\delta e_o}{e_o} = -(A_1^{\gamma ee}(0) + A_3^{\gamma ee}(0) + \frac{1}{2}(Z_\gamma - 1) + 2a_o A^{\gamma Z}(0))$$

$$\frac{\delta G_\mu}{G_\mu} = 2\frac{\delta v_o^{-1}}{v_o^{-1}} = A^{WW}(0) - A_L^{W\mu\nu_\mu}(0) - A_L^{We\nu_e}(0) - A_{LCC}^{box}(0)$$

$$-\frac{\delta\rho_o}{\rho_o} = 2\frac{\delta v_o^{-1}}{v_o^{-1}} - A^{ZZ}(0) + A_L^{Zee}(0) + A_L^{Z\nu\nu}(0) + A_{LNC}^{box}(0)$$

$$\frac{\delta\sin^2\theta_o}{\sin^2\theta_o} = 2M_{W_o}^2\left(\frac{dA^{\gamma Z}}{dq^2}(0) - \left(\frac{A_1^{\gamma\nu\nu}(q^2)-A^{\gamma Z}(0)}{q^2}\right)_{q^2\to 0}\right) + 2c_\theta^2 A_1^{\gamma\nu\nu}(0) - A_\kappa^{Zee}(0) - A_{\kappa NC}^{box}(0). \qquad (2.23)$$

The limit $q^2 \to 0$ exists. The poles at $q^2 \to 0$ from the higher order γ-exchange diagrams cancel, as it should be because the neutrino carries no charge. By definition the fermion wavefunction renormalization factors have been included in the form factors.

Since there are three independent low energy input parameters α, G_μ $\sin^2\theta_o$ there are three independent counterterms only. In the low energy parametrization the mass counterterms are determined by the counterterms (2.23) via the relations (2.11):

$$\frac{\delta M_{W_o}^2}{M_{W_o}^2} = \frac{\delta\alpha}{\alpha} - \frac{\delta G_\mu}{G_\mu} - \frac{\delta\sin^2\theta_o}{\sin^2\theta_o}$$

$$\frac{\delta M_{Z_o}^2}{M_{Z_o}^2} = \frac{\delta M_{W_o}^2}{M_{W_o}^2} - \frac{\delta\rho_o}{\rho_o} + tg^2\theta_o \frac{\delta\sin^2\theta_o}{\sin^2\theta_o} \quad . \tag{2.24}$$

It is often more convenient to use the QED-like parametrization. In this case, the fine structure constant α and the physical particle masses, in particular M_Z and M_W, are the independent parameters. The independent counterterms are $\frac{\delta\alpha}{\alpha}$ in Eq. (2.23) and the mass counterterms (2.18). $\frac{\delta\sin^2\theta_o}{\sin^2\theta_o}$ and $2\frac{\delta v_o^{-1}}{v_o^{-1}}$ are to be replaced by the dependent quantities

$$\frac{\delta\sin^2\theta_W}{\sin^2\theta_W} = ctg^2\theta_W\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right)$$

$$2\frac{\delta v^{-1}}{v^{-1}} = \frac{\delta\alpha}{\alpha} - \frac{\delta M_W^2}{M_W^2} - \frac{\delta\sin^2\theta_W}{\sin^2\theta_W} \quad . \tag{2.25}$$

For the prediction of vector boson processes (scale M_W) a parametrization which avoids large leading log corrections is in terms of G_μ and the physical particle masses. Here $\frac{\delta G_\mu}{G_\mu}$ replaces $\frac{\delta\alpha}{\alpha}$ as an independent counterterm in the previous QED-like scheme.

For the calculation of parameter renormalization like (2.17), one uses the fact that the bare parameters are independent of the parametrization chosen, thus counterterms related to different parametrizations appear in mixed form. Using (2.24), we may write (2.17) in the form

$$M_W^2 = M_{W_0}^2 \left(1 + \frac{\delta M_{W_0}^2}{M_{W_0}^2} - \frac{\delta M_W^2}{M_W^2}\right)$$

$$M_Z^2 = M_{Z_0}^2 \left(1 + \frac{\delta M_{Z_0}^2}{M_{Z_0}^2} - \frac{\delta M_Z^2}{M_Z^2}\right) \qquad (2.17')$$

revealing that the mass shifts are given by the differences of the mass counterterms evaluated in two different parametrizations. Usually the calculation of mass shifts is organized in a way slightly different from the one indicated above. According to (2.5) the masses are given by

$$M_W = \frac{gv}{2}\,, \quad M_Z = \frac{\sqrt{g^2+g'^2}\,v}{2\,\rho^{1/2}}\,.$$

A renormalization group analysis of the leading logarithmic corrections [11] shows that $\sqrt{2}\,G_\mu = \frac{1}{v_0^2}$ and ρ_0 are not running up to a scale set by v_0 in contrast to the known running of the gauge couplings g and g'. Accordingly one keeps

$$G_{\mu\text{eff}} = G_\mu\,, \quad \rho_{\text{eff}} = \rho_0 \qquad (2.26)$$

unrenormalized by definition.[+] The mass shifts are then a consequence of

[+]The adequacy of the corresponding bookkeeping can be tested experimentally. With

$$\hat{G}_\mu = \frac{12\pi\Gamma_{W\ell\nu}}{\sqrt{2}\,M_W^3}\,, \quad \hat{\rho} = \frac{M_W^3}{M_Z^3}\,\frac{2\Gamma_{Z\nu\nu}}{\Gamma_{W\ell\nu}} \qquad (2.26a)$$

we have two observables given in terms of vectorboson parameters, which coincide with the corresponding low energy parameters to lowest order. For the parameter values (2.8) a calculation yields for $M_Z = 92$ GeV [10]

$$\hat{G}_\mu = 1.1655(14) \times 10^{-5}\ \text{GeV}^{-2}\,, \quad \hat{\rho} = 1.0015(14)\,.$$

the renormalization of the gauge couplings and the latter are the objects of primary interest. Instead of g', usually the $U(1)_{em}$ coupling e is considered. An advantage of this procedure is that the renormalization group inspired leading log summation is simplest for the gauge couplings.

The physical definition of the gauge couplings is

$$\alpha = e_o^2/4\pi \ , \quad \alpha_g = g_o^2/4\pi = \frac{\alpha}{\sin^2\theta_o} \tag{2.27a}$$

in the low energy parametrization and

$$\alpha_W = \frac{\sqrt{2}G_\mu}{\pi} M_W^2 \sin^2\theta_W = \frac{\alpha}{1-\Delta r} \ , \ \alpha_{gW} = \frac{\sqrt{2}G_\mu}{\pi} M_W^2 = \frac{\alpha_g}{1-\Delta g} \tag{2.27b}$$

in the "high energy parametrization". Their ratios define the observable parameter shifts Δr and Δg [8]. The shift of $\sin^2\theta_W$ is given by

$$\sin^2\theta_W = \sin^2\theta_o \frac{1-\Delta g}{1-\Delta r} = \sin^2\theta_o(1 + \Delta). \tag{2.28}$$

In the following we consider the three independent observables $\varepsilon = \rho_o - 1$, Δr and $\Delta g = \Delta r - \Delta$. They are calculable measures of quantum fluctuations like the anomalous magnetic moments. From the bare identities

$$1 \equiv \frac{G_{NCb}}{G_{\mu b}} = \frac{\pi \alpha_b}{\sqrt{2}\, G_{\mu b}\, M_{Wb}^2} \ \frac{1}{\sin^2\theta_b} = \frac{\sin^2\theta_b}{\sin^2\theta_b}$$

we immediately obtain, using the counterterms (2.18) and (2.23),

$$\varepsilon = \frac{G_{NC}}{G_\mu} - 1 = \frac{\delta G_\mu}{G_\mu} - \frac{\delta G_{NC}}{G_{NC}} = -\frac{\delta\rho_o}{\rho_o}$$

$$\Delta r = 1 - \frac{\pi\alpha}{\sqrt{2}\, G_\mu M_W^2} \ \frac{1}{\sin^2\theta_W} = \frac{\delta\alpha}{\alpha} - \frac{\delta G_\mu}{G_\mu} - \frac{\delta M_W^2}{M_W^2} - \frac{\delta\sin^2\theta_W}{\sin^2\theta_W}$$

$$\Delta = \frac{\sin^2\theta_W}{\sin^2\theta_o} - 1 = \frac{\delta\sin^2\theta_o}{\sin^2\theta_o} - \frac{\delta\sin^2\theta_W}{\sin^2\theta_W} \ . \tag{2.29}$$

The most significant and most interesting contributions are the gauge boson selfenergies (vacuum polarizations). They are sensitive to all virtual

particles, including possibly existing unknown ones, which couple to the gauge bosons. We thus wirte the expressions in a form exhibiting the self-energy contributions as follows:

$$\varepsilon = A^{WW}(0) - A^{ZZ}(0) + R_{LCC} - R_{LCN}$$

$$\Delta r = -A^{\gamma\gamma}(0) - A^{WW}(0) - \frac{\delta M_W^2}{M_W^2} - ctg^2\theta_W\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + R_{em} - R_{LCC}$$

$$\Delta = 2M_W^2 \frac{dA^{\gamma Z}}{dq^2}(0) - ctg^2\theta_W\left(\frac{\delta M_Z^2}{M_Z^2} - \frac{\delta M_W^2}{M_W^2}\right) + R_{\kappa NC} \qquad (2.30)$$

where the remainders R_i include vertex corrections (we suppose the lepton wave function renormalizations to be added here), box diagram contributions and the γ-Z mixing term $A^{\gamma Z}(0)$ which exhibits vector boson contributions only. Explicitly, the remainders are given by

$$R_{em} = \left(\frac{\delta\alpha}{\alpha}\right)' = -2\left(A_1^{\gamma ee} + A_3^{\gamma ee} + 2a_o A^{\gamma Z}\right)$$

$$R_{LCC} = \left(\frac{\delta G_\mu}{G_\mu}\right)' = -\left(A_L^{W\mu\nu_\mu} + A_L^{We\nu_e} + A_{LCC}^{box}\right)$$

$$R_{LNC} = \left(\frac{\delta G_{NC}}{G_{NC}}\right)' = -\left(A_L^{Zee} + A_L^{Z\nu\nu} + A_{LNC}^{box}\right)$$

$$R_{\kappa NC} = \left(\frac{\delta \sin^2\theta_o}{\sin^2\theta_o}\right)' = -\left\{2M_W^2\left(\frac{A_1^{\gamma\nu\nu}(q^2) - A^{\gamma Z}(0)}{q^2}\right)_{q^2\to 0} - 2c_\theta^2 A_1^{\gamma\nu\nu}(0) + A_\kappa^{Zee} + A_{\kappa NC}^{box}\right\} \qquad (2.31)$$

where all amplitudes are evaluated at $q^2 = 0$.

In the $\overline{MS}$ scheme for the 't Hooft-Feynman gauge we have the simple expressions

$$R_{em} = K\left\{4s_\theta^2 \ln\frac{M_W^2}{\mu^2}\right\}$$

$$R_{LCC} = K\left\{4\ln\frac{M_W^2}{\mu^2} - \frac{4c_\theta^2+3}{2s_\theta^2}\ln c_\theta^2 - 6\right\}$$

$$R_{LNC} = K\left\{4c_\theta^2\ln\frac{M_W^2}{\mu^2} - \frac{24c_\theta^4 - 20c_\theta^2 + 15}{4c_\theta^2}\right\}$$

$$R_{\kappa NC} = K\left\{-4c_\theta^2\ln\frac{M_W^2}{\mu^2} + \frac{2}{3}\left(\ln\frac{M_W^2}{m_\mu^2}+1\right) + \frac{24c_\theta^4 - 14c_\theta^2 + 9}{4c_\theta^2}\right\} \tag{2.32}$$

if we neglect terms of $O(\alpha\frac{m_\ell^2}{M_W^2})$. In (2.32) $K = \frac{\alpha}{4\pi s_\theta^2}$, $s_\theta^2 = \sin^2\theta_W$ and $c_\theta^2 = \cos^2\theta_W$. Of course, the splitting into selfenergies plus remainders is neither gauge invariant nor finite. If we evaluate the expressions (2.32) at scale $\mu^2 = M_W^2$ they are relatively small since no large logs are involved, with one exception. $R_{\kappa NC}$, which contributes to the renormalization Δ of $\sin^2\theta_o$, includes a large logarithmic term proportional to $\ln(M_W^2/m_\mu^2)$ originating from the neutrino charge radius [12]. This infrared log cancels a substantial part of the sum of large ultraviolet logs from the light fermion contributions to $\frac{dA^{\gamma Z}}{dq^2}(0)$. This cancellation is the reason why $\sin^2\theta_W \simeq \sin^2\theta_o$ remains almost unrenormalized.[+] The infrared log carries over to the renormalization of the $SU(2)_L$ coupling α_g, since $\Delta g = \Delta r - \Delta$. Δr is free from infrared logs.

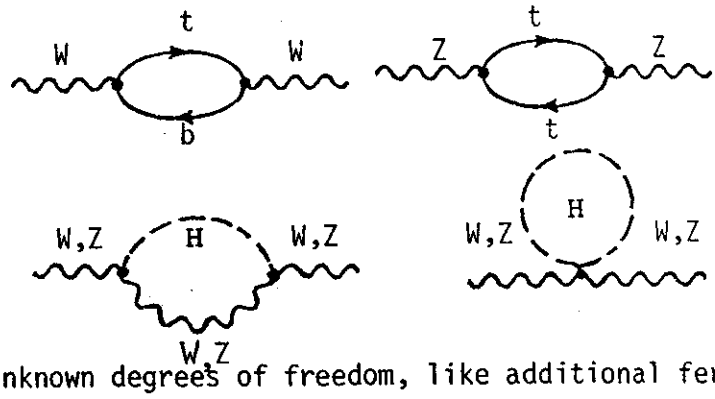
[+]Notice that the renormalization group prediction

$$\sin^2\theta(\mu^2) = \sin^2\theta_o\left\{1 + \frac{\alpha}{3\pi}\sum_{m_f<\mu}\left(Q_f^2 - \frac{1}{4\sin^2\theta_o}|Q_f|\right)\ln\frac{\mu^2}{m_f^2}\right\};\ \mu \lesssim M_W$$

would yield for $\mu = M_Z = 92$ GeV and $\sin^2\theta_o = 0.23$ $\Delta_{UV} \simeq -0.040$ whereas the infrared log contributes $\Delta_{IR} \simeq 0.022$. By inclusion of the "nonleading" terms we finally obtain for the sum $\Delta \simeq 0.005$!

In principle, we are ready now to compute the parameter shifts from the formulae (2.30), (2.32) and the selfenergy functions given in the Appendix.

For the calculation of the selfenergy functions we must specify some values for the as yet unknown parameters m_t and m_H, which are needed for the evaluation of the diagrams



Additional unknown degrees of freedom, like additional fermions, Higgses or SUSY-particles, could be included as well here. It is the sensitivity of parameter shifts to these unknown contributions which make precision measurements so important.

These interesting effects, typically of the order of 100 MeV in the mass shifts (for reasonable assumptions about the values of the unknown parameters), could be obscured by uncertainties in the theoretical predictions which we are going to discuss in the next two sections.

At SLC/LEP I the Z-boson mass will be determined very accurately (see Table 1) such that the best set of parameters for theoretical predictions will be

$$\alpha,\ G_\mu \quad \text{and} \quad M_Z\ .$$

The predictions for M_W and $\sin^2\theta_W$ are

$$\begin{aligned} M_W^2 &= M_Z^2 \cdot \frac{1}{2}\left(1 + \sqrt{1 - 4A^2/M_Z^2}\right) \\ \sin^2\theta_W &= 1 - \frac{M_W^2}{M_Z^2} = \frac{1}{2}\left(1 - \sqrt{1 - 4A^2/M_Z^2}\right) \end{aligned} \tag{2.33}$$

where $A^2 = \frac{\pi\alpha}{\sqrt{2}G_\mu}\ \frac{1}{1-\Delta r}$. Ignoring the small errors in α and G_μ we have

$$\frac{\delta M_W}{M_W} = \frac{\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W} \left(\left[\frac{\delta M_Z}{M_Z}\right]^2 + \left[\frac{\sin^2\theta_W}{\cos^2\theta_W} \frac{\delta \Delta r}{2(1-\Delta r)}\right]^2 \right)^{1/2}$$

$$\frac{\delta \sin^2\theta_W}{\sin^2\theta_W} = \frac{2\cos^2\theta_W}{\cos^2\theta_W - \sin^2\theta_W} \left(\left[\frac{\delta M_Z}{M_Z}\right]^2 + \left[\frac{\delta \Delta r}{2(1-\Delta r)}\right]^2 \right)^{1/2} \qquad (2.34)$$

for the propagation of the experimental error δM_Z in M_Z and of the theoretical error $\delta\Delta r$ in Δr. The errors are assumed to be added in quadrature. For $\sin^2\theta_W = 0.23$ we get

$$\frac{\delta M_W}{M_W} = 1.43 \left(\left[\frac{\delta M_Z}{M_Z}\right]^2 + (0.16\delta\Delta r)^2 \right)^{1/2}$$

$$\frac{\delta \sin^2\theta_W}{\sin^2\theta_W} = 2.85 \left(\left[\frac{\delta M_Z}{M_Z}\right]^2 + (0.54\delta\Delta r)^2 \right)^{1/2} \qquad (2.35)$$

or $\delta M_W \simeq 25$ MeV and $\delta \sin^2\theta_W \simeq 0.00014$ for $\delta M_Z = 20$ MeV $\delta\Delta r \simeq 0$ and $M_Z = 92$ GeV.

3. Estimate of Hadronic Contributions and Hadronic Uncertainties

According to (2.30) the parameter renormalizations for α and α_g

$$\Delta r = -A^{\gamma\gamma}(0) + \ldots = e^2 \pi^{\gamma\gamma}(0) + \ldots$$

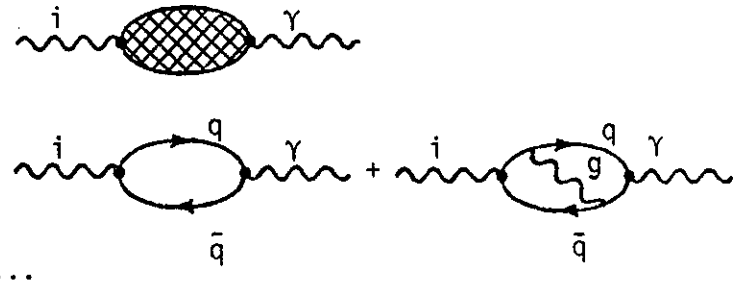
$$\Delta g = -A^{\gamma\gamma}(0) - 2M_W^2 \frac{dA^{\gamma Z}}{dq^2}(0) + \ldots = g^2 \pi^{3\gamma}(0) + \ldots \qquad (3.1)$$

include light quark contributions at low q^2 where non-perturbative strong interaction effects cannot be ignored. Since no reliable theoretical predictions are available for these contributions experimental data must be used for their evaluation. As we have indicated in (3.1), we may represent the relevant self-energy functions, using (2.1) and (2.2), in terms of vacuum polarization amplitudes of the hadronic parts of the conserved

electromagnetic current J^γ and the $SU(2)_L$ vector current component J^3:

$$\Pi^{i\gamma}_{\mu\nu}(q^2) = i\int d^4x\, e^{iqx} \langle 0|T\, J^i(x)\, J^\gamma(0)|0\rangle$$

$$= (q_\mu q_\nu - q^2 g_{\mu\nu})\, \pi^{i\gamma}(q^2)\ ;\quad i = \gamma, 3 \qquad (3.2)$$



The amplitudes

$$\Delta\pi^{i\gamma}(q^2) = \pi^{i\gamma}(q^2) - \pi^{i\gamma}(0) \qquad (3.3)$$

satisfy convergent dispersion relations

$$\Delta\pi^{i\gamma}(q^2) = \frac{q^2}{12\pi^2}\left(\int_{4m_\pi^2}^{E_1^2} + \int_{E_1^2}^{\infty}\right) \frac{ds\, R^{i\gamma}(s)}{s(s-q^2)}\ . \qquad (3.4)$$

Since

$$R^{\gamma\gamma}(s) = \frac{\sigma_{tot}(e^+e^- \to \gamma^* \to \text{hadrons})}{\sigma(e^+e^- \to \gamma^* \to \mu^+\mu^-)} \qquad (3.5)$$

is a fairly well-known experimental quantity $\Delta\pi^{\gamma\gamma}(q^2)$ can be computed using

i) e^+e^- -data up to an energy E_1

ii) perturbative QCD for the high energy tail $E > E_1$.

E_1 should be chosen not too large such that the one-photon exchange approximation is valid. For the calculation of Δr and Δg a natural choice for q^2 is $q^2 = M_Z^2$. $\pi^{i\gamma}(M_Z^2)$ can be calculated using perturbative QCD such that $\pi^{i\gamma}(0) = \pi^{i\gamma}(M_Z^2) - \Delta\pi^{i\gamma}(M_Z^2)$ is determined by the hadronic data

up to the experimental errors. In Ref. [9] I have presented a detailed updated calculation of the quantities

$$\Delta r_{QED,had} = -e^2\,\Delta\pi^{\gamma\gamma}(M_Z^2), \quad \Delta g_{3\gamma,had} = -g^2\,\Delta\pi^{3\gamma}(M_Z^2) \qquad (3.6)$$

taking into account the e^+e^- - data up to $E_1 = 40$ GeV. A short account of the results is listed in Tab. 2. These results show that the only possibility to reduce the hadronic error would be to measure $\sigma_{tot}(e^+e^- \to \text{had})$ with higher precision in the energy range from 1 GeV to M_T. Notice that a reduction of the relative total error to 5% would improve the result from 2.6% to 2% relative error only, because the background contribution below the charm threshold which carries the largest error is relatively small.

Table 2: Contributions to the dispersion integral (3.4) with statistical and systematic errors. Errors from different regions are added in quadrature.

Contribution	$-\Delta\pi^{\gamma\gamma}(M_Z^2) \times 10^3$	relative error (%)
Resonances	46.89(49)(1.13)	2.6
Background $E < M_{J/\psi}$	35.58(73)(3.60)	10.3
$M_{J/\psi} < E < M_T$	78.13(2.52)(4.74)	6.9
$M_T < E < 40$ GeV	103.21(77)(4.40)	4.3
e^+e^- - data	263.80(2.79)(7.48)	3
Perturbative QCD 40 GeV < E	48.33(18)(37)	1
Total	312.13(2.80)(7.50)	2.6

In contrast to $\Delta\pi^{\gamma\gamma}$ a direct evaluation of $\Delta\pi^{3\gamma}$ from e^+e^- - data is not possible, because the various flavors are weighted differently in $\Delta\pi^{3\gamma}$. However, there is an approxiamte relation

$$\Delta\pi^{3\gamma} = 1/2\ \Delta\pi^{\gamma\gamma}_{(uds)} + 3/8\ \Delta\pi^{\gamma\gamma}_{(c)} + 3/4\ \Delta\pi^{\gamma\gamma}_{(b)} \qquad (3.7)$$

which assumes $SU(3)_{flavor}$ symmetry for the light quarks (uds) and the Okubo-Zweig-Iizuka rule for the heavy quarks. One therefore can safely evaluate $\Delta\pi^{3\gamma}$ if we are able to separate charm and bottom contributions from the e^+e^--data. This is not a serious problem. We may collect corresponding resonance contributions and separate the remaining background according to RG improved perturbation theory. The results are given in Tab. 3. As a final result for 5 flavors, we have

$$\Delta\pi^{\gamma\gamma}(M_Z^2) = -0.31213(280)(749) + \frac{11}{3} K_s \ln \frac{M_{Z_o}^2}{M_Z^2}$$

$$\Delta\pi^{3\gamma}(M_Z^2) = -0.14873(125)(354) + \frac{7}{4} K_s \ln \frac{M_{Z_o}^2}{M_Z^2} \qquad (3.8)$$

where $M_{Z_o} = 93.7$ GeV is the value for which we have computed the dispersion integral and $K_s = \frac{1}{12\pi^2}\left(1 + \frac{\alpha_s(M_Z)}{\pi}\right)$. We thus obtain

$$\Delta r^{(5)}_{QED,had} = 0.0286 \pm 0.0007, \quad \Delta g^{(5)}_{3\gamma,had} = 0.0619 \pm 0.0016 \qquad (3.9)$$

or, including all perturbative (5) quark contributions in (2.30),

$$\Delta r^{(5)}_{had} = 0.0326 \pm 0.0007, \quad \Delta g^{(5)}_{had} = 0.0602 \pm 0.0016 \qquad (3.10)$$

at $M_Z = 93$ GeV for $\sin^2\theta_o = 0.22$.

Table 3: Contributions of different flavors to $\Delta\pi^{\gamma\gamma}(M_Z^2)$ (with statistical and systematic errors)

	$-\Delta\pi^{\gamma\gamma}_{(uds)} \times 10^3$	$-\Delta\pi^{\gamma\gamma}_{(c)} \times 10^3$	$-\Delta\pi^{\gamma\gamma}_{(b)} \times 10^3$
$E < M_{J/\psi}$	82.47(88)(3.77)		
$M_{J/\psi} \le E < M_\Upsilon$	44.81(1.74)(3.71)	33.32(1.82)(2.95)	
$M_\Upsilon \le E < 40$ GeV	55.70(63)(3.62)	37.21(42)(2.42)	10.30(12)(60)

For a comparison with previous results, see Ref. [9]. Two remarks are in order concerning the result for $\Delta r^{(5)}_{QED,had}$. Firstly, the result perfectly agrees with earlier estimates by Paschos and Penso [13]. Secondly, the hadronic error $\delta_h = 0.0007$ (= 2.5%) has been reduced by about a factor of 2 to 3 relative to $\delta_h = 0.002$ (= 7%) given by Marciano and Sirlin [7], for example. This improvement has been possible because experimental data have been improved, particularly in the region above the Υ-threshold which gives a large contribution to $\Delta r^{(5)}_{QED,had}$. Frequently [14], the integral (3.4) is evaluated by splitting it according to $\Delta(M_Z^2) = \Delta(-Q_0^2) + [\Delta(M_Z^2) - \Delta(-Q_0^2)]$. The first term, which is sensitive to low energy contributions, is evaluated at spacelike momenta from e^+e^- - data. For the second term lowest order perturbation theory is used. This yields [14], for $Q_0^2 = 79\ \mathrm{GeV}^2$,

$$\Delta r^{(5)}_{QED,had} = 0.0145 \pm 0.0013 + 0.0129 = 0.0274 \pm 0.0013.$$

This result slightly underestimates the hadronic effects, since (in this approach) the large perturbative term is sensitive to higher order QCD-corrections, which have been neglected.

The errors in the predictions of M_W and $\sin^2\theta_W$ (2.33), due to the hadronic ambiguity in Δr, are $\delta M_W \simeq 13(24)$ MeV and $\delta \sin^2\theta_W \simeq 0.00025(0.00046)$ for $\delta\Delta r = 0.0007(0.0013)$.

4. Estimates of Higher Order Uncertainties

We now consider uncertainties due to unknown higher order effects. For example, $1 + \Delta r$ may be compared with its geometrically summed form $1/(1 - \Delta r)$. Naively, an uncertainty $\delta = (\Delta r)^2 \approx 0.005$, yielding $\delta M_W \simeq 92$ MeV, is obtained since the two-loop contribution to Δr is not yet known. This large uncertainty may be reduced, however, by summing the leading logarithms according to the renormalization group (RG) [11].

In the leading log approximation

$$\Delta\alpha^{-1} = \frac{1}{\alpha(0)} - \frac{1}{\alpha(\mu^2)} = \frac{1}{3\pi} \sum_{m_f<\mu} Q_f^2 \ln(\mu^2/m_f^2) \ ; \ \mu \le M_W \qquad (4.1)$$

the r.h.s. is independent of electroweak couplings and hence the l.h.s. is renormalization scheme independent. Eq. (2.27b) which may be written as

$$\Delta\alpha^{-1} = \frac{1}{\alpha} - \frac{1}{\alpha_W} = \frac{\Delta r}{\alpha}(\alpha, M_W, M_Z) \tag{4.2}$$

is the generalization of (4.1) to the case where Δr includes, besides leading logs, also finite terms determined by physical on-shell renormalization conditions. The r.l.s. of (4.2) is weakly scheme dependent in this case and this scheme dependence may be used for an estimate of higher order uncertainties (educated guess). I use four different physical parametrizations and determine $\Delta\alpha^{-1}$ as an average and take the square deviation as the uncertainty. An example is presented in Tab. 4. The same method applies to the $SU(2)_L$ coupling where

$$\Delta\alpha_g^{-1} = \frac{1}{\alpha_g} - \frac{1}{\alpha_{gW}} = \frac{\Delta g}{\alpha_g}(\alpha, M_W, M_Z) \simeq \frac{1}{12\pi} \sum_{m_f<\mu} |Q_f| \ln(\mu^2/m_f^2) \tag{4.3}$$

is also weakly scheme dependent. For the parameters used in Tab. 4 I obtain

$$\Delta\alpha^{-1} = 9.74(15)(10), \quad \Delta\alpha_g^{-1} = 2.068(17)(47)$$

Table 4: Scheme dependence (M_Z = 93 GeV, m_H = 100 GeV, m_t = 35 GeV)

Input Parameters	$\Delta\alpha^{-1}$	$\Delta r = \alpha\Delta\alpha^{-1}(\alpha_W\Delta\alpha^{-1})$	δM_W(MeV)
$\alpha, G_\mu, \sin^2\theta_o$	9.671	0.0706	-9.2
α, G_μ, M_Z	9.872	0.0720	+16.6
α, M_W, M_Z	9.713	0.0709	-3.7
G_μ, M_W, M_Z	9.713	(0.0763)	-3.7
average	9.742	0.0711(0.0765)	0
square deviation	0.153	0.0011	±20
hadronic error	0.102	0.0007	±13

where the first error is due to higher order effects, the second one is the hadronic ambiguity. From $\Delta\alpha^{-1}$ and $\Delta\alpha_g^{-1}$ one can compute α_W/α and α_{gW}/α_g, respectively, which together with the relation (2.27, 2.28) admit to calculate any quantity within the estimated errors for any set of input parameters. Notice that (2.28) provides the proper leading log summation in terms of Δg and Δr. The reader may find my estimate of "uncertainties due to higher order effects" to be too "pessimistic". Therefore, let me point out the precise meaning of using "scheme averaged" values for $\Delta\alpha^{-1}$ and $\Delta\alpha_g^{-1}$ and corresponding square deviation errors in theoretical predictions, which is the following: whatever choice is made for the input parameters (an independent set of quantities measured at scales below, say, $v \simeq 250$ GeV) the (one-loop, leading log summed) prediction for an observable must lie within the estimated errors given. There is no proof, of course, that an actual two-loop calculation could not reveal some "surprises". For another discussion of the scheme dependence of theoretical predictions, in particular for the asymmetries, see Ref. [15].

5. Results of Parameter Shift Calculations

Parameter shifts have been calculated by many authors [7]. Results of calculations including the error estimates discussed in the previous sections may be found in Refs. [9,10]. The consensus reached by different groups active in this field is remarkable by now. Here we present, in Tables 5 and 6, values for Δr and Δg calculated for various values of M_Z, m_H and m_t. In Figures 2 and 3, we have plotted Δr and Δg as functions of m_t. Accurate predictions for the masses M_Z and M_W from the low energy parameters heavily rely on a careful analysis of the NC data because a good value for $\sin^2\theta_W$ is needed. This issue is discussed in detail by P. Langacker in these proceedings.

The predictions

$$\begin{aligned} M_W &= 80.2 \pm 1.1 \text{ GeV} \\ M_Z &= 91.6 \pm 0.9 \text{ GeV} \qquad \text{(Amaldi et al. [4])} \\ \sin^2\theta_W &= 0.233 \pm 0.006 \end{aligned} \tag{5.1}$$

and

$$M_W = 80.99 \pm 0.72$$

$$M_Z = 92.18 \pm 0.59 \qquad \text{(Costa et al. [4])}$$

$$\sin^2\theta_W = 0.2281 \pm 0.004 \qquad (5.2)$$

are in perfect agreement with the experimental values [3]

UA1:
$$M_W = 82.7 \pm 1.0 \pm 2.7 \text{ GeV}$$
$$M_Z = 93.1 \pm 1.0 \pm 3.1 \text{ GeV}$$
$$\sin^2\theta_W = 0.214 \pm 0.006 \pm 0.015$$

UA2:
$$M_W = 80.2 \pm 0.6 \pm 1.6 \text{ GeV}$$
$$M_Z = 91.5 \pm 1.2 \pm 1.7 \text{ GeV}$$
$$\sin^2\theta_W = 0.232 \pm 0.003 \pm 0.008$$

or, with the weighted average

$$M_W = 80.76 \pm 1.72 \text{ GeV}$$

$$M_Z = 91.59 \pm 2.14 \text{ GeV}$$

$$\sin^2\theta_W = 0.2279 \pm \begin{matrix} 0.0089 \\ 0.0084 \end{matrix} . \qquad (5.3)$$

If we assume M_Z to be given (from SCL/LEP I) precise predictions are possible, independent of the inaccuracies of neutral current data. In particular, the W-mass and $\sin^2\theta_W$ can be calculated very precisely using Eqs. (2.33). In Figs. 4 and 5, we have depicted M_W and $\sin^2\theta_W$ as functions of M_Z together with the data points. The importance of including radiative corrections may be judged by a glance at these curves. Improved upper bounds on the top mass and on large doublet mass splittings [16] can be obtained. In Tab. 7 we compare "theoretical" errors, <u>including</u> the indicated input error of M_Z, with the changes of parameters caused by variations of m_H and m_t. The conclusions are the following:

Table 7: Uncertainties versus parameter shifts caused by variation of m_H and m_t (errors added quadratically)

M_Z = 93 GeV	δM_W	$\delta \sin^2\theta_W$	$\delta \sin^2\theta_o$	$\delta \rho_o$
reference value	82004 (MeV)	0.2225	0.2218	1.0053
M_Z = ± 0 MeV	±24	±0.0004	±0.0004	±0.0004
M_Z = ±50 MeV	±65	±0.0006	±0.0005	±0.0004
M_Z = ±20 MeV	±33	±0.0005	±0.0004	±0.0004
m_H = 10 (GeV)	+83	-0.0016	-0.0008	+0.0004
500	-112	+0.0021	+0.0008	-0.0012
1000	-171	+0.0032	+0.0011	-0.0020
m_t = 60 (GeV)	-18	+0.0003	-0.0002	+0.0008
90	+224	-0.0042	-0.0008	+0.0024
120	+418	-0.0080	-0.0016	+0.0046
150	+631	-0.0120	-0.0025	+0.0074
300	+2219	-0.0426	-0.0098	+0.0308

i) Hadronic and higher order errors are of the same size and comparable to uncertainties due to the experimental error of M_Z achievable with a polarized e^--beam at LEP.

ii) The combinded errors are small enough, such that important information on genuine "weak" corrections, as given in Tab. 7, can be obtained at LEP.

Table 5: Δr for various m_t and m_H.Input parameters : α, G_μ, M_Z.
The error includes hadronic and higher order uncertainties.

	M_Z	m_H=10	m_H=100	m_H=500	m_H=1000
m_t=30	89	0.0651(10)	0.0694(12)	0.0750(17)	0.0778(19)
	90	0.0654(10)	0.0698(13)	0.0755(17)	0.0785(20)
	91	0.0657(10)	0.0701(13)	0.0761(17)	0.0791(20)
	92	0.0659(11)	0.0705(13)	0.0766(17)	0.0798(20)
	93	0.0662(11)	0.0709(13)	0.0772(18)	0.0804(20)
	94	0.0665(11)	0.0713(13)	0.0777(18)	0.0810(21)
	95	0.0667(11)	0.0716(14)	0.0782(18)	0.0817(21)
m_t=60	89	0.0647(16)	0.0689(19)	0.0745(23)	0.0773(25)
	90	0.0653(17)	0.0697(20)	0.0754(24)	0.0784(26)
	91	0.0660(18)	0.0704(21)	0.0763(26)	0.0794(28)
	92	0.0667(19)	0.0712(22)	0.0773(27)	0.0804(29)
	93	0.0674(20)	0.0720(23)	0.0783(28)	0.0815(30)
	94	0.0681(21)	0.0728(25)	0.0792(30)	0.0826(32)
	95	0.0688(22)	0.0737(26)	0.0802(31)	0.0837(33)
m_t=90	89	0.0536(15)	0.0580(14)	0.0638(11)	0.0668(08)
	90	0.0535(15)	0.0580(14)	0.0640(12)	0.0671(09)
	91	0.0534(15)	0.0581(14)	0.0642(12)	0.0674(09)
	92	0.0534(16)	0.0581(15)	0.0644(12)	0.0677(09)
	93	0.0534(16)	0.0582(15)	0.0646(13)	0.0680(09)
	94	0.0534(17)	0.0583(16)	0.0649(14)	0.0684(09)
	95	0.0534(17)	0.0584(17)	0.0652(15)	0.0688(10)
m_t=120	89	0.0439(22)	0.0484(22)	0.0544(20)	0.0575(20)
	90	0.0432(22)	0.0479(22)	0.0540(21)	0.0572(20)
	91	0.0425(23)	0.0473(22)	0.0536(21)	0.0569(21)
	92	0.0418(23)	0.0467(23)	0.0532(22)	0.0567(21)
	93	0.0412(23)	0.0461(23)	0.0528(22)	0.0564(21)
	94	0.0405(23)	0.0456(23)	0.0525(22)	0.0561(22)
	95	0.0398(23)	0.0450(23)	0.0521(23)	0.0559(22)
m_t=150	89	0.0327(27)	0.0374(28)	0.0435(29)	0.0468(31)
	90	0.0314(27)	0.0362(28)	0.0426(30)	0.0460(32)
	91	0.0301(27)	0.0351(28)	0.0416(30)	0.0451(33)
	92	0.0288(27)	0.0339(28)	0.0406(30)	0.0443(34)
	93	0.0275(27)	0.0327(29)	0.0396(31)	0.0434(35)
	94	0.0262(28)	0.0315(29)	0.0386(31)	0.0425(36)
	95	0.0248(28)	0.0303(29)	0.0376(32)	0.0416(37)

Table 6: Δg for various m_t and m_H.Input parameters : α, G_μ, M_Z.
The error includes hadronic and higher order uncertainties.

	M_Z	m_H=10	m_H=100	m_H=500	m_H=1000
m_t=30	89	0.0591(15)	0.0605(15)	0.0611(15)	0.0613(15)
	90	0.0612(15)	0.0626(15)	0.0632(15)	0.0634(15)
	91	0.0632(16)	0.0646(15)	0.0653(16)	0.0655(16)
	92	0.0652(16)	0.0667(16)	0.0674(16)	0.0676(16)
	93	0.0672(17)	0.0688(16)	0.0695(17)	0.0697(17)
	94	0.0692(17)	0.0708(17)	0.0716(17)	0.0718(17)
	95	0.0713(18)	0.0729(18)	0.0737(18)	0.0739(18)
m_t=60	89	0.0579(14)	0.0593(14)	0.0599(14)	0.0601(14)
	90	0.0598(15)	0.0612(14)	0.0618(14)	0.0620(14)
	91	0.0616(15)	0.0631(15)	0.0637(15)	0.0639(15)
	92	0.0635(15)	0.0650(15)	0.0657(15)	0.0659(15)
	93	0.0653(16)	0.0669(16)	0.0676(16)	0.0678(16)
	94	0.0672(16)	0.0688(16)	0.0695(16)	0.0697(16)
	95	0.0690(17)	0.0707(17)	0.0715(17)	0.0717(17)
m_t=90	89	0.0605(14)	0.0619(14)	0.0625(14)	0.0627(14)
	90	0.0624(15)	0.0639(14)	0.0645(14)	0.0647(14)
	91	0.0644(15)	0.0658(15)	0.0665(15)	0.0667(15)
	92	0.0663(15)	0.0678(15)	0.0685(15)	0.0687(15)
	93	0.0682(16)	0.0697(16)	0.0704(16)	0.0706(16)
	94	0.0701(16)	0.0717(16)	0.0724(16)	0.0726(16)
	95	0.0720(17)	0.0736(17)	0.0744(17)	0.0746(17)
m_t=120	89	0.0616(14)	0.0630(14)	0.0636(14)	0.0638(14)
	90	0.0636(15)	0.0650(15)	0.0656(15)	0.0658(15)
	91	0.0655(15)	0.0670(15)	0.0677(15)	0.0678(15)
	92	0.0675(15)	0.0690(15)	0.0697(15)	0.0699(15)
	93	0.0694(16)	0.0710(16)	0.0717(16)	0.0719(16)
	94	0.0714(16)	0.0730(16)	0.0737(16)	0.0739(16)
	95	0.0734(17)	0.0750(17)	0.0758(17)	0.0760(17)
m_t=150	89	0.0624(14)	0.0638(14)	0.0644(14)	0.0646(14)
	90	0.0644(15)	0.0658(15)	0.0664(15)	0.0666(15)
	91	0.0664(15)	0.0678(15)	0.0685(15)	0.0687(15)
	92	0.0683(15)	0.0699(15)	0.0705(15)	0.0707(15)
	93	0.0703(16)	0.0719(16)	0.0726(16)	0.0728(16)
	94	0.0723(16)	0.0739(16)	0.0747(16)	0.0749(16)
	95	0.0743(17)	0.0759(17)	0.0767(17)	0.0769(17)

Figure 2

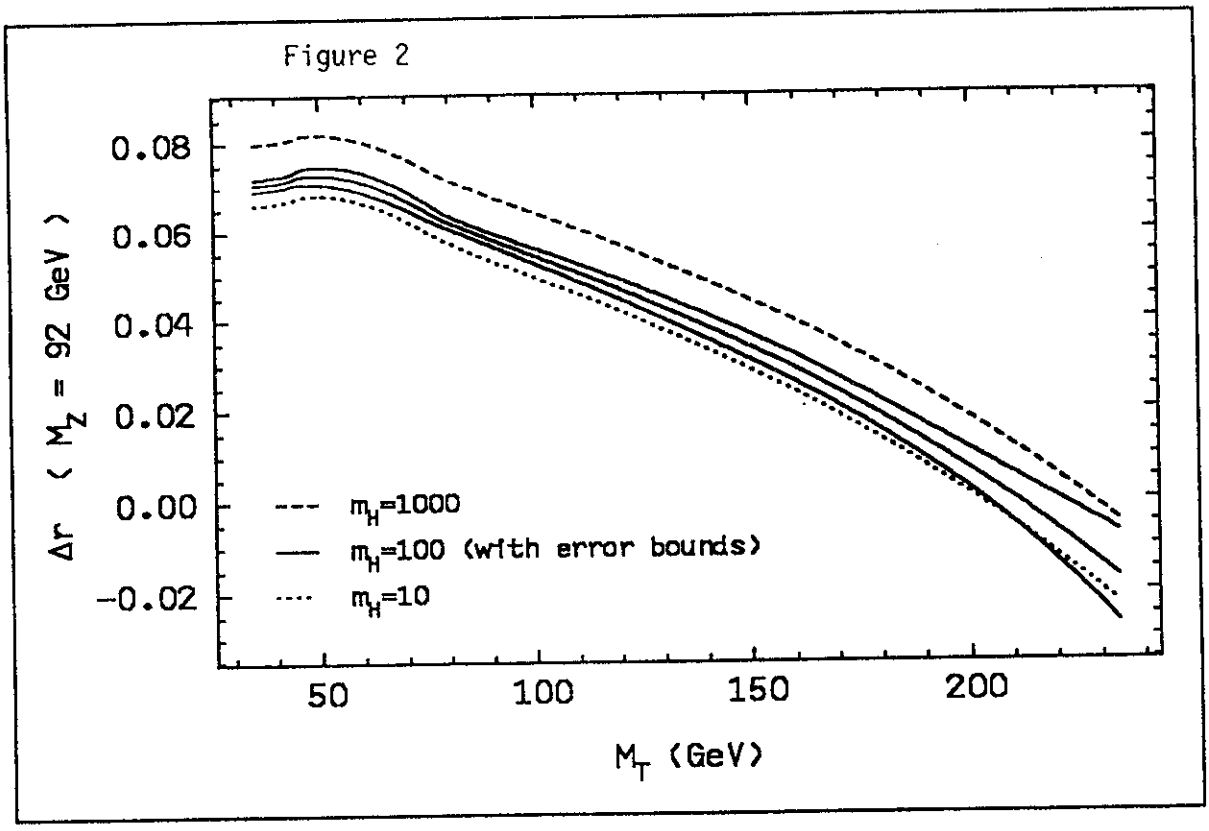


Figure 3

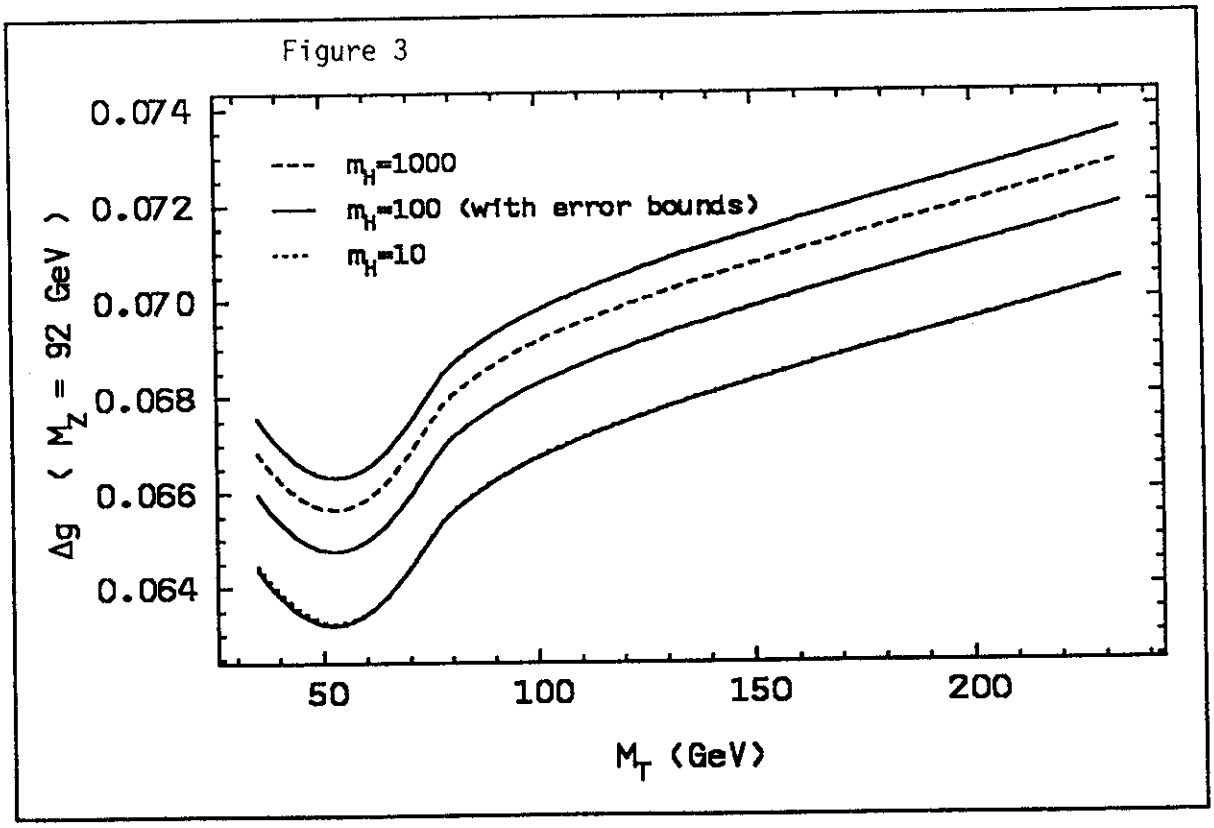


Figure 4a

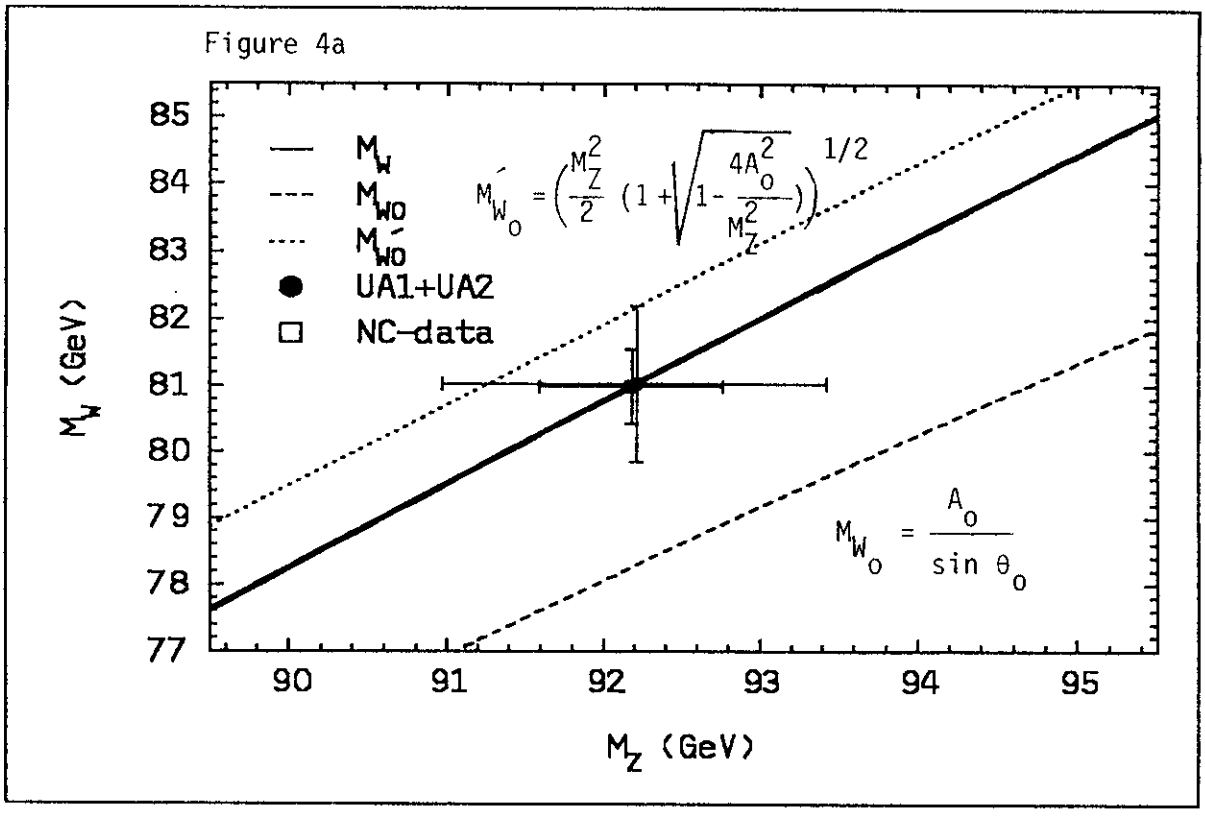


Figure 4b

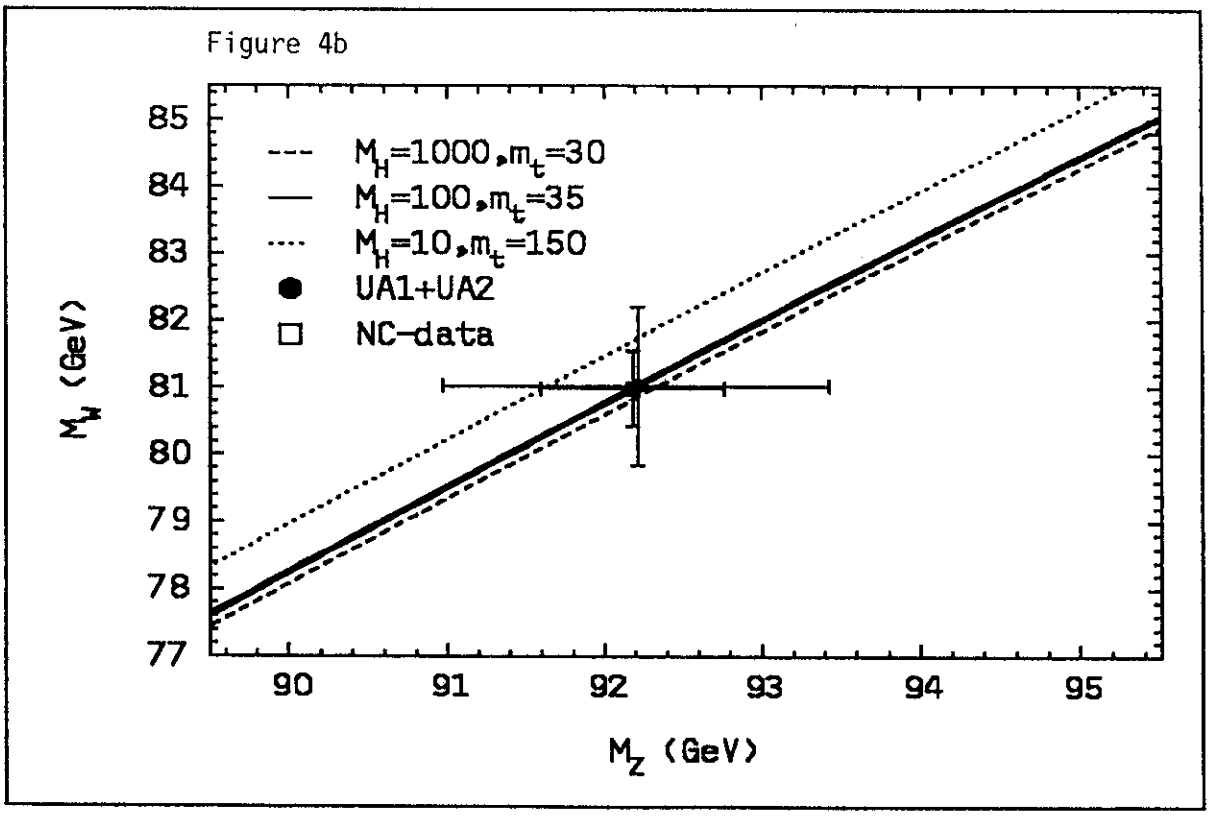


Figure 5a

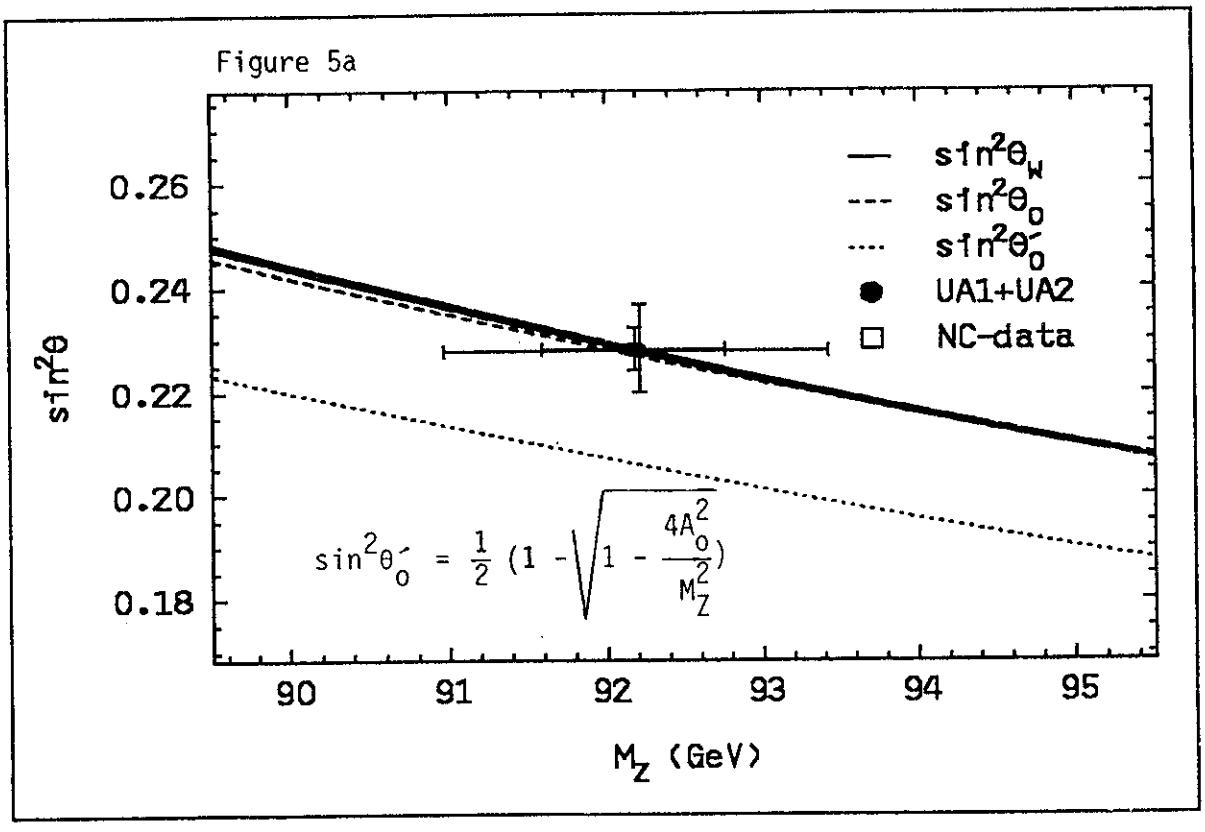
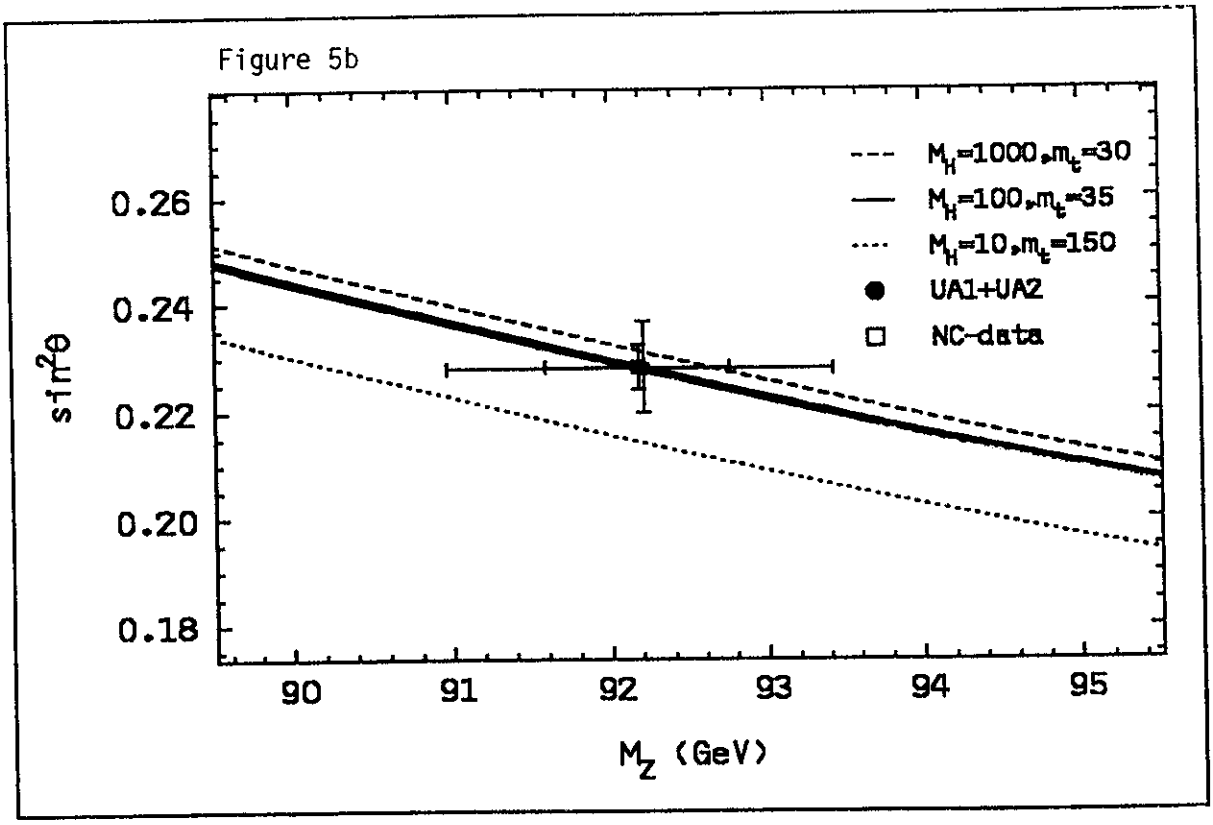


Figure 5b



6. The Vector Boson Widths

We expect that the total widths of the vector bosons will be determined at SLC/LEP with a precision $\delta\Gamma_V \simeq M_V$ ($V = Z,W$). Thus a precision of 8×10^{-3} for Γ_Z could be reached which is by M_Z/Γ_Z less precise than the precision of 2×10^{-4} in the measurement of M_Z. A massless neutrino contibutes about 170 MeV (for $M_Z = 92$ GeV) to Γ_{Ztot}, hence, a precise measurement of the Z-width provides an important tool for neutrino counting, a subject discussed in detail by E. Simopoulos and M. Caffo [17] in these proceedings. A precise knowledge of the leptonic widths would open the possibility to test experimentally the "non-renormalization" of the low energy parameters G_μ and ρ_0 in comparison to the corresponding "vector boson parameters" $\hat{G}_\mu$ and $\hat{\rho}$ defined in (2.26a). Unfortunately, a direct determination of $\Gamma_{Z\nu\nu}$ is not possible and an indirect determination as a difference $\sum_\nu \Gamma_{Z\nu\nu} = \Gamma_{Z\,tot} - \Gamma_{Z\,visible}$ between the total width (determined from the Z-line shape) and the visible leptonic and hadronic channels will not be very precise. A better measure for $\hat{\rho}$ may be obtained if we replace $\Gamma_{Z\nu\nu}$ by $\Gamma_{Z\ell\ell}/f_\ell(s_\theta^2)$ in (2.26a) (see below).

The predominant decay modes of the vector bosons are the two body decays

$$W^- \to \bar{f}_1 f_2\,, \quad W^+ \to \bar{f}_1 f_2 \quad \text{and} \quad Z \to \bar{f}f.$$

The lowest order widths for the decays $V \to \bar{f}_1 f_2$ ($V = W,Z$) are given by

$$\Gamma_{oV\to\bar{f}_1 f_2} = \frac{1}{3\sigma_V}\,\frac{M_V^3}{8\pi v^2}\left(\left[1-\frac{(m_1+m_2)^2}{M_V^2}\right]\left[1-\frac{(m_1-m_2)^2}{M_V^2}\right]\right)^{1/2}$$

$$\cdot\left\{16(a_{Vf}^2+b_{Vf}^2)\left[1-\frac{1}{2}\,\frac{m_1^2+m_2^2}{M_V^2}-\frac{1}{2}\,\frac{(m_1^2-m_2^2)^2}{M_V^4}\right]\right.$$

$$\left. + 16(a_{Vf}^2-b_{Vf}^2)\cdot 3\,\frac{m_1 m_2}{M_V^2}\right\} \qquad (6.1)$$

$$(\sigma_W = 1,\ \sigma_Z = 2)$$

(see Eq. (2.1c) for the definition of the couplings a_f and b_f).

For the decays into light fermions ($m_1 + m_2 \ll M_V$) we have the following simple results:

i) W^--decays:

$$\Gamma_{o\,W\ell\nu} = \frac{\sqrt{2}\, G_\mu\, M_W^3}{12\,\pi}\,; \quad \Gamma_{o\,W\left(\substack{d\bar{u}\\ s\bar{c}}\right)} = \Gamma_{oW\ell\nu}\; 3\left(1 + \frac{\alpha_s(M_W)}{\pi}\right) \cdot |U_{KM}|^2 . \tag{6.2}$$

ii) Z-decays

$$\Gamma_{o\,Z\nu\nu} = \frac{\sqrt{2}\, G_\mu \rho_{oo} M_Z^3}{24\,\pi}\,; \quad \Gamma_{oZ\ell\ell} = \Gamma_{o\,Z\nu\nu} \cdot f_\ell(s_\theta^2)$$

$$\Gamma_{oZqq} = \Gamma_{oZ\nu\nu} \cdot f_q(s_\theta^2)\; 3\left(1 + \frac{\alpha_s(M_Z)}{\pi}\right) \tag{6.3}$$

with $f_f(s_\theta^2) = 8(a_f^2 + b_f^2)$ or, explicitly, $f_\ell(s_\theta^2) = 1 - 4s_\theta^2 + 8s_\theta^4$,
$f_u(s_\theta^2) = 1 - \frac{8}{3}\, s_\theta^2 + \frac{32}{9}\, s_\theta^4$ and $f_d(s_\theta^2) = 1 - \frac{4}{3}\, s_\theta^2 + \frac{8}{9}\, s_\theta^4$.

The factor 3 is the color factor and $(1 + \alpha_s/\pi)$ the leading QCD correction. For $\Lambda^{(4)}_{\overline{MS}} = 200$ MeV we have $(1 + \alpha_s/\pi) \simeq 1.035$.

The one-loop electroweak corrections include photonic corrections C_{em} and a "weak" part C_w

$$\Gamma = \Gamma_o(1 + \Delta C) = \Gamma_o(1 + C_w + C_{em}) \; . \tag{6.4}$$

Disregarding real photon emission the corrections are given by
$2\mathrm{Re}\, A_{ren}^{Vff'}(M_V^2)$, where $A_{ren}^{Vff'}(M_V^2)$ are the renormalized on-shell form factors. Considering the leptonic case first, we have

$$C_{W\ell\nu} = 2\mathrm{Re}\, A_L^{W\ell\nu}(M_W^2) + \delta Z_W + \frac{\delta M_W^2}{M_W^2} + \frac{\delta G_\mu}{G_\mu}$$

$$C_{Z\nu\nu} = 2\mathrm{Re}\, A_L^{Z\nu\nu}(M_Z^2) + \delta Z_Z + \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta G_\mu}{G_\mu}$$

$$C_{Z\ell\ell} = 2\mathrm{Re}\, A_\Gamma^{Z\ell\ell}(M_Z^2) + \delta Z_Z + \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta G_\mu}{G_\mu}$$

$$- \frac{4s_\theta^2 |Q_\ell| (1-4|Q_\ell| s_\theta^2)}{f_\ell(s_\theta^2)} \left(\frac{\delta \sin^2\theta_W}{\sin^2\theta_W} - 2c_\theta^2 A^{\gamma Z}(M_Z^2) \right) . \qquad (6.5)$$

Here we have chosen G_μ, M_W and M_Z for the input parameters. The corresponding parameter renormalization counterterms are given by (2.18) and (2.23). $\delta Z_V = Z_V - 1$ are the on-shell wavefunction renormalization factors of the gauge bosons. The lepton wavefunction factors are included in the vertex corrections.

The amplitude A_Γ^{Zff} derives from the one-loop contributions A_1 and A_2 to the Zff-vertex

$$\frac{2M_Z}{v} \gamma^\mu ((a + A_1) + (b + A_2)\gamma_5).$$

The renormalized amplitudes are given by

$$A_{1r} = A_1 + \frac{a}{2} (\delta Z_Z + \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta G_\mu}{G_\mu}) + \delta a + 2Q_f\, s_\theta^2 c_\theta^2\, A^{\gamma Z}(M_Z^2)$$

$$A_{2r} = A_2 + \frac{b}{2} (\delta Z_Z + \frac{\delta M_Z^2}{M_Z^2} + \frac{\delta G_\mu}{G_\mu}) \qquad (6.6)$$

where $\delta a = -Q_f \delta \sin^2\theta_W$. For light fermions the width is proportional to $a^2 + b^2$ or upon inclusion of the corrections and expansion to one-loop order

$$(a^2 + b^2) \to (a^2 + b^2)(1 + 2\mathrm{Re}\, A_\Gamma)$$

$$A_\Gamma = \frac{aA_1 + bA_2}{a^2 + b^2} . \qquad (6.7)$$

The mixing contribution, which is due to the renormalization of $\sin^2\theta_W$ and the γ-Z mixing term (see (6.6)) is treated separately.

For the hadronic decay modes Eqs. (6.5) apply if we replace the corresponding form factor amplitudes, $A_L^{W\ell\nu} \to A_L^{Wdu}$, $A_\Gamma^{Z\ell\ell} \to A_\Gamma^{Zuu}$ etc., and

$(Q_\ell, f_\ell) \to (Q_q, f_q)$ in the coefficient of the mixing contribution.

The selfenergy terms in (6.5) may be written in the form

$$\Delta C^V = \frac{\mathrm{Re}\,\Pi^{VV}(M_V^2)}{M_V^2} - \frac{\Pi^{VV}(0)}{M_V^2} - \mathrm{Re}\,\dot{\Pi}^{VV}(M^2)$$

$$= -\mathrm{Re}\,\Pi^{VV}_{ren}(0)/M_V^2 = -M_V^2\,\mathrm{Re}\,\dot{\pi}^{VV}(M_V^2) \qquad (6.8)$$

where $\Pi^{VV}(q^2) = \Pi^{VV}(0) + q^2\,\pi^{VV}(q^2)$ are the transversal selfenergy functions (2.18a) and $\dot{\Pi} = \frac{d}{dq^2}\Pi$ denotes the derivative. For the mixing term we have

$$\Delta C^{mix} = \mathrm{ctg}^2\theta_W(A^{WW}(0) - A^{ZZ}(0)) - 2c_\theta^2\,A^{\gamma Z}(0) +$$

$$+ \mathrm{ctg}^2\theta_W\,\mathrm{Re}\left[\pi^{ZZ}(M_Z^2) - \pi^{WW}(M_W^2) - \frac{s_\theta}{c_\theta}\,\pi^{\gamma Z}(M_Z^2)\right]. \qquad (6.9)$$

In order to obtain an infrared finite answer we have to include real photon emission. Integration over the full photon phase space yields the well-known results [18] for the QED correction C_{em}. For the total correction in Eq. (6.4) we finally have (A´: the QED part subtracted)

$$\Delta C_{W\ell\nu} = 2\mathrm{Re}\,A_L'^{W\ell\nu} + \Delta C^W + R_{LCC} + \frac{\alpha}{\pi}\left(\frac{77}{24} - \frac{\pi^2}{3} + \frac{3}{4}\,Q_\nu Q_\ell\right)$$

$$\Delta C_{Z\nu\nu} = 2\mathrm{Re}\,A_L^{Z\nu\nu} + \Delta C^Z + R_{LNC} + \varepsilon$$

$$\Delta C_{Z\ell\ell} = 2\mathrm{Re}\,A_L'^{Zee} + \Delta C^Z + R_{LNC} + \varepsilon$$

$$- \frac{4s_\theta^2|Q_\ell|(1 - 4|Q_\ell|s_\theta^2)}{f_\ell(s_\theta^2)}\,\Delta C^{mix} + \frac{\alpha}{\pi}\,\frac{3}{4}\,Q_\ell^2 \qquad (6.10)$$

where C_{em} is the last term in $\Delta C_{W\ell\nu}$ and $\Delta C_{Z\ell\ell}$[+]. The remainders R ... have been given in Eq. (2.32) and ε is the correction to the ρ-parameter (see (2.30)). Notice that also ΔC^{mix} exhibits a term $\mathrm{ctg}^2\theta_W\,\varepsilon^{(SE)}$ where

[+]Footnote: see page

$\varepsilon^{(SE)}$ is the selfenergy contribution to ε.

Radiative corrections to the leptonic decays have been analyzed systematically by Consoli et al. [7]. Tabulated results may be found in the more recent articles [10,19].[++] Using G_μ, M_Z and M_W as input parameters the corrections are generally at the per mille level unless the top is very heavy. In Table 8 some results are given, which illustrate the smallness of the electroweak corrections.

Table 8: Partial widths for decays into light fermions for M_Z = 92 GeV (M_W and $\sin^2\theta_W$ calculated for fixed α, G_μ and M_Z)

(m_t, M_H)	$\Gamma_{W\ell\nu}$	$\Gamma_{Z\nu\nu}$	$\Gamma_{Z\ell\ell}$
(35,100)	0.2306(0.02)	0.1704(0.05)	0.0857(-0.02)
(35,1000)	0.2291(0.02)	0.1701(-0.14)	0.0855(-0.11)
(180,100)	0.2373(-0.33)	0.1722(1.09)	0.0868(-0.37)
	Γ_{Wdu}	Γ_{Zuu}	Γ_{Zdd}
(35,100)	0.7136(-0.02)	0.3036(0.02)	0.3904(-0.11)
(35,1000)	0.7090(-0.02)	0.3023(0.17)	0.3890(-0.06)
(180,100)	0.7345(-0.37)	0.3091(-1.31)	0.3967(-0.63)

[+] The splitting of the electroweak radiative corrections into $C_{QED}^{virtual} + C_{weak}$ is somewhat ambiguous (neither finite nor gauge invariant) in case of the $W\ell\nu_\ell$-vertex. According to [18] ultraviolet divergent and constant terms from diagrams exhibiting virtual photons are included in C_{weak}. $C_{QED}^{virtual}$ thus only includes the logarithmic terms

$$C_{QED}^{virtual} = \frac{\alpha}{2\pi}\left\{-2\left(\ln\frac{M_W^2}{m_\ell^2} - 2\right)\left(\ln\frac{M_W^2}{m_\gamma} - \frac{1}{4}\ln\frac{M_W^2}{m_\ell^2}\right) + \frac{3}{2}\ln\frac{M_W^2}{m_\ell^2}\right\}.$$

[++] In Ref. [10] the results in Tab. 5c for $\Gamma_{Z\ell\ell}$ are errorness due to an error in the coefficient of the mixing term in Eq. (5.2) where c_θ^2 is is to be replaced by $(1-4s_\theta^2)$.

The dependence of the widths over a wide range of values of m_t and m_H is of the order of a few MeV only. This has to be confronted with the experimental errors

$$\delta\Gamma_V = 3\Gamma_V \frac{\delta M_V}{M_V} \tag{6.11}$$

yielding $\delta\Gamma_Z \simeq 1.6$ MeV for $M_Z = 92$ GeV, $\Gamma_Z = 2.5$ GeV and $\delta M_Z = 20$ MeV and $\delta\Gamma_W \simeq 8$ MeV for $M_W = 82$ GeV, $\Gamma_W = 2.2$ GeV and $\delta M_W = 100$ MeV.

For an evaluation of the total widths we have to include the decays into b- and t-flavored states and mass effects become nonnegligible. We distinguish three kinds of mass effects which we consider in turn.

i) Kinematical mass dependence given by Eq. (6.1). Disregarding higher order effects and U_{KM}-mixing for the moment we have

$$\Gamma_{Wbt} = \begin{cases} \Gamma_{Wdu} & m_t, m_b \ll M_W \\ 0 & m_t \geq M_W - m_b \end{cases}$$

$$\Gamma_{Ztt} = \begin{cases} \Gamma_{Zuu} & m_t,\ m_b \ll M_W \\ 0 & m_t \geq M_Z/2. \end{cases} \tag{6.12}$$

One thus obtains m_t-independent bounds (up to higher order effects) for the total widths (N_ν = number of light neutrinos).

$$3\Gamma_{W\ell\nu} + 2\Gamma_{Wdu} < \Gamma_{Wtot} < 3(\Gamma_{W\ell\nu} + \Gamma_{Wdu})$$

$$N_\nu\Gamma_{Z\nu\nu} + 3\Gamma_{Z\ell\ell} + 2\Gamma_{Zuu} + 3\Gamma_{Zdd} < \Gamma_{Ztot} < N_\nu\Gamma_{Z\nu\nu} + 3(\Gamma_{Z\ell\ell} + \Gamma_{Zuu} + \Gamma_{Zdd}) \tag{6.13}$$

and a strong dependence on m_t below the t-production thresholds, depicted in Fig. 6 for $N_\nu = 3$. At LEP/SLC it will be easy to establish the existence of a top with mass $m_t \lesssim M_Z/2$. It seems unlikely that another 4th family fermion with mass $m_f < M_Z/2$ exists except for additional neutrinos.

ii) Mass dependence through strong interaction effects. The QCD cor-

rection $\frac{\alpha_s}{\pi}$ is valid for light quarks only and must be replaced by mass dependent corrections which differ for vector and axial-vector currents. It is simplest to consider the contribution of a particular decay channel to be given by a contribution to the imaginary part of the vector boson selfenergies, according to the relation

$$M_V \Gamma_V = - \text{Im } \Pi^{VV}(M_V^2). \tag{6.14}$$

The vector boson selfenergies may be decomposed into vector (V) and axial vector (A) vacuum polarization amplitudes (3.2) of the currents (2.1):

$$\Pi^{WW} = g^2(\hat{\Pi}_V^{+-} + \hat{\Pi}_A^{+-})$$

$$\Pi^{ZZ} = \frac{g^2}{c_\theta^2}(\hat{\Pi}_V^{33} + \hat{\Pi}_A^{33}) - 2\frac{e^2}{c_\theta^2}\Pi_V^{3\gamma} + \frac{e^2 s_\theta^2}{c_\theta^2}\hat{\Pi}_V^{\gamma\gamma}$$

$$\Pi^{\gamma Z} = \frac{eg}{c_\theta}\Pi_V^{3\gamma} - \frac{e^2 s_\theta}{c_\theta}\hat{\Pi}_V^{\gamma\gamma}$$

$$\Pi^{\gamma\gamma} = e^2 \hat{\Pi}_V^{\gamma\gamma} . \tag{6.15}$$

Using the higher order perturbative QCD corrections of the different vacuum polarization functions [20] we may write

$$\Gamma_{Zff} = \Gamma_{Z\nu\nu}\, 8N_c \left\{a_f^2 R_V^f + b_f^2 R_A^f\right\} \tag{6.16}$$

with $\left(\beta = \left(1 - \frac{4m_f^2}{M_Z^2}\right)^{1/2}, \quad \beta^2 > 0\right)$

$$R_V^f = \frac{\beta}{2}(3-\beta^2)\left\{1 + \frac{\alpha_s}{\pi} f(\beta) + c_2(\frac{\alpha_s}{\pi})^2\right\}$$

$$R_V^f = \beta^3 \left\{1 + \frac{\alpha_s}{\pi} g(\beta) + c_2(\frac{\alpha_s}{\pi})^2\right\} . \tag{6.17}$$

We have included the $O(\alpha_s^2)$ term in the massless approximation $c_2 = 1.986 - 0.115\, N_f$ in the $\overline{MS}$ scheme (N_f the number of open

flavor channels at scale M_V). The exact results for f and g are lengthly and will not be given here. Simple approximation formulae mostly used are

$$f(\beta) = \frac{2\pi^2}{3\beta} - (3+\beta)\left(\frac{\pi^2}{6} - \frac{1}{4}\right)$$

$$g(\beta) = \frac{2\pi^2}{3\beta} - 4\left(\frac{19}{10} - \frac{22}{5}\beta + \frac{7}{2}\beta^2\right)\left(\frac{\pi^2}{6} - \frac{1}{4}\right) \qquad (6.17a)$$

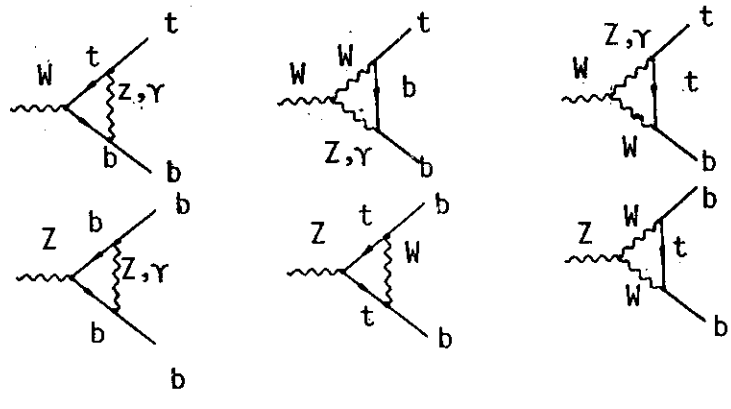
The terms proportional to β^{-1} diverge at threshold and a simple resummation [21]

$$1 + x \rightarrow \frac{2x}{1 - e^{-2x}}\,; \qquad x \doteq \frac{2\pi\alpha_s}{3\beta} \qquad (6.18)$$

of higher order singular terms improves the validity of the result near threshold.

Furthermore, in QCD the coupling and the quark masses are running parameters and one has to use renormalization group improved perturbation theory throughout. For details we refer to Ref. [22,9]. Notice that the standard use of $O(\alpha_s^2)$ running parameters, requires to take into account the $O(\alpha_s^2)$ term in Eq. (6.17) for consistency. The importance of the mass effects for intermediate top masses is obvious from Fig. 6. A shift of Γ_Z and Γ_W by up to +100 MeV is obtained, consistently with the result of Chang et al. [20].

iii) Mass dependence of the electroweak radiative corrections. Besides the known heavy particle effects showing up in the selfenergy contributions ΔC^V and C^{mix} the vertex corrections A_L^{Wbt}, A_L^{Zbb} A_L^{Ztt} are dependent on m_t in an essential way. The relevant diagrams are



and those obtained with $t \leftrightarrow b$. An essential m_t dependence of Γ_{Zbb} has been pointed out in Ref. [19]. Neglecting the mass effects of the form factors we estimate (assuming $m_t \lesssim M_W$)[+]

$$2.117 \pm 0.006 \text{ GeV} < \Gamma_{W\,tot} < 2.832 \pm 0.006 \text{ GeV}$$

$$2.551 \pm 0.006 \text{ GeV} < \Gamma_{Z\,tot} < 2.863 \pm 0.006 \text{ GeV}$$

for $M_Z = 92$ GeV $\pm$ 50 MeV. For $m_t = 35$ GeV we find

$$\Gamma_{Wtot} = 2.661 \pm 0.006 \text{ GeV}, \qquad \Gamma_{Ztot} = 2.680 \pm 0.006 \text{ GeV}$$

$$\Gamma(W \to e\nu_e,\ \mu\nu_\mu,\ \tau\nu_\tau) = 692 \pm 2 \text{ MeV}$$

$$\Gamma(Z \to \nu_e\bar{\nu}_e,\ \nu_\mu\bar{\nu}_\mu,\ \nu_\tau\bar{\nu}_\tau) = 511 \pm 1 \text{ MeV}$$

$$\Gamma(Z \to e^+e^-,\ \mu^+\mu^-,\ \tau^+\tau^-) = 257 \pm 1 \text{ MeV}$$

$$\Gamma(W \to \text{"udcs"}) = 1432 \pm 4 \text{ MeV}$$

$$\Gamma(Z \to \text{"udcs"}) = 1400 \pm 4 \text{ MeV}$$

A final remark is important. We use the t-quark mass $m_t(m_t)$ as an input parameter where $M_{t\bar{t}} = 2m_t(m_t)\left(1 + \frac{4\alpha_s\,(M_{t\bar{t}})}{3\,\pi}\right)$ is supposed to be close to the toponium mass [22]. $m_t(\mu)$ is the $O(\alpha_s^2)$ running current quark mass. The effective t-quark mass used for the evaluation of perturbative expressions is $m_{t\,eff}(\mu) = m(\mu)\left(1 + \frac{4}{3}\frac{\alpha_s(2\mu)}{\pi}\right)$ in order to get (approximately) the correct toponium threshold. Results obtained for a given value of m_t differ if m_t is considered to be a free quark mass.

[+]Since $\Gamma_V \propto M_V^3$ all numerical values strongly depend on the value of M_Z chosen. M_W is calculated using (2.33) from α, G_μ and M_Z and thus depends on m_H and m_t.

Figure 6a: Prediction of W-width (M_Z = 92 GeV)

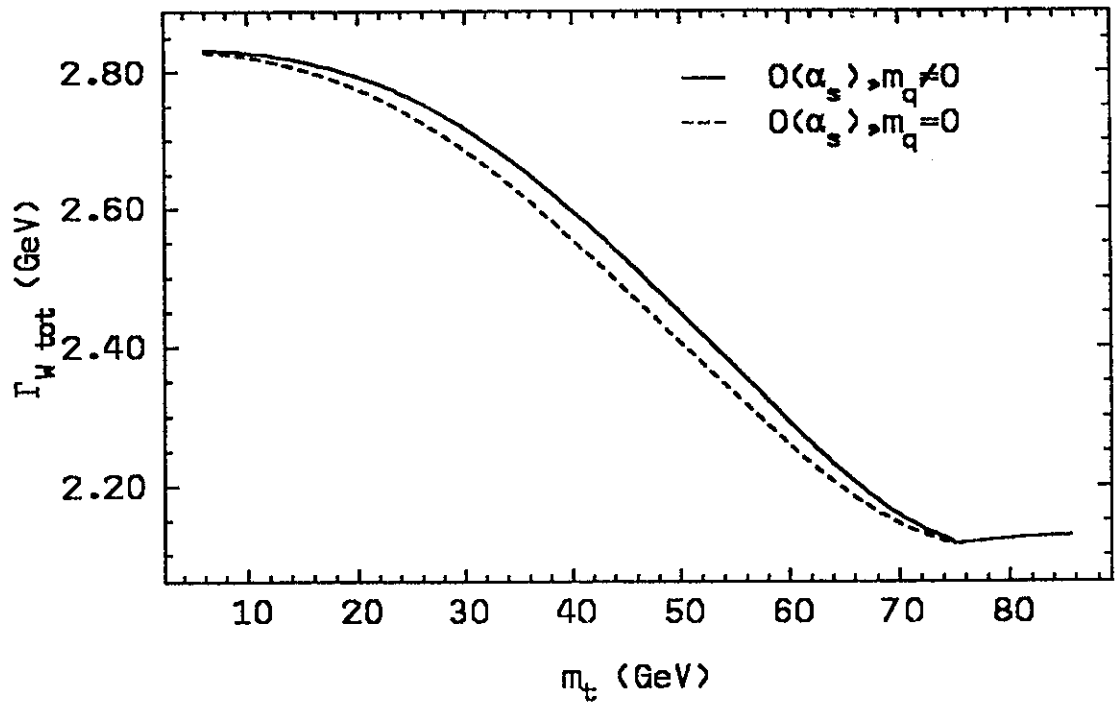
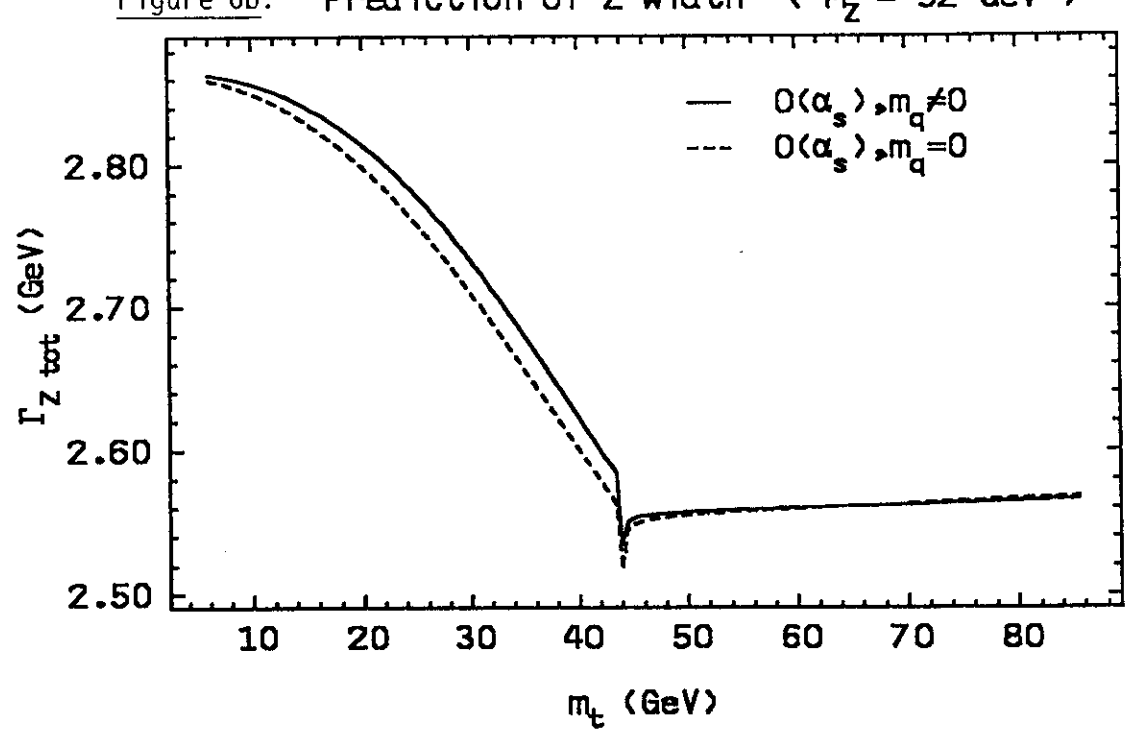


Figure 6b: Prediction of Z-width (M_Z = 92 GeV)



7. Heavy Particle Effects

The major interest in precision tests of higher order effects lies in the possibility of establishing deviations from the standard model predictions which would signal the existence of new degrees of freedom or new interactions. In addition, if the top and the Higgs should turn out to be too heavy to be produced at LEP ($m_t \gtrsim 100$ GeV, $m_H \gtrsim M_Z$), bounds on their masses can be obtained from precise measurements of their virtual contributions.

In electroweak theory, masses are generated by the Higgs mechanism implying mass-coupling relations like

$$M_W = \frac{gv}{2}\,, \quad M_Z = \frac{gv}{2c_\theta}\,, \quad m_H = \sqrt{2\lambda}v, \quad m_f = \frac{G_f}{\sqrt{2}}v\,.$$

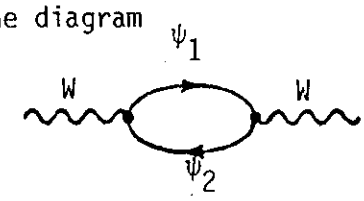
Since the vacuum expectation value v of the Higgs field is fixed, the existence of heavy particles implies the presence of strong interactions unless the mass is due to a large vacuum expectation value of another neutral Higgs field. Otherwise, it is not surprising that heavy particles do not decouple in perturbation theory which necessarily breaks down if a particle gets too heavy (see, for example, the increase of higher order uncertainties in Δr (Fig. 2) for large m_t).

In the fermion-vector boson sector large effects are obtained (at the one-loop level) only in the case of large mass splittings of members of a $SU(2)_L$ multiplet. This has first been observed by Veltman [16] in his investigation of heavy fermion contributions to the ρ-parameter. The quantities which have the largest sensitivity to heavy particle effects are the bosonic selfenergies (having power counting dimension 2) which potentially lead to a m_h^2 behavior for $m_h \to \infty$ in the one-loop approximation. However, only the flavor changing charged current interaction, which mixes members of $SU(2)_L$ multiplets, exhibits this leading effects. In the Higgs sector quadratic mass effects are obtained also in case of mass degenerate multiplets (see Fleischer and Jegerlehner [23]). These effects carry over to the fermion sector at two-loop order.

The key quantity for leading heavy particle effects is the ρ-parameter

$$\delta\rho = \varepsilon = \rho_0 - 1 = \frac{\Pi^{ZZ}(0)}{M_Z^2} - \frac{\Pi^{WW}(0)}{M_W^2} + \ldots$$

to which the diagram



$\begin{pmatrix}\psi_1\\ \psi_2\end{pmatrix}$: $SU(2)_L$ doublet

contributes (N_c = color factor)

$$\delta\rho = \frac{\sqrt{2}G_\mu}{16\pi^2} N_c \left(m_1^2 + m_2^2 + \frac{2m_1^2 m_2^2}{m_1^2 - m_2^2} \ln \frac{m_2^2}{m_1^2}\right)$$

$$\simeq \frac{\sqrt{2}G_\mu}{16\pi^2} N_c \cdot \begin{cases} m_h^2 \; ; & m_h >> m_l \\ 0 \; ; & m_1 = m_2 \end{cases} .$$

Van der Bij and Hoogeveen [24] have computed the two-loop heavy fermion effects to the ρ-parameter. They find

i) for a heavy fermion doublet with small mass difference $m_1 - m_2 = \Delta << m_1 = m_h$

$$\delta\rho = \frac{g^2}{48\pi^2} \frac{N_c}{3} \frac{\Delta^2}{M_W^2} + \frac{g^4}{1024\pi^4} \frac{N_c}{3} \frac{m_h^2 \Delta^2}{M_W^4}$$

ii) for a fermion doublet with large mass difference $m_1 = m_h >> m_2$

$$\delta\rho = \frac{g^2}{64\pi^2} N_c \frac{m_h^2}{M_W^2} + \frac{g^4}{2048\pi^4} N_c \frac{m_h^4}{M_W^4} (10 - \pi^2)$$

iii) for a mass degenerate heavy fermion doublet $m_1 = m_2 = m_h$.

$$\delta\rho = \frac{g^4}{256\pi^4} \frac{N_c}{3} \frac{m_h^2}{M_W^2} \left(-tg^2\theta_W (\ln \frac{m_h^2}{m_H^2} + \frac{11}{12})\right.$$

$$\left. + \frac{M_W^2}{m_H^2 - M_W^2} \ln \frac{M_W^2}{m_H^2} - \frac{M_Z^2}{m_H^2 - M_Z^2} \frac{M_Z^2}{M_W^2} \ln \frac{M_Z^2}{m_H^2}\right) .$$

Numerically, the two-loop terms are small relative to the leading correction if the masses of particles are below a few TeV.

Another important question concerns the manifestation of a very heavy Higgs particle. According to Veltman's screening theorem [25] a heavy Higgs would decouple from the light fermion sector if the "accidental" global $SU(2)_R$ symmetry of the minimal Higgs sector would be unbroken. Since this symmetry is broken by the $U(1)_Y$ couplings, logarithmic heavy Higgs effects which are proportional to $\sin^2\theta_W$ show up in the ρ-parameter. At two-loop order the quadratic term has been calculated by Van der Bij and Veltman [25]. They find the result

$$\delta\rho = \frac{g^2}{16\pi^2}\,\frac{3}{4}\,tg^2\theta_W \ln\frac{m_H^2}{M_W^2} + \frac{g^4}{\pi^4}\frac{m_H^2}{M_W^2}\,tg^2\theta_W \cdot (5.854\times 10^{-4})$$

with a rather small coefficient for the quadratic two-loop correction. The latter is significant only for a Higgs mass above 1 TeV.

Generally, it appears that higher order terms can be ignored if particle masses are below the 1 TeV scale.

How do heavy particles manifest themselves in other observables in the fermion-vector boson sector? It is interesting to notice that all leading heavy fermion effects can be attributed to a change of the ρ-parameter. In order to see this, we must parametrize the theory in a way which treats the ρ-parameter to be an independent parameter. To this end, the following replacements are necessary in the mass-coupling relations:

$$M_Z^2 \to \rho M_Z^2$$

$$\sin^2\theta_W \to \sin^2\theta'_W = 1 - \frac{M_W^2}{\rho M_Z^2} = \frac{\sin^2\theta_W + \rho - 1}{\rho}$$

$$\alpha_W \to \alpha'_W = \frac{\sqrt{2}G_\mu}{\pi} M_W^2 \sin^2\theta'_W = \frac{\alpha}{1-\Delta e}$$

$$\Gamma_{Z\nu\nu} \to \rho\frac{\sqrt{2}\,G_\mu M_Z^3}{24\pi} \quad .$$

with $\Delta e = \Delta r - ctg^2\theta_W \varepsilon$.

From (2.30) one infers that $\Delta e = \Delta r - c\,tg^2\theta_W$ (replacing Δr) and Δg exhibit no quadratic heavy mass effects and hence the ratios

$$\frac{\alpha_{g_W}}{\alpha_g} = \frac{1}{1-\Delta g}\,, \quad \frac{\alpha'_W}{\alpha} = \frac{1}{1-\Delta e}\,, \quad \frac{\sin^2\theta'_W}{\sin^2\theta_o} = \frac{1-\Delta g}{1-\Delta e} \tag{7.7}$$

are not renormalized by the leading effects. Then, the observables depend on the leading heavy particle effects only through their explicit dependence on ρ at the Born term level. Thus, if α, G_μ and M_Z are given by experiment, a quantity $X(\alpha, G_\mu, M_Z, M_W, \sin^2\theta'_W, \rho)$ depends on a ρ-shift by ($s'^2_\theta \doteq \sin^2\theta'_W$)

$$\delta X = \left(\frac{\partial X}{\partial M_W}\right) \delta M_W + \left(\frac{\partial X}{\partial s'^2_\theta}\right) \delta s'^2_\theta + \left(\frac{\partial X}{\partial \rho}\right) \delta\rho \tag{7.8}$$

where the partial derivatives are evaluated for fixed input parameters α, G_μ and M_Z. δM_W and $\delta s'^2_\theta$ follow from (2.33) and the substitution (7.6). We obtain (numerical values are given for $s'^2_\theta = 0.23$)

$$\frac{\delta M_W}{M_W} = \frac{1}{2}\,\frac{c'^2_\theta}{1-2s'^2_\theta}\,\frac{\delta\rho}{\rho} = 0.713\,\delta\rho$$

$$\frac{\delta s'^2_\theta}{s'^2_\theta} = -\,\frac{c'^2_\theta}{1-2s'^2_\theta}\,\frac{\delta\rho}{\rho} = 1.426\,\delta\rho. \tag{7.9}$$

From $\rho\sin^2\theta'_W = \sin^2\theta_W + \rho - 1$ furthermore

$$\frac{\delta s^2_\theta}{s^2_\theta} = -c\,tg^2\theta'_W\,\frac{c'^2_\theta}{1-2s'^2_\theta}\,\frac{\delta\rho}{\rho} = -4.774\,\delta\rho \tag{7.10}$$

and by (7.7) ($s^2_{\theta_o} = \sin^2\theta_o$)

$$\frac{\delta s^2_{\theta_o}}{s^2_{\theta_o}} = -\,\frac{c'^2_{\theta_o}}{1-2s'^2_{\theta_o}}\,\frac{\delta\rho}{\rho} = -1.426\,\delta\rho\,. \tag{7.11}$$

The latter relation implies

$$\frac{\delta R_{\nu\nu}}{R_{\nu\nu}} = -64 \frac{s_{\theta_o}^4 c_{\theta_o}^2}{1+\xi^2+\xi^4} \frac{\delta\rho}{\rho} = -2.590\ \delta\rho \qquad (7.12)$$

for the neutrino electron scattering cross section ratio (2.6b). For the widths (6.2, 6.3) we find, using (7.6) and (7.9),

$$\frac{\delta\Gamma_{Wff'}}{\Gamma_{Wff'}} = \frac{3}{2} \frac{c'^2_\theta}{1-2s'^2_\theta} \frac{\delta\rho}{\rho} \simeq 2.139\ \delta\rho$$

$$\frac{\delta\Gamma_{Z\nu\nu}}{\Gamma_{Z\nu\nu}} = \frac{\delta\rho}{\rho} \qquad (7.13)$$

$$\frac{\delta\Gamma_{Zff}}{Zff} = \left(1 - \frac{\dot{f}_f(s'^2_\theta)}{f_f(s'^2_\theta)} \frac{s'^2_\theta c'^2_\theta}{1-2s'^2_\theta}\right)\frac{\delta\rho}{\rho} = \begin{cases} 1.209\ \delta\rho\ ; & f = \ell \\ 1.588\ \delta\rho\ ; & f = u \\ 1.409\ \delta\rho\ ; & f = d \end{cases}$$

The asymmetries (2.1) are given in terms of

$$A_f = -2\frac{a_f b_f}{a_f^2+b_f^2} = \frac{1-4|Q_f|s'^2_\theta}{f_f(s'^2_\theta)}\ ;\quad f_f = 1-4|Q_f|s'^2_\theta+8Q_f^2 s'^4_\theta$$

for which

$$\frac{\delta A_f}{A_f} = 16Q_f^2 \frac{(1-2|Q_f|s'^2_\theta)}{f_f(s'^2_\theta)(1-4|Q_f|s'^2_\theta)} \frac{s'^4_\theta c'^2_\theta}{1-2s'^2_\theta} \frac{\delta\rho}{\rho}$$

$$= \begin{cases} 16.190\ \delta\rho\ ; & f = \ell \\ 1.673\ \delta\rho\ ; & f = u \\ 0.221\ \delta\rho\ ; & f = d \end{cases} \qquad (7.14)$$

and consequently

$$\frac{\delta A_{LR}}{A_{LR}} = \frac{\delta A_e}{A_e} = 16.190\ \delta\rho$$

$$\frac{\delta A_{FB}^{ff}}{A_{FB}^{ff}} = \frac{\delta A_e}{A_e} + \frac{\delta A_f}{A_f} = \begin{cases} 32.379\ \delta\rho\ ; & f = \ell \\ 17.863\ \delta\rho\ ; & f = u \\ 16.411\ \delta\rho\ ; & f = d \end{cases} . \qquad (7.15)$$

Within the minimal SM with 3 families only the top-quark might give a contribution of the kind for which the above formulae hold with

$$\delta\rho = \frac{\sqrt{2}\, G_\mu}{16\pi^2}\, 3 m_t^2 \ ; \quad m_t >> m_b \ . \qquad (7.16a)$$

Additional doublets of fermions or scalars exhibiting large mass splittings add up like

$$\delta\rho = \frac{\sqrt{2}\, G_\mu}{16\pi^2} \left\{ |m_{\nu'}^2 - m_{\ell'}^2| + N_c |m_{t'}^2 - m_{b'}^2| \right.$$
$$\left. + \frac{1}{2} |m_{\tilde{\nu}}^2 - m_{\tilde{e}}^2| + \frac{N_c}{2} |m_{\tilde{u}}^2 - m_{\tilde{d}}^2| + \ldots \right\} \qquad (7.16b)$$

for leptons, quarks, sleptons, squarks, ...

For an additional Higgs doublet the same kind of quadratic mass effects has been observed [26]. The upper limits on m_t obtained from the measurement of the ρ-parameter [4]

$$m_t \lesssim 180\ \text{GeV}, \qquad \delta\rho \lesssim 0.01 \qquad (7.18)$$

in fact limits the sum of mass squares of heavy particles with a light partner, when the masses are generated by the standard minimal Higgs mechanism. For example, the mass of a 4th family heavy lepton accompanying a massless neutrino is bound by

$$m_{\ell'} \lesssim 300\ \text{GeV}$$

if we assume that no other contribution is present. The bound $\delta\rho \lesssim 0.01$ inserted into Eqs. (7.9) to (7.15) limits corresponding effects in various observables.

The simple rule given above for the evaluation of the leading heavy fermion effects in the different observables does not apply to the logarithmic heavy mass effects like in the case of the Higgs. It is easy, however, to obtain the Higgs mass dependent corrections from the results listed in Appendix C. Good bounds on the Higgs mass are much harder to obtain because of the logarithmic behavior in this case (see also Tab. 7 above).

8. Conclusions and Outlook

We have reviewed the calculation of higher order effects in parameter relations of the electroweak standard model. For a discussion of the asymmetries we refer to W. Hollik's lectures given at this meeting [8]. Radiative corrections are sizable at LEP energies and their precise evaluation is indispensable for the interpretation of future precision experiments. The precision of theoretical predictions, which is about 20 MeV in mass units, essentially matches the experimental precision supposed to be reached at LEP.

The physics interest in electroweak radiative corrections lies in their sensitivity to physics at very different energy scales. Precision experiments thus provide a key to unknown physics. A comparison of Tabs. 1 and 7 may illustrate this point.

The first precision test will be obtained at LEP I/SLC from a measurement of the forward-backward asymmetry $A^{\mu^+\mu^-}$ versus M_Z. It will be possible to determine the number of light neutrinos from the observed total Z-width (from the Z-line shape) and independently from the measurement of the $e^+e^- \to \gamma\nu\bar{\nu}$ cross section at peak [17]. We should mention that the precise determination of mass and width of the Z-boson from its line shape is quite a nontrivial task, mainly because of large photonic corrections which depend in a complicated way on the experimental conditions. Thus, before M_Z and Γ_Z can be determined, the Z-resonance must be disentangled from the "uninteresting" effects. This requires to control QED corrections beyond the one-loop level [27].

Longitudinally polarized beams at LEP would make possible a very precise measurement of the left-right asymmetry A_{LR} [28]. The sensitivity to unknown effects would be substantially better than what can be reached at LEP II by a precision measurement of the W-mass. Of course, only at LEP II it will be possible to establish the precise properties of the W and the existence of the triple gauge couplings and their properties. For a discussion of radiative corrections to W-production in $e^+e^- \to W^+W^-$ we refer to M. Zralek's contribution [29].

Last but not least, we hope with A. Barroso [30] that the Higgs will show up sooner or later at LEP. To have a Higgs in the detector would certainly be more exciting than to have it in some loops. A heavy Higgs, however, could be a loop(-hole) for ever.

Appendix A

In this appendix we present explicit expressions for the counterterms and the invariant amplitudes which have been used in the text. Diagrams are illustrated for the unitary gauge. If possible, amplitudes are normalized relative to the Born terms. For simplicity, results are presented for the 't Hooft-Feynman gauge ($\xi = 1$) in the $\overline{MS}$ scheme ($2/\varepsilon - \gamma + \ln 4\pi \doteq \ln\mu^2$) neglecting terms of order $O(m_\ell^2/M_W^2)$ (m_ℓ a light external fermion mass). Contributions from virtual vector bosons (B), photons (QED), Higgs bosons (H), and fermions (F) are given separately. The sums Σf_s and Σf_d extend, respectively, over the single fermions and the fermion doublets. a_f and b_f are the neutral current fermion couplings eq. (2.1c) and Q_f is the fermion charge. The tadpole terms (Tp) have been included in the definition of the selfenergies in order to obtain gauge invariant mass counterterms. The counterterms are given by the real parts of corresponding expressions which are given below in complex form.

By $\Pi_\pm = \frac{1}{2}(1 \pm \gamma_5)$ we denote the chiral projectors. The integrals B_o and $\dot{B}_o$ are given by

$$B_o(m_1,m_2;q^2) = \ln\mu^2 - \int_0^1 dz\, \ln(-q^2 z(1-z) + m_1^2(1-z) + m_2^2 z - i0)$$

and

$$\dot{B}_o(m_1,m_2;q^2) = \frac{\partial}{\partial q^2} B_o(m_1,m_2;q^2),$$

respectively. Furthermore, we have used the functions[+]

[+]These are obtained from particular integrals B_o and $\dot{B}_o$:

$$B_o(M,m;m^2) = 2 - \ln\frac{m^2}{\mu^2} - \frac{1}{2}\frac{M^2}{m^2}\ln\frac{M^2}{m^2} + \frac{M^2}{m^2}\left(1 - \frac{4m^2}{M^2}\right)F\left(\frac{4m^2}{M^2}\right)$$

$$\dot{B}_o(M,m;m^2) = -m^{-2}\left(1 + \frac{1}{2}\left(1 - \frac{M^2}{m^2}\right)\ln\frac{M^2}{m^2} + \frac{M^2-3m^2}{m^2}F\left(\frac{4m^2}{M^2}\right)\right)$$

$$B_o(M,M;m^2) = 2 - \ln\frac{M^2}{\mu^2} + 2\left(\frac{4M^2}{m^2} - 1\right)G\left(\frac{4M^2}{m^2}\right)$$

$$\dot{B}_o(M,M;m^2) = -m^{-2}\left(1 + \frac{4M^2}{m^2}G\left(\frac{4M^2}{m^2}\right)\right).$$

$$F(y) = \frac{1}{2\sqrt{1-y}} \ln \frac{1+\sqrt{1-y}}{1-\sqrt{1-y}}$$

and

$$G(y) = \frac{1}{2\sqrt{1-y}} \ln \frac{\sqrt{1-y}+1}{\sqrt{1-y}-1} \quad .$$

For real arguments we have

$$F(y) = \frac{1}{\sqrt{y-1}} \operatorname{arctg} \sqrt{y-1}; \; G(y) = -\frac{1}{\sqrt{y-1}} \operatorname{arctg} \frac{1}{\sqrt{y-1}}; \quad (y > 1)$$

and

$$G(y) = F(y) - i \frac{\pi}{2\sqrt{1-y}} \; ; \; F(y) = \frac{1}{\sqrt{1-y}} \operatorname{arctgh} \sqrt{1-y} \; ; \quad (y < 1)$$

and the asymptotic expansions

$$G(y) = -y^{-1}(1 + \frac{2}{3} y^{-1} + \frac{8}{15} y^{-2} + \ldots) \; ; \quad y \to \infty$$

$$F(y) = -\frac{1}{2\sqrt{1-y}} \ln \frac{y}{4} - \frac{y}{4}(1 + \frac{7}{8} y + \frac{37}{48} y^2 + \ldots); \; y \to 0.$$

The spence function is

$$Sp(x) = -\int_0^1 \frac{dt}{t} \ln(1-xt)$$

and satisfies the relations

$$Sp(x) = -Sp(1-x) + \frac{\pi^2}{6} - \ln x \ln(1-x)$$

$$Sp(x) = -Sp(\frac{1}{x}) - \frac{\pi^2}{6} - \frac{1}{2} \ln^2(-x).$$

Special values are

$$Sp(0) = 0, \; Sp(1) = \frac{\pi^2}{6} \; , \; Sp(-1) = -\frac{\pi^2}{12} \; , \; Sp(1/2) = \frac{\pi^2}{12} - \frac{1}{2}(\ln 2)^2$$

$$Sp(2 \pm i0) = \frac{\pi^2}{4} \pm i\pi \ln 2.$$

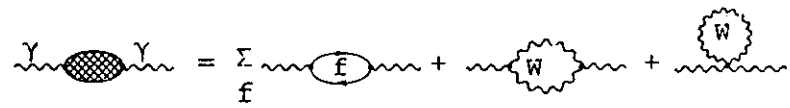
Furthermore

$$Sp(x) = \sum_{k=1}^{\infty} \frac{x^k}{k^2} \simeq x + \frac{x^2}{4} + \ldots \qquad \text{for} \quad x \to 0 .$$

$$|x| \leq 1$$

a) Selfenergies and counterterms

<u>γ-propagator:</u>



$$y_f \doteq \frac{4m_f^2}{q^2} \qquad = ig^{\mu\nu} q^2 A^{\gamma\gamma}(q^2) + \ldots$$

$$A_B^{\gamma\gamma}(q^2) = \frac{e^2}{16\pi^2} \left\{ -3 \ln \frac{M_W^2}{\mu^2} + 6 + \frac{8M_W^2}{q^2} + 2\left(3 + \frac{4M_W^2}{q^2}\right)\left(\frac{4M_W^2}{q^2} - 1\right) G\left(\frac{4M_W^2}{q^2}\right) \right\}$$

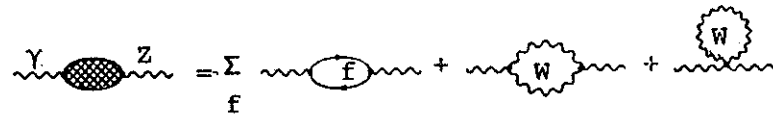
$$A_F^{\gamma\gamma}(q^2) = \frac{e^2}{16\pi^2} \frac{4}{3} \sum_{f_s} Q_f^2 \left[\ln \frac{m_f^2}{\mu^2} - \frac{5}{3} - y_f - 2\left(1 + \frac{y_f}{2}\right)(y_f - 1) G(y_f) \right]$$

$$(Z_\gamma - 1)_B = \frac{e^2}{16\pi^2} \left\{ -3 \ln \frac{M_W^2}{\mu^2} + \frac{2}{3} \right\}$$

$$; \qquad Z_\gamma - 1 = A^{\gamma\gamma}(o)$$

$$(Z_\gamma - 1)_F = \frac{e^2}{16\pi^2} \frac{4}{3} \sum_{f_s} Q_f^2 \ln \frac{m_f^2}{\mu^2} .$$

<u>γ-Z-mixing:</u>



$$= ig^{\mu\nu}(ev) M_Z \, A^{\gamma Z}(q^2) + \ldots$$

$$A_B^{\gamma Z}(q^2) = \frac{-M_Z^2}{16\pi^2 v^2} \left\{ \left(\frac{1}{3} \frac{q^2}{M_Z^2} + 4c_\theta^2 + 6c_\theta^2 \frac{q^2}{M_Z^2} \right) \ln \frac{M_W^2}{\mu^2} \right.$$

$$\left. - 2\left(\frac{4}{9} \frac{q^2}{M_Z^2} + \frac{8}{3} c_\theta^2 + 8c_\theta^4 + 6c_\theta^2 \frac{q^2}{M_Z^2} \right) \right.$$

$$+ 2\left(\frac{1}{3}\frac{q^2}{M_Z^2} + \frac{8}{3}c_\theta^2 + 8c_\theta^4 + 6c_\theta^2\frac{q^2}{M_Z^2}\right)\left(1 - \frac{4M_W^2}{q^2}\right)G\left(\frac{4M_W^2}{q^2}\right)\Big\}$$

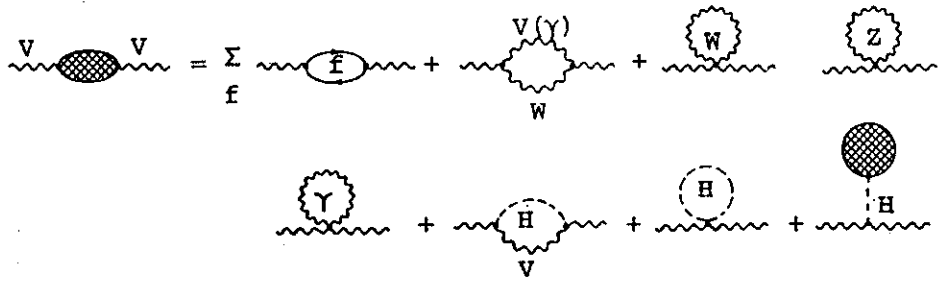
$$A_F^{\gamma Z}(q^2) = \frac{q^2}{16\pi^2 v^2}\frac{2}{3}\sum_{f_s}(4a_f Q_f)\left[\ln\frac{m_f^2}{\mu^2} - \frac{5}{3} - y_f - 2\left(1+\frac{y_f}{2}\right)(y_f-1)G(y_f)\right]$$

$$A_B^{\gamma Z}(0) = \frac{-4M_W^2}{16\pi^2 v^2}\ln\frac{M_W^2}{\mu^2}\ ;\ A_F^{\gamma Z}(0) = 0$$

$$\frac{dA_B^{\gamma Z}}{dq^2}(0) = \frac{-2}{16\pi^2 v^2}\left\{(3c_\theta^2 + \frac{1}{6})\ln\frac{M_W^2}{\mu^2} - \frac{2c_\theta^2+1}{3}\right\}$$

$$\frac{dA_F^{\gamma Z}}{dq^2}(0) = \frac{-1}{16\pi^2 v^2}\frac{2}{3}\sum_{f_s}(4s_\theta^2 Q_f^2 - |Q_f|)\ln\frac{m_f^2}{\mu^2}\ .$$

Vector-boson propagators: (V = Z, W)



$$= ig^{\mu\nu}M_V^2\,A^{VV}(q^2) + \ldots$$

$$A_B^{ZZ}(q^2) = \frac{M_W^2}{3\cdot 16\pi^2 v^2}\Big\{(-2-16c_\theta^2+48c_\theta^4)\ln\frac{M_W^2}{\mu^2}$$

$$+ \left(-\frac{2}{3}\frac{q^2}{M_W^2} + 6 + \frac{8}{3}c_\theta^2\frac{q^2}{M_W^2}\right)$$

$$+ \left(-\frac{q^2}{M_W^2} - 20 + 4c_\theta^2\frac{q^2}{M_W^2} + 32c_\theta^2 + 36c_\theta^4\frac{q^2}{M_W^2} + 48c_\theta^4\right)$$

$$B_0(M_W, M_W; q^2)\Big\}\ .$$

$$A_B^{WW}(q^2) = \frac{M_Z^2}{3\cdot 16\pi^2 v^2}\left\{(-\frac{M_Z^2}{q^2} - 1 - 6\,\frac{M_W^2}{q^2} + 51c_\theta^2 + 15c_\theta^2\frac{M_W^2}{q^2} - 20c_\theta^4 - 8c_\theta^4\frac{M_W^2}{q^2})\ln\frac{M_W^2}{\mu^2}\right.$$

$$+ (\frac{M_Z^2}{q^2} + 3 + 6c_\theta^2\frac{M_Z^2}{q^2} - \frac{2}{3}\frac{q^2}{M_Z^2} - 9c_\theta^2 - 15c_\theta^2\frac{M_W^2}{q^2} + \frac{8}{3}c_\theta^2\frac{q^2}{M_Z^2} + 12c_\theta^4 + 8c_\theta^4\frac{M_W^2}{q^2})$$

$$+ (\frac{M_Z^2}{q^2} + 1 + 7\,\frac{M_W^2}{q^2} - 16c_\theta^2 - 8c_\theta^2\frac{M_W^2}{q^2})\ln c_\theta^2$$

$$+ (-\frac{M_Z^2}{q^2} - 10 - 6\,\frac{M_W^2}{q^2} - \frac{q^2}{M_Z^2} + 54c_\theta^2 + 15c_\theta^2\frac{M_W^2}{q^2}$$

$$\left. + 40c_\theta^2\frac{q^2}{M_Z^2} + 16c_\theta^4 - 8c_\theta^4\frac{M_W^2}{q^2})B_o(M_Z, M_W; q^2)\right\}$$

$$+ \frac{e^2}{3\cdot 16\pi^2}\left\{-(5 + 2\,\frac{M_W^2}{q^2})\ln\frac{M_W^2}{\mu^2}\right.$$

$$+ (\frac{2}{3}\frac{q^2}{M_W^2} + 3 + 2\,\frac{M_W^2}{q^2})$$

$$\left. + (10\,\frac{q^2}{M_W^2} + 4 - 2\,\frac{M_W^2}{q^2})\,B_o(m_\gamma, M_W; q^2)\right\}$$

$$A_H^{VV}(q^2) = \frac{M_V^2}{3\cdot 16\pi^2 v^2}\left\{-\frac{2}{3}\frac{q^2}{M_V^2} + 3 + \frac{(M_V^2 - m_H^2)^2}{q^2 M_V^2}\right.$$

$$+ 2\,\frac{m_H^2}{M_V^2}\ln\frac{m_H^2}{\mu^2} - \ln\frac{M_V^2}{\mu^2}$$

$$+ \frac{(M_V^2 - m_H^2)}{q^2 M_V^2}(m_H^2\ln\frac{m_H^2}{\mu^2} - M_V^2\ln\frac{M_V^2}{\mu^2})$$

$$\left. + (-\frac{q^2}{M_V^2} - 10 - 2\,\frac{m_H^2}{M_V^2} - \frac{(M_V^2 - m_H^2)^2}{q^2 M_V^2})\,B_o(m_H, M_V; q^2)\right\}$$

$$+ Tp$$

where Tp are constant terms originating from tadpoles

$$T_p = \frac{-1}{16\pi^2 v^2}\,\frac{2}{m_H^2}\left\{6M_W^4\left(\ln\frac{M_W^2}{\mu^2}-\frac{1}{3}\right)+3M_Z^4\left(\ln\frac{M_Z^2}{\mu^2}-\frac{1}{3}\right)\right.$$

$$\left.+\frac{3}{2}\,m_H^4\left(\ln\frac{m_H^2}{\mu^2}-1\right)-\sum_{f_s}4m_f^4\left(\ln\frac{m_f^2}{\mu^2}-1\right)\right\}.$$

$$A_F^{ZZ}(q^2) = \frac{-M_Z^2}{3\cdot 16\pi^2 v^2}\sum_{f_s}\left\{16(a_f^2+b_f^2)\left[2\,\frac{m_f^2}{M_Z^2}\ln\frac{m_f^2}{\mu^2}-\frac{1}{3}\,\frac{q^2}{M_Z^2}\right]\right.$$

$$\left.+16\left((a_f^2+b_f^2)\frac{q^2}{M_Z^2}+2(a_f^2-2b_f^2)\frac{m_f^2}{M_Z^2}\right)B_o(m_f,m_f;q^2)\right\}$$

$$A_F^{WW}(q^2) = \frac{-M_W^2}{3\cdot 16\pi^2 v^2}\sum_{f_d}2\left\{2\,\frac{m_1^2}{M_W^2}\ln\frac{m_1^2}{\mu^2}+2\,\frac{m_2^2}{M_W^2}\ln\frac{m_2^2}{\mu^2}-\frac{2}{3}\,\frac{q^2}{M_W^2}\right.$$

$$+\frac{(m_1^2-m_2^2)^2}{q^2M_W^2}-\frac{m_1^2-m_2^2}{q^2}\left(\frac{m_1^2}{M_W^2}\ln\frac{m_1^2}{\mu^2}-\frac{m_2^2}{M_W^2}\ln\frac{m_2^2}{\mu^2}\right)$$

$$\left.+\left(2\,\frac{q^2}{M_W^2}-\frac{m_1^2+m_2^2}{M_W^2}-\frac{(m_1^2-m_2^2)^2}{q^2M_W^2}\right)B_o(m_1,m_2;q^2)\right\}.$$

Selfenergies at $q^2 = 0$:

$$A_B^{ZZ}(0) = \frac{M_W^2}{16\pi^2 v^2}\left\{\,2(3-8c_\theta^2)\,\ln\frac{M_W^2}{\mu^2}+2\,\right\}$$

$$A_B^{WW}(0) = \frac{M_Z^2}{16\pi^2 v^2}\left\{\,(3-13c_\theta^2)\,\ln\frac{M_W^2}{\mu^2}+20c_\theta^2-\frac{3}{2}s_\theta^2-1\right.$$

$$\left.-(20+8c_\theta^2-\frac{17}{s_\theta^2})\,\ln c_\theta^2\right\}$$

$$A_H^{VV}(0) = \frac{M_V^2}{16\pi^2 v^2}\left\{\,3\ln\frac{M_V^2}{\mu^2}-\frac{5}{2}+\frac{1}{2}\,\frac{m_H^2}{M_V^2}-\frac{3m_H^2}{M_V^2-m_H^2}\ln\frac{m_H^2}{M_V^2}\,\right\}+Tp$$

$$A_F^{ZZ}(0) = \frac{-1}{16\pi^2 v^2} \sum_{f_s} \left\{ 2m_f^2 \ln \frac{m_f^2}{\mu^2} \right\}$$

$$A_F^{WW}(0) = \frac{-1}{16\pi^2 v^2} \sum_{f_d} \left\{ 2 \frac{m_1^4 \ln \frac{m_1^2}{\mu^2} - m_2^4 \ln \frac{m_2^2}{\mu^2}}{m_1^2 - m_2^2} - (m_1^2 + m_2^2) \right\} .$$

Counterterms: $\frac{\delta M_V^2}{M_V^2} \doteq -A^{VV}(M_V^2)$; $(Z_V - 1) \doteq M_V^2 \frac{dA^{VV}}{dq^2}(M_V^2)$

$$\left(\frac{\delta M_Z^2}{M_Z^2}\right)_B = \frac{M_W^2}{3 \cdot 16\pi^2 v^2} \left\{ (-c_\theta^{-2} - 14 + 84c_\theta^2) \ln \frac{M_W^2}{\mu^2} \right.$$

$$+ \left(\frac{8}{3} c_\theta^{-2} + \frac{70}{3} - 136c_\theta^2 - 96c_\theta^4\right)$$

$$\left. -2(1-4c_\theta^2)(c_\theta^{-2} + 16 - 68c_\theta^2 - 48c_\theta^4) G(4c_\theta^2) \right\}$$

$$\left(\frac{\delta M_W^2}{M_W^2}\right)_B = \frac{M_Z^2}{3 \cdot 16\pi^2 v^2} \left\{ (-9 + 78c_\theta^2) \ln \frac{M_W^2}{\mu^2} \right.$$

$$+ (c_\theta^{-2} + 23 - 230c_\theta^2)$$

$$- \frac{1}{2c_\theta^2} (-c_\theta^{-2} - 14 + 84c_\theta^2) \ln c_\theta^2$$

$$\left. + \frac{1}{c_\theta^2} (1-4c_\theta^2)(c_\theta^{-2} + 16 - 68c_\theta^2 - 48c_\theta^4) F(4c_\theta^2) \right.$$

$$\left(\frac{\delta M_V^2}{M_V^2}\right)_H = \frac{M_V^2}{3 \cdot 16\pi^2 v^2} \left\{ (-10) \ln \frac{M_V^2}{\mu^2} + \left(\frac{62}{3} - 6 \frac{m_H^2}{M_V^2} + \frac{m_H^4}{M_V^4}\right) \right.$$

$$\left. + \frac{m_H^2}{M_V^2} \left(-9 + 3 \frac{m_H^2}{M_V^2} - \frac{1}{2} \frac{m_H^4}{M_V^4}\right) \ln \frac{m_H^2}{M_V^2} \right.$$

$$+\frac{m_H^2}{M_V^2}\,(1-4\,\frac{M_V^2}{m_H^2})\,(12-4\,\frac{m_H^2}{M_V^2}+\frac{m_H^4}{M_V^4})\,F(4\,\frac{M_V^2}{m_H^2})\Big\}-Tp$$

$$(\frac{\delta M_Z^2}{M_Z^2})_F=\frac{M_Z^2}{3\cdot 16\pi^2 v^2}\sum_{f_s}\Big\{16(a_f^2+b_f^2)\Big[2\frac{m_f^2}{M_Z^2}\ln\frac{m_f^2}{\mu^2}-\frac{1}{3}\Big]$$

$$+16\,((a_f^2+b_f^2)+2(a_f^2-2b_f^2)\frac{m_f^2}{M_Z^2})\,B_o(m_f,m_f;M_Z^2)\Big\}$$

$$(\frac{\delta M_W^2}{M_W^2})_F=\frac{M_W^2}{3\cdot 16\pi^2 v^2}\sum_{f_d}2\Big\{2\frac{m_1^2}{M_W^2}\ln\frac{m_1^2}{\mu^2}+2\frac{m_2^2}{M_W^2}\ln\frac{m_2^2}{\mu^2}-\frac{2}{3}+\frac{(m_1^2-m_2^2)^2}{M_W^4}$$

$$-\frac{m_1^2-m_2^2}{M_W^2}\Big(\frac{m_1^2}{M_W^2}\ln\frac{m_1^2}{\mu^2}-\frac{m_2^2}{M_W^2}\ln\frac{m_2^2}{\mu^2}\Big)$$

$$+\,(2-\frac{m_1^2+m_2^2}{M_W^2}-\frac{(m_1^2-m_2^2)^2}{M_W^4})\,B_o(m_1,m_2;M_W^2)\Big\}$$

$$(Z_Z-1)_B=\frac{M_W^2}{3\cdot 16\pi^2 v^2}\Big\{(c_\theta^{-2}-4-36c_\theta^2)\ln\frac{M_W^2}{\mu^2}$$

$$+(-\frac{5}{3}c_\theta^{-2}+\frac{80}{3}+4c_\theta^2-48c_\theta^4)$$

$$+2(c_\theta^{-2}-6+12c_\theta^2+8c_\theta^4-96c_\theta^6)\,G(4c_\theta^2)\Big\}$$

$$(Z_W-1)_B=\frac{M_Z^2}{3\cdot 16\pi^2 v^2}\Big\{-c_\theta^2(15+24)\ln\frac{M_W^2}{\mu^2}$$

$$+(2c_\theta^{-2}+22-43c_\theta^2)$$

$$+(c_\theta^{-4}+\frac{19}{2}c_\theta^{-2}-57+42c_\theta^2+24c_\theta^4)\ln c_\theta^2$$

$$+(2c_\theta^{-4}+15c_\theta^{-2}-156+268c_\theta^2-48c_\theta^4)\,F(4c_\theta^2)\Big\}$$

$$(Z_W-1)_{QED}=\frac{e^2}{3\cdot 16\pi^2}\Big\{6\ln\frac{M_W^2}{m_\gamma^2}\Big\}$$

$$(Z_V - 1)_H = \frac{M_V^2}{3\cdot 16\pi^2 v^2}\left\{\ln\frac{M_V^2}{\mu^2} + \left(\frac{31}{3} - 6\,\frac{m_H^2}{M_V^2} + 2\,\frac{m_H^4}{M_V^4}\right)\right.$$

$$+\left(6 - 9\,\frac{m_H^2}{M_V^2} + \frac{9}{2}\,\frac{m_H^4}{M_V^4} - \frac{m_H^6}{M_V^6}\right)\ln\frac{m_H^2}{M_V^2}$$

$$\left.+\left(-36+32\frac{m_H^2}{M_V^2} - 13\frac{m_H^4}{M_V^4} + 2\frac{m_H^6}{M_V^6}\right)\,F\left(4\frac{M_V^2}{m_H^2}\right)\right\}$$

$$(Z_Z - 1)_F = \frac{-M_Z^2}{3\cdot 16\pi^2 v^2}\sum_{f_s}\left\{16\,(a_f^2+b_f^2)\left[-\frac{1}{3} + B_o(m_f, m_f; M_Z^2)\right]\right.$$

$$\left.+16\left((a_f^2+b_f^2) + 2\,(a_f^2-2b_f^2)\frac{m_f^2}{M_Z^2}\right)M_Z^2\;\dot{B}_o(m_f, m_f; M_Z^2)\right\}$$

$$(Z_W - 1)_F = \frac{-M_W^2}{3\cdot 16\pi^2 v^2}\sum_{f_d} 2\left\{-\frac{2}{3} - \frac{(m_1^2-m_2^2)^2}{M_W^4} + \frac{m_1^2-m_2^2}{M_W^2}\left(\frac{m_1^2}{M_W^2}\ln\frac{m_1^2}{\mu^2} - \frac{m_2^2}{M_W^2}\ln\frac{m_2^2}{\mu^2}\right)\right.$$

$$+\left(2 + \frac{(m_1^2-m_2^2)^2}{M_W^4}\right)\,B_o(m_1, m_2; M_W^2)$$

$$\left.+\left(2 - \frac{m_1^2+m_2^2}{M_W^2} - \frac{(m_1^2-m_2^2)^2}{M_W^4}\right)\,M_W^2\;\dot{B}_o(m_1, m_2; M_W^2)\right\}$$

Remark: In the limit $s_\theta^2 \to 0$ of mass degenerate multiplets

$$M_Z = M_W = M\,, \quad m_1 = m_2 = m$$

$$\frac{\delta M_Z^2}{M_Z^2} = \frac{\delta M_W^2}{M_W^2} = \frac{\delta M^2}{M^2}\;;\quad (Z_Z - 1) = (Z_W - 1) = (Z - 1)$$

with

$$\left(\frac{\delta M^2}{M^2}\right)_B = \frac{M^2}{3\cdot 16\pi^2 v^2}\left\{\,69\ln\frac{M^2}{\mu^2} - 206 + 33\cdot\sqrt{3}\,\pi\right\}$$

$$(Z-1)_B = \frac{M^2}{3\cdot 16\pi^2 v^2}\left\{-39\ln\frac{M^2}{\mu^2} - 19 + 81\,\frac{\pi}{3\sqrt{3}}\right\}$$

$$\left(\frac{\delta M^2}{M^2}\right)_F = \frac{M^2}{3\cdot 16\pi^2 v^2} \sum_{f_d} 4\left\{\left(3\frac{m^2}{M^2} - 1\right)\ln\frac{m^2}{\mu^2} + \frac{5}{3} - 2\frac{m^2}{M^2} - 2\left(1-\frac{y}{4}\right)(1-y)G(y)\right\}$$

$$(Z-1)_F = \frac{M^2}{3\cdot 16\pi^2 v^2} \sum_{f_d} 4\left\{\ln\frac{m^2}{\mu^2} - \frac{2}{3} - \frac{m^2}{M^2} + \left(2-y-\frac{y^2}{4}\right)G(y)\right\}$$

$$y = \frac{4m^2}{M^2}$$

Lepton and neutrino wave function renormalization

$(m_\ell \to 0)$

$$Z_\ell = 1 + z_a + z_b\,\gamma_5$$

$$z_a = \frac{M_W^2}{16\pi^2 v^2}\left\{\left(\frac{5}{2c_\theta^2} - 6 + 4c_\theta^2\right)\left(\ln\frac{M_Z^2}{\mu^2} + \frac{1}{2}\right) + \left(\ln\frac{M_W^2}{\mu^2} + \frac{1}{2}\right)\right\}$$

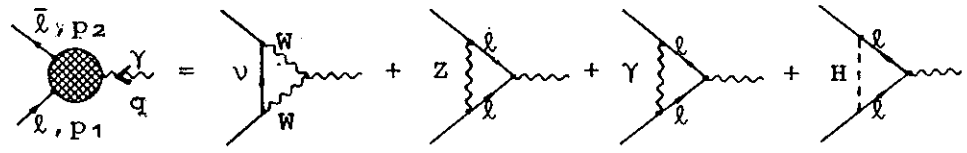
$$(z_a)_{QED} = \frac{e^2}{16\pi^2}\left\{2\ln\frac{m_\ell^2}{m_\gamma^2} + \ln\frac{m_\ell^2}{\mu^2} - 4\right\}$$

$$z_b = \frac{M_W^2}{16\pi^2 v^2}\left\{\left(\frac{3}{2c_\theta^2} - 2\right)\left(\ln\frac{M_Z^2}{\mu^2} + \frac{1}{2}\right) - \left(\ln\frac{M_W^2}{\mu^2} + \frac{1}{2}\right)\right\}$$

$$Z_\nu = 1 + z_\nu(1-\gamma_5)$$

$$z_\nu = \frac{M_W^2}{16\pi^2 v^2}\left\{\frac{1}{2c_\theta^2}\left(\ln\frac{M_Z^2}{\mu^2} + \frac{1}{2}\right) + \left(\ln\frac{M_W^2}{\mu^2} + \frac{1}{2}\right)\right\}$$

b) Form factors for on-shell fermions including fermion wave function renormalizations (z_a, z_b and z_ν) in the approximation $m_f \to 0$. Amplitudes which vanish in this limit (as well as longitudinal terms (owing to $\varepsilon_\alpha(q,s)q^\alpha = 0$)) are omitted in the covariant decomposition of amplitudes.



$$= -ie \left\{ \gamma^\mu (A_{10} + A_{20}\gamma_5) + \frac{(p_2+p_1)^\mu}{2m_\ell} A_3 \right\}$$

$$= -ie \left\{ \gamma^\mu ((A_{10} + A_3) + A_{20}\gamma_5) - i\sigma^{\mu\alpha} \frac{q_\alpha}{2m_\ell} A_3 \right\}$$

i) The renormalized lepton form factor at $q^2 = 0$ is ($m_\ell \neq 0$)

$$A_{10}^{\gamma\ell\ell}(0) + A_3^{\gamma\ell\ell}(0) + z_a + \frac{\delta e}{e} + \frac{1}{2}(Z_\gamma - 1) + 2aA^{\gamma Z}(0) = 0$$

$$A_{20}^{\gamma\ell\ell}(0) + z_b + 2bA^{\gamma Z}(0) = 0 \quad \text{(e.m. WT-identity)}$$

$$A_3^{\gamma\ell\ell}(0) = -\frac{\alpha}{2\pi} \quad (a_e = -A_3(0) \text{ anomalous magnetic moment}).$$

This is the anomalous magnetic moment of the lepton.
The first equation corresponds to eq. (2.19i) and determines the charge renormalization with the flavor independent result

$$\frac{\delta e}{e} = -(A_{10}^{\gamma\ell\ell}(0) + A_3^{\gamma\ell\ell}(0) + z_a + \frac{1}{2}(Z_\gamma - 1) + 2aA^{\gamma Z}(0))$$

$$= \frac{e^2}{16\pi^2} \left(\frac{7}{2} \ln \frac{M_W^2}{\mu^2} - \frac{1}{3} - \frac{2}{3} \sum_{f_s} Q_f^2 \ln \frac{m_f^2}{\mu^2} \right)$$

(charge universality).

ii) QED contributions for $q^2 \neq 0$ $(y \doteq \frac{4m_\ell^2}{q^2})$

$$A_{1QED}^{\gamma\ell\ell} = (A_{10} + z_a)_{QED} = \frac{e^2}{2\pi^2}\left\{\frac{1}{2}\ln\frac{m_\ell}{m_\gamma} - \frac{1}{2} + \frac{3}{4}(1-y)F_3(y)\right.$$

$$\left. + \frac{1}{2}(1-\frac{y}{2})F_3(y)\ln\frac{m_\gamma^2}{q^2} + \frac{1}{4}(1-\frac{y}{2})F_4(y)\right\}^+$$

$$A_{2QED}^{\gamma\ell\ell} = (A_{20} + z_b)_{QED} = 0$$

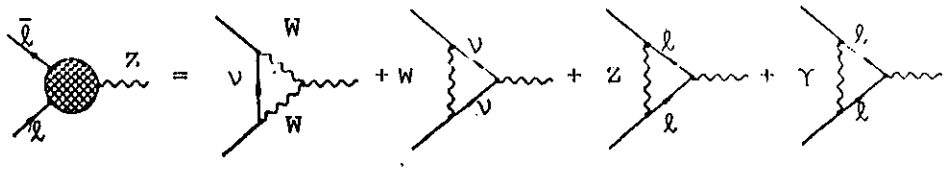
$$A_{3QED}^{\gamma\ell\ell} = \frac{2e^2}{16\pi^2}\, y\, F_3(y)$$

where (with $\xi = \frac{\sqrt{1-y}-1}{\sqrt{1-y}+1}$

$$F_3(y) = -\frac{1}{\sqrt{1-y}}\ln\xi$$

$$F_4(y) = \frac{1}{2\sqrt{1-y}}\left\{4\,Sp(-\xi) + \frac{\pi^2}{3} - \ln^2\xi + 4\ln\xi\ln(1+\xi)\right\}.$$

<u>$Z\ell\bar{\ell}$-vertex:</u>



$$\doteq i\,\frac{2M_Z}{v}\left\{\gamma^\mu((a+A_{10}) + (b+A_{20})\gamma_5) + \frac{(p_2+p_1)^\mu}{2m_\ell}\,A_3\right\}$$

$$A_1^{Z\ell\ell} = A_{10} + a\,z_a + b\,z_b = A_{1n} + A_c + a\,A_{1QED}^{\gamma\ell\ell}$$

$$A_2^{Z\ell\ell} = A_{20} + b\,z_a + a\,z_b = A_{2n} - A_c + b\,A_{1QED}^{\gamma\ell\ell} + A_{3QED}^{\gamma\ell\ell}$$

$$A_3^{Z\ell\ell} = A_{3w} + a\,A_{3QED}^{\gamma\ell\ell}$$

+For $m_\ell^2 \ll q^2 = s$

$$A_{1QED}^{\gamma\ell\ell} = \frac{\alpha}{2\pi}\left\{-\ln\frac{s}{m_\ell^2}\ln\frac{s}{m_\gamma^2} + \ln\frac{m_\ell^2}{m_\gamma^2} + \frac{1}{2}\ln^2\frac{s}{m_\ell^2} + \frac{3}{2}\ln\frac{s}{m_\ell^2} - 2 + 2\,\frac{\pi^2}{3}\right\}.$$

i) zeor momentum transfer: $q^2 = 0$

$$A_{1n} = A_{2n} = 0$$

$$A_c = \frac{M_W^2}{16\pi^2 v^2} \left\{ 2c_\Theta^2 \ln \frac{M_W^2}{\mu^2} \right\}$$

$$A_{3w} = 0.$$

The neutral current amplitudes used in sec. 2 are:

$$A_L^{Z\ell\ell} = -4A_c \quad \text{and} \quad A_K^{Z\ell\ell} = 4A_c.$$

ii) On-shell: $q^2 = M_Z^2$

$$A_{1n} = \frac{M_Z^2}{16\pi^2 v^2} \left\{ -2(1 - 6s_\Theta^2 + 12s_\Theta^4 - 16s_\Theta^6)\left(-\frac{11}{16} + \frac{\pi^2}{12}\right) \right\}$$

$$A_{2n} = \frac{M_Z^2}{16\pi^2 v^2} \left\{ 2(1 - 6s_\Theta^2 + 12s_\Theta^4)\left(-\frac{11}{16} + \frac{\pi^2}{12}\right) \right\}$$

$$A_c = \frac{M_Z^2}{16\pi^2 v^2} \left\{ 2c_\Theta^4 \ln \frac{M_W^2}{\mu^2} - \frac{c_\Theta^2}{4}(7 + 14c_\Theta^2 - 8c_\Theta^4) \right.$$

$$- \frac{c_\Theta^2}{2}(3 + 2c_\Theta^2) \ln c_\Theta^2 - 2c_\Theta^4(1 + 2c_\Theta^2)\left(\sqrt{4c_\Theta^2 - 1}\ \operatorname{arctg} \frac{1}{\sqrt{4c_\Theta^2 - 1}}\right)$$

$$+ 4c_\Theta^6\left(1 + \frac{1}{2}c_\Theta^2\right) \cdot \left(2\operatorname{arctg} \frac{1}{\sqrt{4c_\Theta^2 - 1}}\right)^2$$

$$\left. + c_\Theta^2(1 + c_\Theta^2)^2 \cdot \left(\mathrm{Sp}\left(1 + \frac{1}{c_\Theta^2}\right) - \frac{\pi^2}{6}\right) \right\}$$

$$A_{3w} = 0.$$

For the reader's convenience we write down the expression also for the linear combination eq. (6.7) which shows up as a correction term for the Z-decay width. We define

$$A_\Gamma = \frac{aA_1 + bA_2}{a^2 + b^2} \; ; \quad f(s_\Theta^2) = 8(a^2 + b^2)$$

and use the relation

$$\frac{M_Z^2}{16\pi^2 v^2}\,\frac{(a,b)}{a^2+b^2} = \frac{\alpha}{8\pi s_\Theta^2 f(s_\Theta^2)}\,\left(\frac{3}{c_\Theta^2} - 4\ ,\ \frac{1}{c_\Theta^2}\right) .$$

for the evaluation of A_Γ .

As a result we obtain

$$A_\Gamma^{Z\ell\ell} = \frac{\alpha}{8\pi s_\Theta^2 f(s_\Theta^2)} \left\{ \frac{4}{c_\Theta^2}\,(1 - 8s_\Theta^2 + 24s_\Theta^4 - 32s_\Theta^6 c_\Theta^2)\,(\frac{\pi^2}{12} - \frac{11}{16}) \right.$$

$$+ (1 - \frac{1}{2c_\Theta^2}) \left[-8c_\Theta^4 \ln \frac{M_W^2}{\mu^2} + c_\Theta^2(7 + 14c_\Theta^2 - 8c_\Theta^4) \right.$$

$$+ 2c_\Theta^2(3 + 2c_\Theta^2) \ln c_\Theta^2$$

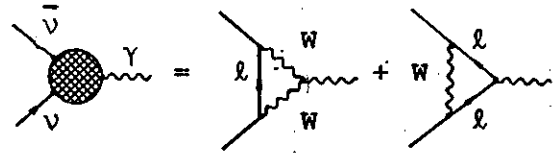
$$+ 8c_\Theta^4(1 + 2c_\Theta^2)\,(\sqrt{4c_\Theta^2 - 1}\ \mathrm{arctg}\ \frac{1}{\sqrt{4c_\Theta^2 - 1}})$$

$$- 16c_\Theta^6(1 + \tfrac{1}{2}c_\Theta^2)\,(2\,\mathrm{arctg}\frac{1}{\sqrt{4c_\Theta^2 - 1}})^2$$

$$\left. \left. - 4c_\Theta^2(1 + c_\Theta^2)^2\ \ (\mathrm{Sp}(1 + \frac{1}{c_\Theta^2}) - \frac{\pi^2}{6}) \right] \right\}$$

in agreement with Ref. [7].

<u>γνν-vertex:</u>



$$= ie\,\gamma^\mu\,\Pi_-\,A_{Lo}\ ,\quad A_L^{\gamma\nu\nu} = A_{Lo}$$

i) $q^2 \ll M_W^2$:

$$A_L = -\frac{4M_W^2}{16\pi^2 v^2} \left\{ \ln \frac{M_W^2}{\mu^2} + \frac{1}{3}\frac{q^2}{M_W^2} \left\{ (\ln \frac{M_W^2}{m_\ell^2} + 1) + \frac{5}{3} + \frac{4m_\ell^2}{q^2} \right.\right.$$

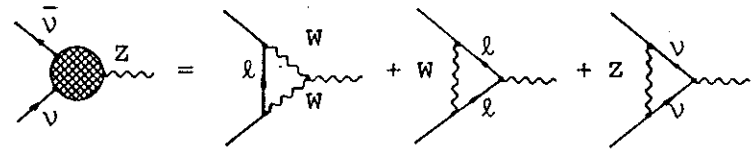
$$\left.\left. + 2(1 + \frac{2m_\ell^2}{q^2})\,(\frac{4m_\ell^2}{q^2} - 1)\ G(\frac{4m_\ell^2}{q^2}) \right\} \right\}$$

The neutral current amplitude used in sec. 2 is:

ii) $q^2 = 0$: $A_1^{\gamma\nu\nu} = A_L$ (γee-coupling at the electron vertex)

$$A_L = -\frac{4M_W^2}{16\pi^2 v^2} \ln \frac{M_W^2}{\mu^2} .$$

<u>Z$\nu\nu$-vertex</u>:



$$= i \frac{M_Z}{v} \gamma^\mu \Pi_- (1 + A_{LO}); \quad A_L^{Z\nu\nu} = A_{LO} + 2z_\nu$$

i) zero momentum transfer $q^2 = 0$:

$$A_L = \frac{4M_W^2}{16\pi^2 v^2} \left\{ - 2c_\Theta^2 \ln \frac{M_W^2}{\mu^2} \right\} .$$

ii) On-shell: $q^2 = M_Z^2$

$$A_L = \frac{2M_W^2}{16\pi^2 v^2} \left\{ - 4c_\Theta^2 \ln \frac{M_W^2}{\mu^2} + \frac{4}{c_\Theta^2}\left(- \frac{11}{16} + \frac{\pi^2}{12}\right) \right.$$

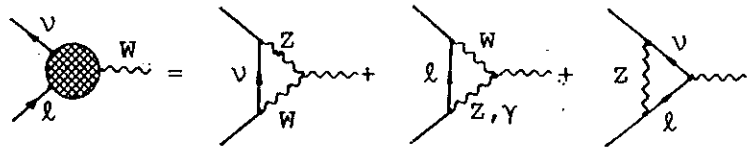
$$- \frac{7}{2} + 10c_\Theta^2 + 2c_\Theta^2 \left(1 - \frac{1}{2c_\Theta^2}\right)(3 + 2c_\Theta^2) \ln c_\Theta^2$$

$$+ 4c_\Theta^2 (1+2c_\Theta^2) \left(\sqrt{4c_\Theta^2 - 1} \operatorname{arctg} \frac{1}{\sqrt{4c_\Theta^2 - 1}}\right)$$

$$- 8c_\Theta^4 \left(1 + \frac{1}{2}c_\Theta^2\right) \left(2 \operatorname{arctg} \frac{1}{\sqrt{4c_\Theta^2 - 1}}\right)^2$$

$$\left. - 4c_\Theta^2 \left(1 - \frac{1}{2c_\Theta^2}\right)(1 + c_\Theta^2)^2 \left(\mathrm{Sp}\left(1 + \frac{1}{c_\Theta^2}\right) - \frac{\pi^2}{6}\right) \right\}$$

$W\ell\nu_\ell$-vertex:



$$= i\sqrt{2}\,\frac{M_W}{v}\,\gamma^\mu \Pi_-\,(1 + A_{Lo})\,;$$

$$A_L^{W\ell\nu} + A_{QED}^{W\ell\nu} = A_{Lo} + \left(\frac{z_a - z_b}{2}\right)_\ell + z_\nu$$

i) zero momentum transfer $q^2 = 0$:

$$A_L = \frac{-4M_W^2}{16\pi^2 v^2}\left\{(2 + \frac{s_\Theta^2}{2})\,\ln\frac{M_W^2}{\mu^2} + (\frac{1}{2} - \frac{3}{s_\Theta^2})\,c_\Theta^2 \ln c_\Theta^2 + (\frac{s_\Theta^2}{4} - 3)\right\}$$

$$A_{QED} = +\frac{e^2}{16\pi^2}\left\{\ln\frac{m_\ell^2}{m_\gamma^2} + \frac{1}{2}\ln\frac{m_\ell^2}{\mu^2} - 2\right\}.$$

Only the virtual photon contributions to the lepton wave function renormalizations are included in A_{QED}. The infrared finite term $-\frac{4M_W^2}{16\pi^2 v^2}\,s_\Theta^2(3\ln\frac{M_W^2}{\mu^2} - \frac{5}{2})$ from the diagram with the photon line is contained in A_L.

ii) On-shell $q^2 = M_W^2$:

$$A_L = \frac{2M_W^2}{16\pi^2 v^2}\left\{-4\ln\frac{M_W^2}{\mu^2} - \frac{1}{c_\Theta^4} + \frac{1}{4c_\Theta^2} + 4 + \frac{c_\Theta^2}{2}\right.$$

$$+ (\frac{1}{c_\Theta^4} - \frac{3}{2c_\Theta^2} - 3 + 3c_\Theta^2)\ln c_\Theta^2$$

$$+ 4(1 + \frac{1}{2c_\Theta^2})(4c_\Theta^2 - 1)\;F(4c_\Theta^2)$$

$$- 2(1 - \frac{1}{2c_\Theta^2})(1 + \frac{1}{c_\Theta^2})^2\;(Sp(1 + c_\Theta^2) - \frac{\pi^2}{6})$$

$$\left. - 8(1 + \frac{1}{2}c_\Theta^2)(\frac{1}{4}\ln^2 c_\Theta^2 + (\text{arctg}\,\sqrt{4c_\Theta^2 - 1})^2)\right\}$$

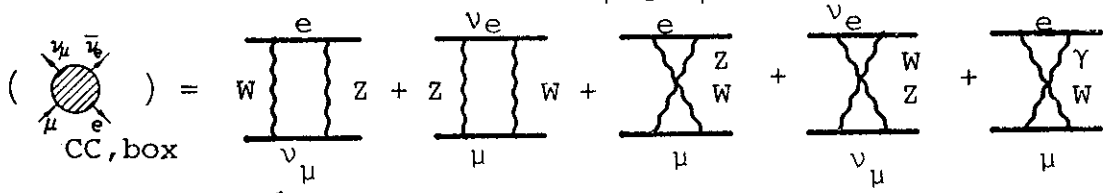
, in agreement with Ref. [7].

$$A_{QED} = \frac{e^2}{16\pi^2}\left\{\ln\frac{m_\ell^2}{m_\gamma^2} + \frac{3}{2}\ln\frac{M_W^2}{m_\ell^2} - \frac{1}{2}\ln\frac{M_W^2}{m_\ell^2}\left(\ln\frac{m_\ell^2}{m_\gamma^2} + \ln\frac{M_W^2}{m_\gamma^2}\right)\right\} .$$

The UV-divergent term $-\frac{e^2}{16\pi^2}\left(\frac{5}{2}\ln\frac{M_W^2}{\mu^2} - 1\right)$ of the virtual photon corrections has been included in A_L. This splitting of terms is chosen in accordance with the result of Marciano and Sirlin [18] for the photonic corrections to W-decay.

c) Box diagrams

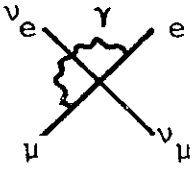
i) Charged current (CC): $\mu^- \to e^-\nu_\mu\bar{\nu}_e$ $(\nu_\mu e^- \to \mu^-\nu_e)$



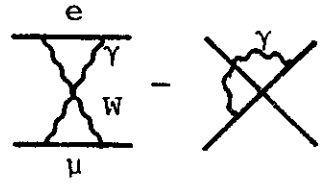
$$= -\frac{1}{2v^2}\,(\bar{\mu}\gamma^\alpha(1-\gamma_5)\nu_\mu)\,(\bar{\nu}_e\gamma_\alpha(1-\gamma_5)e)\cdot(A_{LCC}^{box} + R_{QED}^{Fermi})$$

$$A_{LCC}^{box} = \frac{-4M_W^2}{16\pi^2v^2}\,\frac{1}{2s_\Theta^2}\,(-3 + 6c_\Theta^2 + 2c_\Theta^4)\ln c_\Theta^2 + R_W .$$

The terms R_W and R_{QED}^{Fermi} follow from the last diagram. R_{QED}^{Fermi} is defined as the QED correction to the local four fermion interaction

$R_{QED}^{Fermi} =$ 

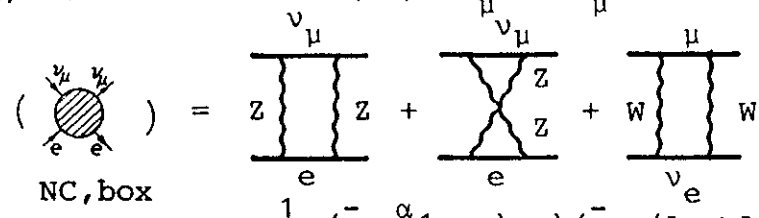
and correspondingly one finds

$R_W =$ 

$= \frac{4M_W^2}{16\pi^2v^2}\,\frac{s_\theta^2}{2}\left\{2\ln\frac{M_W^2}{\mu^2} + 1\right\}$

for the remainder.

ii) Neutral current (NC): $\nu_\mu e \to \nu_\mu e$



NC,box

$$= -\frac{1}{v^2}\,(\bar{\nu}_\mu \gamma^\alpha (1-\gamma_5)\nu_\mu)\,(\bar{e}\gamma_\alpha (A_1 + A_2\gamma_5) e)$$

$$A_1 = \frac{M_W^2}{16\pi^2 v^2}\,(-28 + 9c_\Theta^{-2})/4$$

$$A_2 = \frac{M_W^2}{16\pi^2 v^2}\,(24c_\Theta^2 - 20 + 15c_\Theta^{-2})/4$$

$$A_{LNC}^{box} = 4\,A_2 = \frac{4M_W^2}{16\pi^2 v^2}\;\frac{24c_\Theta^4 - 20c_\Theta^2 + 15}{4\,c_\Theta^2}$$

$$A_{\kappa NC}^{box} = \frac{A_1 - (4s_\Theta^2 - 1)A_2}{s_\Theta^2} = \frac{-4M_W^2}{16\pi^2 v^2}\;\frac{24c_\Theta^4 - 14c_\Theta^2 + 9}{4\,c_\Theta^2}$$

For $\nu_\mu q \to \nu_\mu q$ on isoscalar targets "$q = \frac{u+d}{2}$"

$$A_{LNC}^{qbox} = \frac{4M_W^2}{16\pi^2 v^2}\,c_\Theta^{-4}\left(\frac{5}{2} - \frac{15}{4}\,s_\Theta^2 - \frac{1}{5}\,s_\Theta^4 + \frac{14}{9}\,s_\Theta^6\right)$$

$$A_{\kappa NC}^{qbox} = \frac{-4M_W^2}{16\pi^2 v^2}\,c_\Theta^{-4}\left(\frac{5}{2} - \frac{61}{20}\,s_\Theta^2 - \frac{9}{10}\,s_\Theta^4 + \frac{14}{9}\,s_\Theta^6\right)$$

Appendix B: $(\nu_\mu e)$-scattering at small momentum transfer [4]

The definition of $\sin^2\theta_o$ used in the text looks pretty academic since neutrino scattering cross sections are measured at finite momentum transfer $-q^2 = Q^2 << M_Z^2$. Thus the deviation from the local current-current interaction must be taken into account. The q^2-dependence to be considered is already indicated in Eq. (2.22). We may include it into effective amplitudes at the electron vertex

$$\gamma^\mu(a+b\gamma_5)_{eff} = \gamma^\mu(A(q^2) + B(q^2)\gamma_5) + C(q^2)\,\frac{(p_e' + p_e)^\mu}{2m_e} \quad .$$

The amplitude $C(q^2)$ is added for completeness here, it vanishes in the limit $m_e \to 0$. The differential cross section is given by

$$\frac{d\sigma}{dt} = \frac{1}{2\pi}\left\{|A \mp B|^2 + |A \pm B|^2\left(\frac{u-m_e^2}{s-m_e^2}\right)^2 + 2(|A|^2 - |B|^2)\,\frac{m_e^2 t}{(s-m_e^2)^2}\right.$$
$$\left. + 4\mathrm{Re}(AC^*)\left(1 + \frac{st}{(s-m_e^2)^2}\right)\right\} \qquad (B.1)$$

for $(\nu_\mu e)$- and $(\bar{\nu}_\mu e)$-scattering, respectively, with $s = (p_e + p_\nu)^2$, $t = q^2 = (p_e - p_e')^2$ and $u = 2m_e^2 - s - t$.

The kinematical constraints on t read

$$2m_e^2 - s - \frac{1}{2}\frac{m_e^4}{s} \leq t \leq \frac{1}{2}\frac{m_e^4}{s} \quad .$$

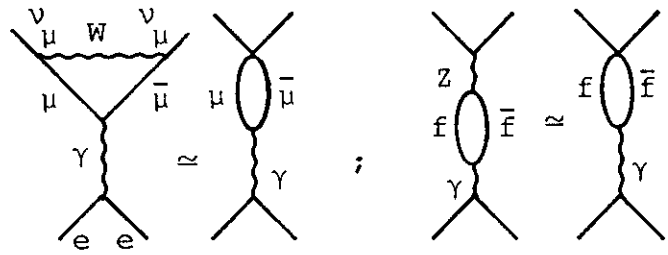
For small momentum transfers $Q^2 << M_Z^2$, terms of the order $O(\alpha\frac{q^2}{M_W^2})$ may be ignored. The remaining q^2-dependence is owing to

i) the Z-boson exchange in the Born term giving rise to the correction

$$\frac{M_Z^2}{M_Z^2-q^2} \simeq 1 + \frac{q^2}{M_Z^2}$$

of G_{no}^{eff} , and

ii) the $\gamma\nu\nu$ neutrino charge radius and the γZ-mixing diagrams, which in the limit $m_f^2, -q^2 << M_W^2, M_Z^2$ reduce to penguin diagrams



and lead to a q^2-dependent correction of $\sin^2\theta_o^{eff}$.

Thus, in this approximation, the q^2-dependence may be fully accounted for by using the running effective NC-parameters

$$\hat{G}_{NC}(Q^2) = G_{NC}(1 - \frac{Q^2}{M_Z^2}) = \hat{\rho}_{NC}(Q^2) G_\mu$$

$$\hat{\rho}_{NC}(Q^2) = \rho_o \ (1 - \frac{Q^2}{M_Z^2}) \qquad (B.2)$$

and

$$\sin^2\hat{\theta}\ (Q^2) = \sin^2\theta_o \ (1 + \frac{4M_W^2}{16\pi^2 v^2} \ (F(q^2) - F(0))) \qquad (B.3)$$

where

$$F(q^2) = F^{\gamma\nu\nu}(q^2) + F^{\gamma Z}(q^2)$$

with

$$F^{\gamma\nu\nu}(q^2) = \frac{16\pi^2 v^2}{2} \frac{1}{q^2} \ (A_1^{\gamma\nu\nu}(q^2) - A^{\gamma Z}(0))$$

$$F^{\gamma Z}(q^2) = -\frac{16\pi^2 v^2}{2} \frac{1}{q^2} \ (A^{\gamma Z}(q^2) - A^{\gamma Z}(0)).$$

The term $A^{\gamma Z}(0)$ has been added and subtracted in the definitions of $F^{\gamma\nu\nu}$ and $F^{\gamma Z}$, respectively, such that both functions exist separately in the limit $q^2 \to 0$.

In the $\overline{MS}$ scheme for the 't Hooft-Feynman gauge we have

$$F^{\gamma\nu\nu}(0) = -\frac{2}{3}\left(\ell n \frac{M_W^2}{m_\mu^2} + 1\right)$$

$$F^{\gamma Z}(0) = \left(3 c_\theta^2 + \frac{1}{6}\right) \ell n \frac{M_W^2}{\mu^2} - \frac{2c_\theta^2+1}{3} - \frac{1}{3} \sum_{f_s} (4 a_f Q_f) \ell n \frac{m_f^2}{\mu^2} \qquad (B.4)$$

where the last term in $F^{\gamma Z}(0)$ are the fermion contributions to the γZ mixing. The sum $\sum_{f_s}$ extends over the single fermions (including color), a_f is the neutral current vector coupling (2.1c) and Q_f the charge.

In the approximation $m_f^2, -q^2 << M_W^2$, then, the contribution $\Delta F = F(q^2) - F(0)$ to (B.3) is given by

$$\Delta F^{\gamma\nu\nu}(q^2) = -\frac{2}{3} G_\mu(y_\mu)$$

$$\Delta F^{\gamma Z}(q^2) = \frac{1}{3} \sum_{f_s} (4 a_f Q_f) G_f(y_f) \qquad (B.5)$$

where

$$G_i(y_i) = \frac{5}{3} + y_i + \left(1 + \frac{y_i}{2}\right)\sqrt{1-y_i}\, \ell n \frac{\sqrt{1-y_i}-1}{\sqrt{1-y_i}+1} \; ; \; y_i = \frac{4m_i^2}{q^2} .$$

By definition $G_i(\infty) = 0$ (value at $q^2 = 0$) and

$$G_i(y_i) \simeq - \ell n \frac{Q^2}{m_i^2} + \frac{5}{3} \; ; \quad Q^2 = -q^2 >> m_i^2 .$$

Appendix C: Higgs- and top-quark contributions (asymptotic forms).

The asymptotic contributions to the parameter shifts (2.29) and (6.10) for a heavy Higgs particle and for a heavy top-quark are given by $(K = \frac{3}{16}\frac{g^2}{12\pi^2})$

(i) for $m_H \gg M_W$ $[m_H \ll M_W]$

$$ctg^2\theta_W \varepsilon^H \simeq -3K\,(\ln\frac{m_H^2}{M_W^2} - \frac{5}{6}) \qquad [3K\,(\frac{\ln c_\theta^2}{s_\theta^2} + \frac{5}{6})]$$

$$\Delta r^H + ctg^2\theta_W \varepsilon^H \simeq \frac{2K}{3}\,(\ln\frac{m_H^2}{M_W^2} - \frac{5}{6}) \qquad [-\frac{K}{3}\,(\frac{\ln c_\theta^2}{s_\theta^2} + \frac{79}{3})]$$

$$\Delta g^H \simeq \frac{K}{3}\,(\ln\frac{m_H^2}{M_W^2} - \frac{5}{6}) \qquad [-\frac{K}{3}\,\frac{79}{6}\,]$$

$$\Delta C^{VH} \simeq 0 \qquad [2K(\ln\frac{m_H^2}{M_V^2} + \frac{47}{12})]$$

$$\Delta C^{mixH} - ctg^2\theta_W \varepsilon^H \simeq -\frac{K}{3}\,(\ln\frac{m_H^2}{M_W^2} - \frac{5}{6}) \qquad [\frac{K}{3}\,(\frac{\ln c_\theta^2}{s_\theta^2} + \frac{79}{6})]$$

(ii) for $m_t \gg M_W$ $[m_t \ll M_W]$

$$\varepsilon^{top} \simeq 3K\,\frac{m_t^2}{M_W^2} \qquad [\ 0\ ]$$

$$\Delta r^{top} + ctg^2\theta_W \varepsilon^{top} \simeq \begin{cases} -K(2(ctg^2\theta_W - \frac{1}{3})\ln\frac{m_t^2}{M_W^2} + \frac{4}{3}\ln c_\theta^2 + ctg^2\theta_W - \frac{7}{9}) \\ [-K(\frac{64}{9}\,s_\theta^2\,(\ln\frac{m_t^2}{M_Z^2} + \frac{5}{3}) + 2(ctg^2\theta_W - 1)\ln c_\theta^2)] \end{cases}$$

$$\Delta g^{top} \simeq \frac{2K}{3}\,(2\ln\frac{m_t^2}{M_W^2} + 3) \qquad [\frac{2K}{3}\,(\ln\frac{M_W^2}{m_t^2} - 5)]$$

$$\Delta C^{W\,top} \simeq -2K \qquad [2K]$$

$$\Delta C^{Z\,top} \simeq 0 \qquad [\frac{2K}{c_\theta^2}(1 - \frac{8}{3}s_\theta^2 + \frac{32}{9}s_\theta^4)]$$

$$\Delta C^{mix\ top} - ctg^2\theta_W \varepsilon^{top} \simeq \begin{cases} K(2(ctg^2\theta_W + \frac{1}{3})\ln\frac{m_t^2}{M_W^2} + \frac{2}{3}\ln c_\theta^2 + ctg^2\theta_W + \frac{1}{9}) \\ [2K\ ctg^2\theta_W\ \ln c_\theta^2] \end{cases}$$

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