The effective fine structure constant and other SM running couplings

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people.physik.hu-berlin.de/~fjeger/alphaQEDc26.tar.gz

*

October 2025 Addendum α and α_2 complex simplified + June 2026 update

Status June 2026:

$\gamma - \rho^0$ mixing subtraction on isovector part of ρ resonance data implemented

☆☆☆ now getting closer to lattice QCD results and isovector τ data ☆☆☆

Latest news: see note on a_μ and a_e Chapt. 10
(the final Muon $g-2$ report [1] reports $a_\mu^{\rm exp} = 1165920715(145) \times 10^{-12}$ (124ppm))

Check new multi flavor correlators $n_f = 2, 3, 4, 5$ see Chapt. 9:

./intRdatx_gnu.sh 19 for $\Delta\alpha_{\rm had}^{(n_f)}(s)$ $(s < 0)$ X

./intRdatx_gnu.sh 20 or ./ECintR_gnu.sh 0 for Eucl. Corr. $\hat{\Pi}_{\rm had}^{(n_f)}(t)$ X

Abstract

Description of FORTRAN package `alphaQEDc26`[1]. For latest changes read CHANGES and installation details INSTALLandRUN.

*for href's to work use `evince` or `Adobe Reader` for display

[1] For more background see and the more recent contribution to arXiv:1905.05078, arXiv:2601.08101

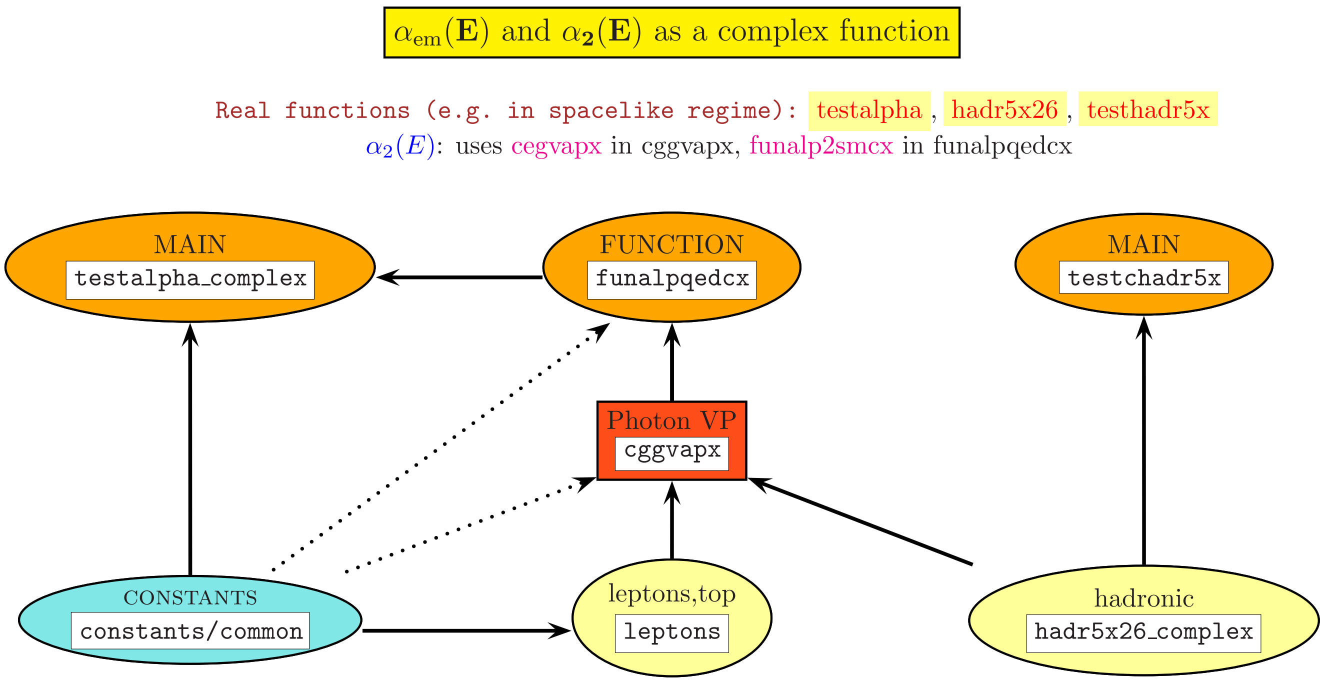
For recent updates go-to »2023 extensions and updates« and 2016 Updates below.

✲⋙ see also the PDF Flavorseparation

Outline of Instructions:

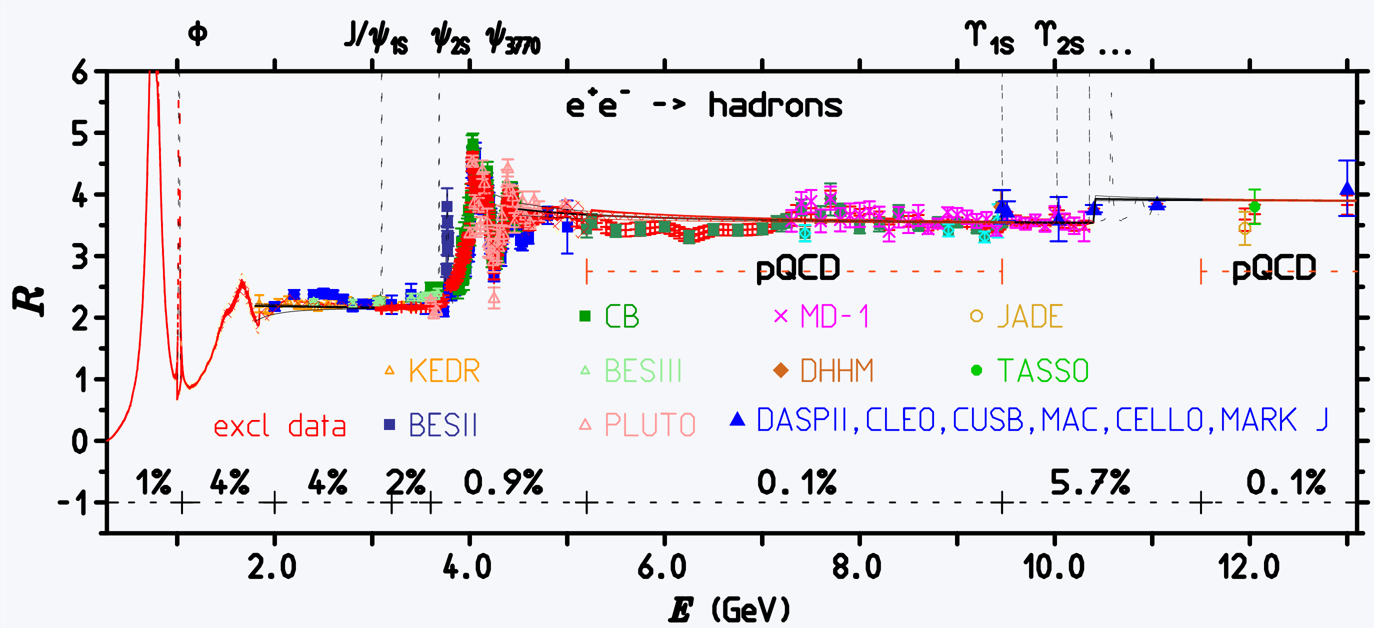
❒ New FORTRAN Code Set alphaSMsimple

June 2026: now updated to alphaQEDc26 version, implementing $\gamma - \rho^0$ mixing of the I=1 part of the ρ resonance data. New simplified version of alphaQEDc26 to calculate α, α_2 and $\sin^2\Theta_{\rm eff}$: imaginary parts together with real parts of hadronic contributions now are tabulated in the file chadr5x26_complex.f (no need for rhad package, intRdatx and all that). The tabulated hadronic part is combined with the lepton code leptons.f in cggvapx.f. $\alpha(s)$ in provided by the function funalpqedcx.f, $\alpha_2(s)$ via funalp2smcx.f (code included in funalpqedcx.f) and $\sin^2\Theta_{\rm eff}$ via funsin2theta.f. Three test programs testhadr5x.f and testalpha_complex.f allows one to test the hadronic part chadr5x26_complex.f and the functions providing α and α_2, respectively. The effective weak mixing angle can be tested using testsin2theta_complex. Makefile: make_testalpha, Shell script: test_hadr5x_gnu.sh, testalpha_gnu.sh, testsin2theta_gnu.sh , all collected in alphaSMsimple.tar. Output files for plotting to be customized to the users need. The *real part only* version hadr5x26.f is included for comparison with the new complex version. The 2026 update of the previous 23-versions concerns `hadr5x26_complex.f`, `hadr5x26.f` including $\gamma-\rho^0$ mixing subtraction, for comparison I added `hadr5x26a_complex.f` and `hadr5x26a.f` the standard versions not corrected for mixing.



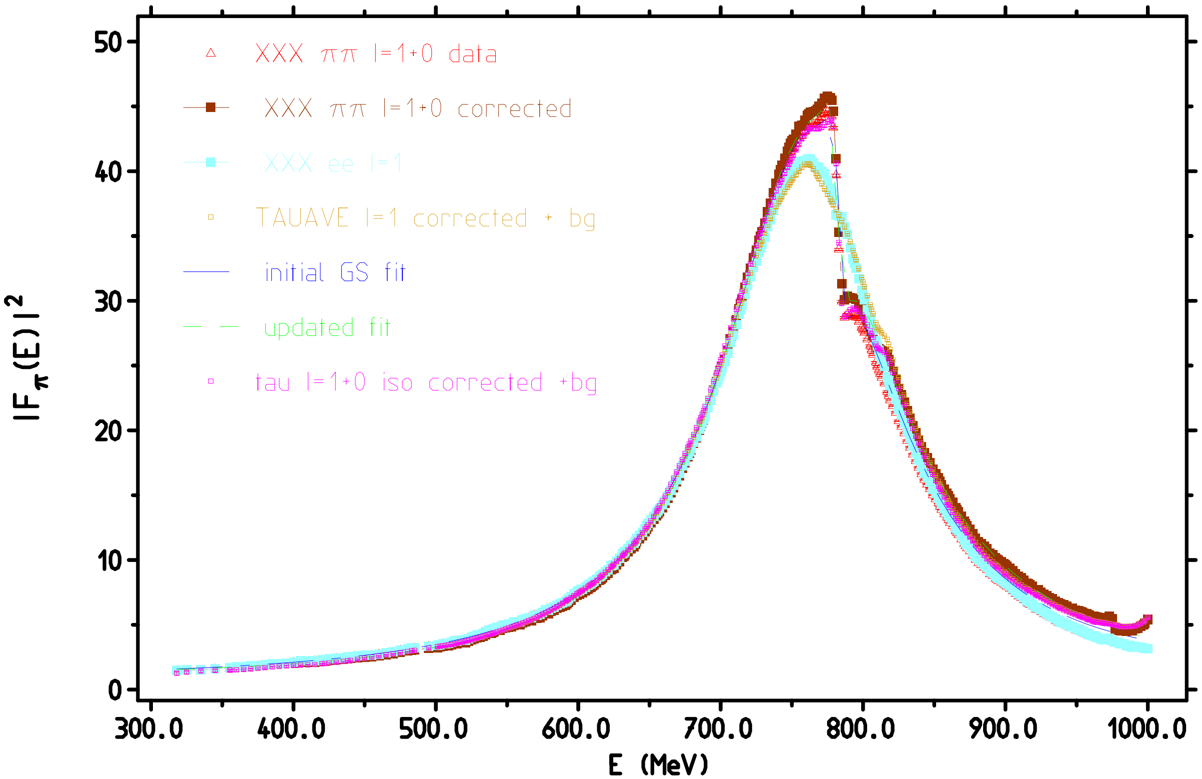
0. My $R_\gamma(s)$-Compilation and Basic Applications

Analyses based on $e^+e^- \to \gamma^* \to$ hadrons data.



World average $R(E)$ data compilation

New option IUNMIX=1 : to remove $\gamma - \rho^0$ mixing from e^+e^- data (see [2, 8][2]).
IUNMIX=0 yields the standard e^+e^- data driven dispersive result. The QED-QCD interference effect is shown in the following plot:



[2]A similar extended version of the VMD II+sQED model has been investigated recently in [9]

IUNMIX=2 replaces the isovector part I=1 in the ρ range by the isospin corrected $\tau^- \to \pi^0\pi^-\nu_\tau$ $\pi\pi$ data.

Tab.1 Impact of $\gamma-\rho^0$ mixing correction and using τ data

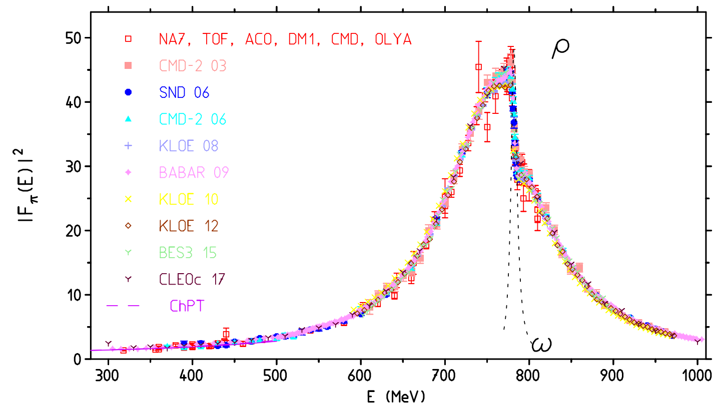
IUNMIX	$a_\mu^{\rm had} \times 10^{10}$ (stat)(syst)[tot]	
1	e^+e^- corrected for $\gamma-\rho^0$ mixing	704.85(0.81)(3.89)[3.97]
2	τ for ρ I=1 part	702.37(0.84)(4.25)[4.33]
0	e^+e^- standard dispersive	697.93(0.81)(3.93)[4.01]
-1	$\gamma-\rho^0$ corr of full (I=1+0) $e^+e^- \to \pi\pi$ channel	705.71(0.81)(3.93)[4.01]

Specify IUNMIX in script intRdatx_gnu.sh and run ./intRdatx_gnu.sh 16

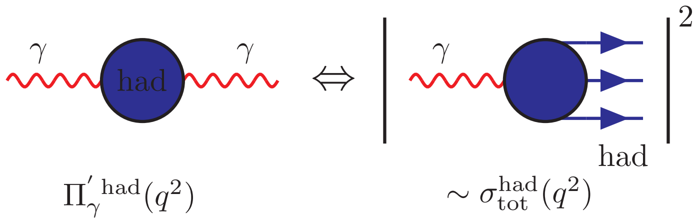
R-data based dispersive $\alpha(M_Z^2)$ determination

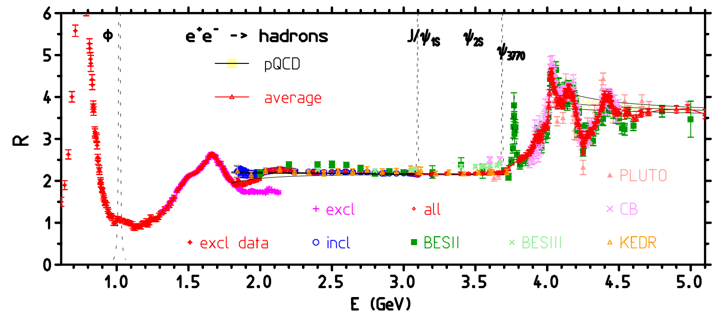
Non-perturbative hadronic contributions $\Delta\alpha^{(5)}_{\rm had}(s) = -\left(\Pi'_\gamma(s) - \Pi'_\gamma(0)\right)$ **can be evaluated in terms of** $\sigma(e^+e^- \to \mathrm{hadrons})$ **data via dispersion integral:**

$$\Delta\alpha^{(5)}_{\rm had}(s) = -\frac{\alpha\, s}{3\pi}\left(\oint_{4m_\pi^2}^{E^2_{\rm cut}} ds' \frac{R^{\rm data}_\gamma(s')}{s'(s'-s)} + \oint_{E^2_{\rm cut}}^{\infty} ds' \frac{R^{\rm pQCD}_\gamma(s')}{s'(s'-s)} \right)$$

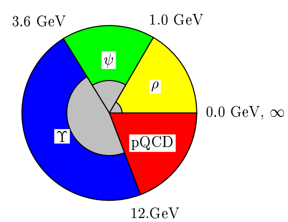


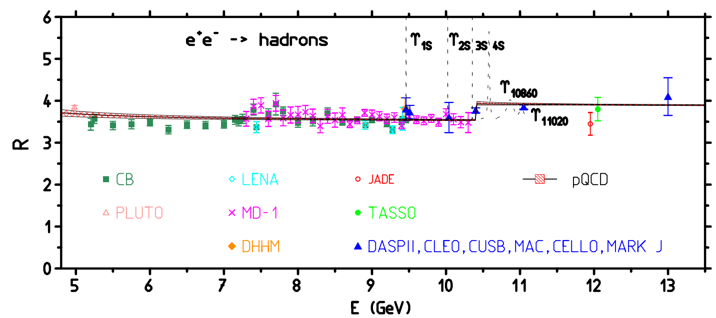
where $R_\gamma(s) \equiv \dfrac{\sigma^{(0)}(e^+e^-\to\gamma^*\to\mathrm{hadrons})}{\frac{4\pi\alpha^2}{3s}}$





hadronic vacuum polarization





$$\alpha(s) = \frac{\alpha}{1-\Delta\alpha(s)}\ ; \quad \Delta\alpha(s) = \Delta\alpha_{\rm lep}(s) + \Delta\alpha^{(5)}_{\rm had}(s) + \Delta\alpha_{\rm top}(s)$$

$\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2)$	$=$	0.027756 ± 0.000157	
		0.027563 ± 0.000120	Adler
$\alpha^{-1}(M_Z^2)$	$=$	128.916 ± 0.022	
		128.953 ± 0.016	Adler

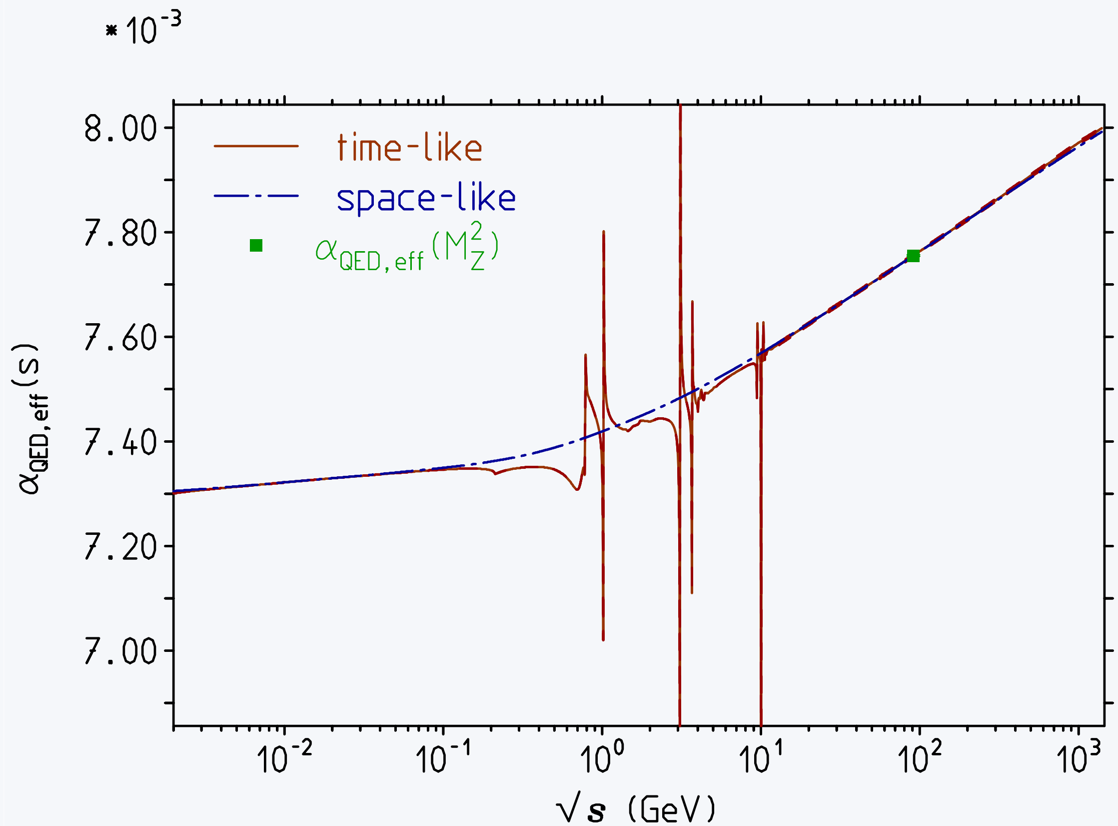
$\alpha_{\rm em}(M_Z^2)$ by the "Adler function controlled" approach hep-ph/9812521,arXiv:0807.4206 [hep-ph]

$$\alpha(M_Z^2) \quad = \quad \alpha^{\rm data}(-s_0) + \left[\alpha(-M_Z^2) - \alpha(-s_0)\right]^{\rm pQCD} + \left[\alpha(M_Z^2) - \alpha(-M_Z^2)\right]^{\rm pQCD}$$

❑ the space-like $-s_0$ is chosen such that pQCD is well under control for $-s < -s_0$; offset $\alpha^{\rm data}(-s_0)$ integrated $R(s)$ data. A good choice is $\sqrt{s_0} \simeq 2$ GeV.

New! new LQCD: $\Delta\alpha^{(5)}_{\rm hadrons}(M_Z^2) = 0.02773(15)$! Mainz Group

❐ $\alpha_{\rm QED,eff}$: time-like vs. space-like

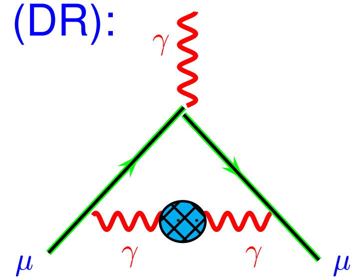


$\alpha_{\rm QED,eff}$ duality: $\alpha_{\rm QED,eff}(s)$ is varying dramatically near resonances, but agrees quite well in average with space-like version. Locally ill-defined near OZI suppressed meson decays: $J/\psi, \psi_1, \Upsilon_{1,2,3}$! Dyson series not convergent.

R-data based dispersive $a_\mu^{\rm had}$ determination

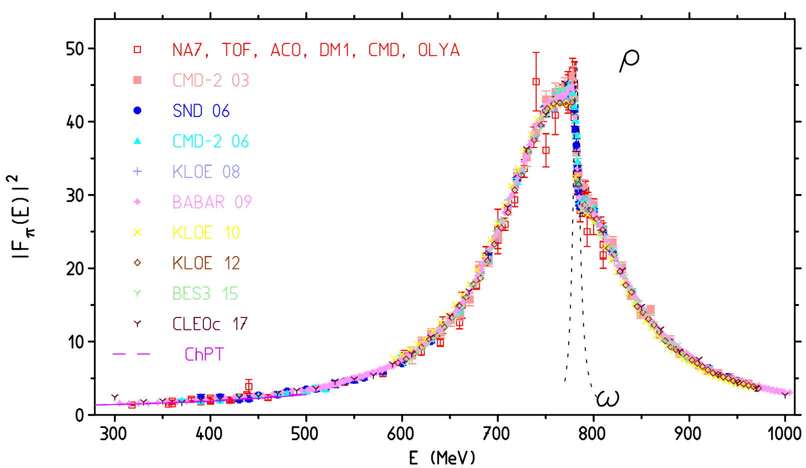
Leading non-perturbative hadronic contributions $a_\mu^{\rm had}$ can be obtained in terms of

$R_\gamma(s) \equiv \sigma^{(0)}(e^+e^- \to \gamma^* \to \text{hadrons})/\frac{4\pi\alpha^2}{3s}$ data via Dispersion Relation (DR):



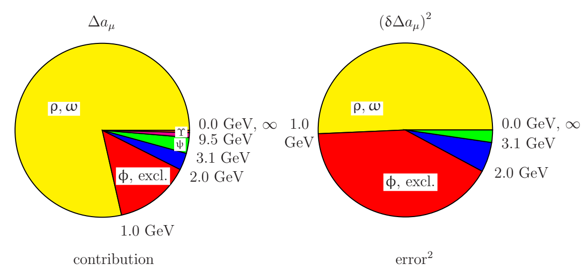
$$a_\mu^{\rm had} = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \left(\int\limits_{4m_\pi^2}^{E^2_{\rm cut}} ds \frac{R_\gamma^{\rm data}(s)\ \hat{K}(s)}{s^2} + \int\limits_{E^2_{\rm cut}}^{\infty} ds \frac{R_\gamma^{\rm pQCD}(s)\ \hat{K}(s)}{s^2} \right)$$

Data: NSK, KLOE, BaBar, BES3, CLEOc



$$a_\mu^{\rm had(1)} = (697.93 \pm 4.01)\ 10^{-10}$$

e^+e^-–data based



- Experimental error implies theoretical uncertainty!
- Low energy contributions enhanced like $1/E^4$: $\sim 75\%$ come from region $4m_\pi^2 < m_{\pi\pi}^2 < M_\Phi^2$

1. What is new? (alphaQEDc19 updated)

What is new/updated: as a main mode the $\pi^+\pi^-$ data I also use local averaging in place of averaging integrals in overlapping ranges. I now include SND 2020 $\pi\pi$ data and the new 2022 $R(s)$ measurement by BESIII [range 2.232-3.671 GeV]. Exclusive channels from BaBar are completed. Up to the lowest KEDR inclusive data point I include exclusive channels only (skipping old Frascati inclusive data like $\gamma\gamma 2$ Collab. data etc.) from 2 GeV KEDR data and BESII are in fair agreement and it is justified to combine the inclusive data there. From 1.84 to 2 GeV where we observe a clash between exclusive and inclusive data I take the weighted average including χ^2 mismatch S-factor rescaled errors. This increases the 1.05 GeV to 2.00 GeV range result by $+1.63 \times 10^{-10}$. In including SND20 enhances the $\pi^+\pi^-$ channel integral by $+0.67 \times 10^{-10}$.

The pQCD result for the range 5.20 to 9.46 GeV has changed by -0.07×10^{-10}. This changes

$$688.07(1.11)(3.99)[4.14] \times 10^{-10}$$

from my book to

$$697.93(0.81)(3.93)[4.01] \times 10^{-10}\ .$$

However, the main difference comes from replacing the Breit-Wigner + PDG parametrization of the ω and ϕ by the data for 3π, K^+K^- and $K^0\bar{K}^0$, including new Novosibirsk data (NSK) which are more consistent now (as analyzed thoroughly in `arXiv:2105.13018 [hep-ph]` [**?**]). The CMD-3 K^+K^- `arXiv:1710.02989 [hep-ph]` [3] for consistency with BaBar have to be scaled down by a normalization factor 0.967. In addition the two CMD-2 set undergo a rescaling by 1.094 ± 0.04 [3]. Shifts are within the reported energy-scale uncertainties.

New options: iintRd=1 switch of narrow resonances from data and fits in `Rdat_fun.f`, `Rdat_fit.f`, `RS3G_fun.f`, `RS3G_fit.f`, etc.

iomegaphidat=	-1,0,1,2		options for including narrow resonances
		0	use BW with PDG parameters for ω and ϕ
		1	use ω and ϕ data contribution to $R(s)$
		2	replace BW resonances of option 0 by resonance data, $\omega \approx 3\pi$, $\phi \approx K^+K^- \& K^0\bar{K}^0$
		-1	background only

R-data integration routine intRdatx.f takes into account narrow resonances separately via subroutine resonances so set iintRd=1 in intRdatx.f ! never change this setting (else double counting narrow resonances ω, ϕ, J/ψ's and Υ's) and iomegaphidat=2 for proper error handling for ω and ϕ contributions (i.e. systematic errors for 3π, K^+K^-, K_LK_S and background taken to be uncorrelated as channels).

One main unsolved problem concerns the unexplained differences in the $\pi^+\pi^-$ cross-section measurements. In fact, lattice QCD calculations of the hadronic vacuum polarization reveal a larger value than what we usually get in the e^+e^- data-driven dispersive approach. Therefore, I no longer take the KLOE data into account, since its results are lower than those of all other experiments. I also do not take into account the most recent CMD3 result, which is significantly higher than the other results. While the results of the lattice QCD simulations agree quite well with the dispersive results that include the τ data, the long-standing discrepancy with the standard dispersive results based on e^+e^- data suggests that the problem lies in the extraction of the hadronic component in the $e^+e^- \to \mathrm{hadrons}$ experiments. It appears that the QED corrections in hadronic processes are more complex than usually anticipated. In the dominating $\pi^+\pi^-$ channel, substantial photon radiation contamination from $\rho^0 - \gamma$ mixing is present in e^+e^--annihilation data, as discussed in [arXiv:1101.2872 [hep-ph]] [8] (see also [**?**] Sect. 21.3, evaluating higher order effects as well). The correction of $(5.1 \pm 0.5) \times 10^{-10}$ is calculated to be subtracted from the τ data to bring it in agreement with the e^+e^-. This correction must actually be added to the e^+e^- $\pi^+\pi^-$ spectra to remove QED-QCD mixing inherent only in this data. This mixing is absent in the τ-decay spectra and in lattice QCD calculations based on the Euclidean hadronic current-current correlator. Thus, the correct result is:

$$703.03(0.81)(3.93)(0.50)[4.04] \times 10^{-10} \; .$$

New is the direct implementation of the $\gamma - \rho^0$ interference correction to the $e^+e^- \to \pi^+\pi^-$ spectra in the range 0.592-0.972-GeV. In this range, the VMD II+sQED effective model investigated in [8] is expected to work rather well. The scope of validity can be verified, for example, using the data for the transition $\gamma\gamma \to \pi^+\pi^-, \pi^0\pi^0$. This model does not contain a new fitting parameter that is not already showing up in the widely used VMD I Gounaris-Sakurai model.

New options here: IUNMIX=1,2,0-1 switch of $\gamma - \rho^0$ mixing related correction of data and fits in `Rdat_fun.f`, `Rdat_fit.f`, `RS3G_fun.f`, `RS3G_fit.f`, etc.

IUNMIX=	1,2,0,-1	options for applying $\gamma - \rho^0$ mixing or not or using τ data
	1	correcting the I=1 part of the WA ρ resonance data
	2	use isospin corrected τ data in place of the I=1 part of the e^+e^- data
	0	use standard $e^+e^- \to \pi^+\pi^-$ data,
	-1	apply the mixing correction to the full (I=1+0) ρ^0 resonance data

The impact of the different options for the $a_\mu^{\rm had}$ results is listed in Tab.1

A modified package for calculating the effective electromagnetic fine structure constant is available in real (for space-like regime) and complex (for time-like regime) version. It provides statistical and systematic errors separately now. A number of functions have been **renamed** as `name.f -> namex.f` and/or `name.f -> name26.f` in order not to conflict with older versions, which provided total error only for the hadronic shifts `der` for $\Delta\alpha^{(5)}_{\rm had}(s)$ and `deg` for $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ provided by `hadr5n.f`. The World Average (WA) compilation for $R(s)$ provided by `Rdat_fun.f` always included statistical and systematic errors separately (also in earlier versions).

Also included is an integration routine intRdatx.f, which allows you to calculate weighted integrals of $R(s)$ with appropriate error estimates.

Note that $R(s)$ data are sorted in typical energy ranges $n = 1, ...$, originally corresponding to different generations of experiments. Data are considered to be 100% correlated within these ranges and independent otherwise.

The **covariance matrix** for each range n is given by

$$C^n_{ij} = \begin{cases} (\delta^n_{i\,\mathrm{sta}})^2 + (\delta^n_{i\,\mathrm{sys}})^2 & \text{for} \quad j = i \\ \delta^n_{i\,\mathrm{sys}} \cdot \delta^n_{j\,\mathrm{sys}} & \text{for} \quad j \neq i \end{cases} \quad , \quad i, j = 1, \cdots, N_n \,,$$

where $\delta^n_{i\,\mathrm{sta}}$ and $\delta^n_{i\,\mathrm{sys}}$ denote the statistical and systematic error, respectively, of R^n_i, where i labels the data points. This has been available in previous versions of `alphaQED` since 2009 and later.

List of main routines: cp Makefile.complex ==> Makefile

Rdat_fun.f	$R(s)$	
RS3G_fun.f	$\mathrm{Im}\,\Pi_{3,\gamma}$	via flavor separation of $R(s)$
test_Rdat.f	test of Rdat_fun.f	Makefile_Rdat
hadr5n26.f	$\Delta\alpha^{(5)}_{\mathrm{had}}(s)$ & $\Delta\alpha^{(5)}_{2\,\mathrm{had}}(s)$	
alphaQEDr26.f	$\alpha(s)$ real and related	Makefile.real
alphaQEDc26.f	$\alpha(s)$ complex and related	Makefile
alpha2SMc26.f	$\alpha_2(s)$ complex and related	Makefile prog=alpha2SMc26
ACWMsin2theta.f	$\sin^2\theta_{\mathrm{eff}}(s)$ and related	Makefile prog=ACWMsin2theta
intRdatx.f	integrals of $R(s)$: a_μ^{had}, $\Delta\alpha^{(5)}_{\mathrm{had}}(s)$	Makefile_iRdx
ECintR1.f	Euclidean correlators	Makefile_ERintR1
amuviaEC.f	$a_\mu^{\mathrm{HVP-LO}}$ via Euclidean correlator	Makefile_amuviaEC

Drivers (providing run specific parameters if required):

test_Rdat.f	test_Rdat_gnu.sh	testRXYgnuplot.in
alphaQEDr26.f	./alphaQEDr26	
alphaQEDc26.f	alphaQEDc26_gnu.sh	alphaQEDplot_gnu.in0
alpha2SMc26.f	./alpha2SMc26_gnu.sh	alphaQEDplot_gnu.in0
ACWMsin2theta.f	./ACWMsin2theta_gnu.sh	ACWMsin2theta_gnu.in
intRdatx.f	intRdatx_gnu.sh	intRdat.inp, intRdat.inp0
	intRdatx_upd.sh, intRdatx_all.sh	
	intRdatx_amuHO.sh, intRdatx_aelHO.sh	
ECintr1.f	ECintR_gnu.sh	ECintR.inp0, ECintR_gnu.in
amuviaEC.f	./amuviaEC.sh	

Note: the important parameters related to handling resonances

IUNMIX, iomegaphidat, iintRd, iresonances and IRESON have to be set as flags in the fortran code file intRdatx.f

Important note concerning VP subtraction: in obtaining the undressed $R(s)$ [imaginary part of Im $\Pi_\gamma(s)$]. For narrow resonances ω, ϕ, etc I use PDG parameters for masses and width (which are the physical ones) and one has to undress the corresponding physical Breit-Wigner and the undressed BW has of course different parameters. However, this is not unambiguous. I have performed VP subtraction using space-like $\alpha(-s)$ because for narrow resonances iterating VP subtraction with the time-like $\alpha(s)$ does not converge for some J/ψ and Υ resonances and as long as one is interested in integrals like $a_\mu^{\rm had}$ away from any resonance position the effect from over and under-shooting at resonances should cancel out. This can be checked for the cases where the time-like subtraction iteration converges. I have always pointed out that local resonance shapes are not unambiguous and for any change of the resonance parameters the calculation has to be redone from scratch in principle. Another difference one obtains if one is using the resonance data directly and not the PDG. In earlier evaluations I decided to trust the PDG parametrization in my analysis. This I have changed now, because earlier inconsistencies between different experiments in the ω and ϕ dominating channels $\pi^+\pi^-\pi^0$ and $K^+K^-, K^0\bar{K}^0$ by including new Novosibirsk data (NSK) which are more consistent now (as analyzed thoroughly in [**?**]). Some of the points I have discussed in my Capri talk, where I also say what precisely I have done. For details also see: F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, STMP **274**, 1 (2017); EPJ Web

Conf. **166**, 00022 (2018) arXiv:1705.00263; ibid. **118**, 01016 (2016) arXiv:1511.04473.

2. Download and Installation

First read INSTALLandRUN for latest implementation.

New: I include the rhad FORTRAN package by Harlander and Steinhauser in the alphaQEDc26 package on a subdirectory ./rhad. This should simplify including pQCD $R(s)$ results where needed. This supersedes instructions concerning rhad implementation given elsewhere.

Download link: ✻⋙ [alphaQEDc26.tar.gz]

➠ alphaQEDc26 [FUNCTION funalpqedcx] providing the real and imaginary parts of the subtracted photon vacuum polarization including hadronic, leptonic and top quark contributions as well as the weak part (relevant at ILC energies and beyond).

To take full advantage of the functionality, I recommend using the package `alphaQEDc26`, which provides $\alpha(s)$ and $\alpha_2(s)$ as complex functions of the square c.m. energy s. Statistical and systematic uncertainties are specified separately.

A much simpler real-valued version, `alphaQEDr26`, is also provided. I recommend using this version exclusively for space-like momentum transfers, where $\alpha(s)$ and $\alpha_2(s)$ are real in every case.

Based on my R-compilation, tables for $\Delta\alpha^{\rm had}(s)$ and $\Delta\alpha_2^{\rm had}(s)$ were calculated using the program intRdatx.f included in the package. The integration routine allows for the calculation of many different interesting quantities.

The **real parts** of the hadronic contributions to the vacuum polarization shift (renormalized VP function) are accessible by linking

hadr5x26.f as a standalone version including all tables

To integrate $R(s)$-related data and determine the complex renormalized VP functions, the complete alphaQEDc26 package must be installed, whereas to calculate the running SM parameters $\alpha(s)$, $\alpha_2(s)$, and $\sin^2\Theta_{\rm eff}(s)$ only, a much simpler collection of routines is alphaSMsimple

which contains the file hadr5x26_complex.f (generated with complexifytables.f) with tabulated real and imaginary parts, as outlined already [alphaSMsimple].

alphaQEDc26 the complex version

This version includes an $R(s)$ compilation (representing the imaginary part) in addition to $\Delta\alpha^{\rm had}(s)$ (often taken to be real). This requires first to install the `rhad` package written by Harlander and Steinhauser (FORTRAN package version rhad-1.01 (March 2009 issue)). For actualization see INSTALLandRUN. Note that rhad is now included on subdirectory ./rhad

- untar file `alphaQEDc26.tar.gz`
- go to subdirectory ./rhad/ and compile them make & [gfortran -c rqcdHSn.f]. The updated version includes the subroutine rqcdHS3x(...) which allows for flavor recombination needed for the complex version of `alpha2SMc26` providing $\Delta\alpha_2^{\rm had}(s)$.
- edit `Makefile.complex`; adjust directories, load path [compiler?] etc.

- copy `Makefile.complex` to `Makefile`
- make
- run ./alphaQEDc26 $\longrightarrow$ writes results to files fort.1 - fort.4 (various real parts) and fort.11 - fort.14 (various imaginary parts)

  which you may plot with your favorite plot program[3]

- in user program use one of the function calls

  Delta_alpha = cggvapx(s,cerror,cerrorsta,cerrorsys)

[3] My plot-file headers contain the following information

```
number of lines following before first data entry
number data sets (curves)
number of data points
comment lines
```

data set

change the `intRdatx.f` code for the needs of your favorite plot program alpha_complex=funalpqedcx(s,cerror,cerrorsta,cerrorsys,contributionflag)

requires to include `funalpqedcx.h` as a header in your calling program. Check and edit, e.g., to change reference value of st2 (weak mixing parameter) required to define reference input $\alpha_2(0)$ in terms of $\alpha(0)$. Also applies to subroutine `alphaQEDcx_sub.f`.

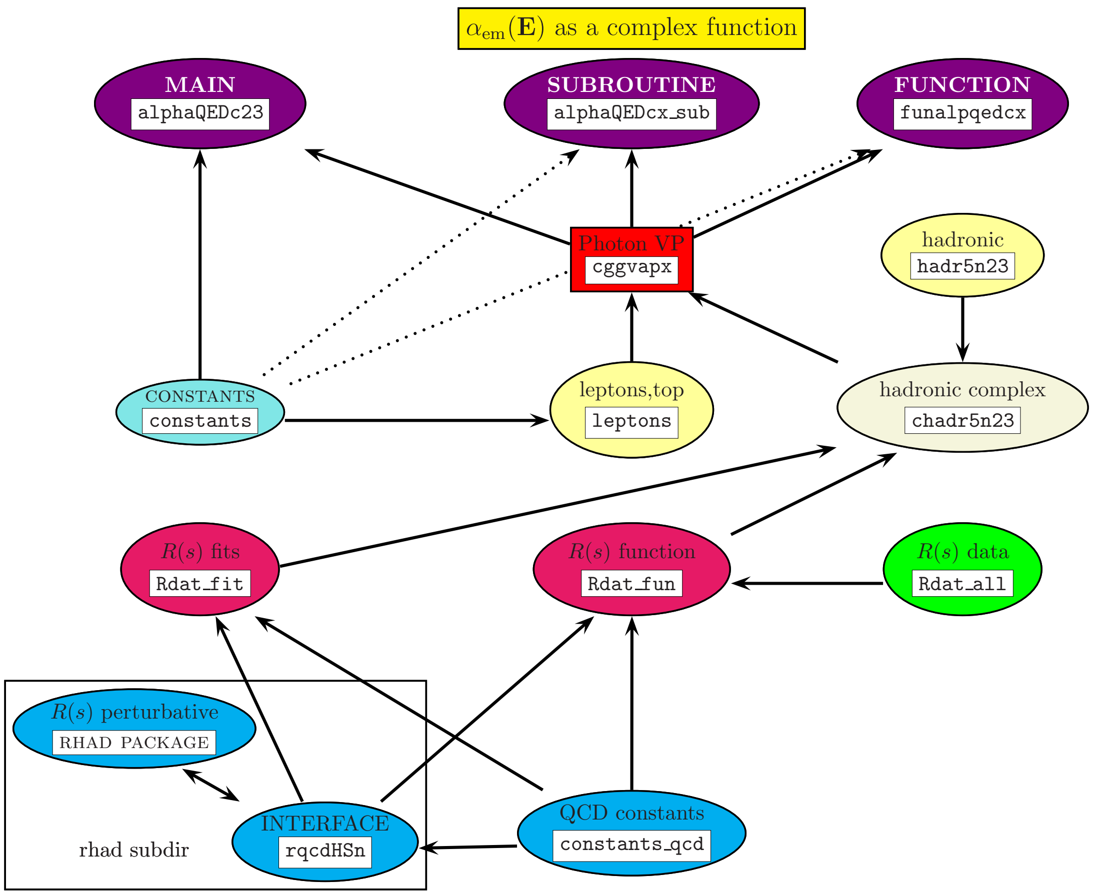
Available variants: Main program `alphaQEDc26.f` and subroutine `alphaQEDcx_sub.f` to be called [Input: s ; $s = \pm E^2$; E in GeV]

```
call alphaQEDcx(s,cvpt_new,cvpt_hig,cvpt_low,alphac,alphah
 & ,alphal,alphact,alphaht,alphalt,calept,cahadr,caltop)
```

`cvpt_new`	$\Delta\alpha^{(5)}_{\rm had}(s)$ central value
`cvpt_hig`	$\Delta\alpha^{(5)}_{\rm had}(s)$ + total error
`cvpt_low`	$\Delta\alpha^{(5)}_{\rm had}(s)$ - total error
`alphac`	$\alpha(s)$ central value no top quark
`alphah`	high value
`alphal`	low value
`alphact`	$\alpha(s)$ central value incl. top quark
`alphaht`	high value
`alphalt`	low value
`calept`	$\Delta\alpha$ leptonic
`cahadr`	$\Delta\alpha$ hadronic 5 quarks
`caltop`	$\Delta\alpha$ top quark

For an extended version including resonances data see corresponding header of `alphaQEDc26.f`
Note: rhad-package and interfaces now included in subdirectory ./rhad/
For gnuplot display of results run:
alphaQEDc26_gnu.sh which uses alphaQEDplot_gnu.in0 for gnuplot settings

The program structure is displayed in the following diagram:



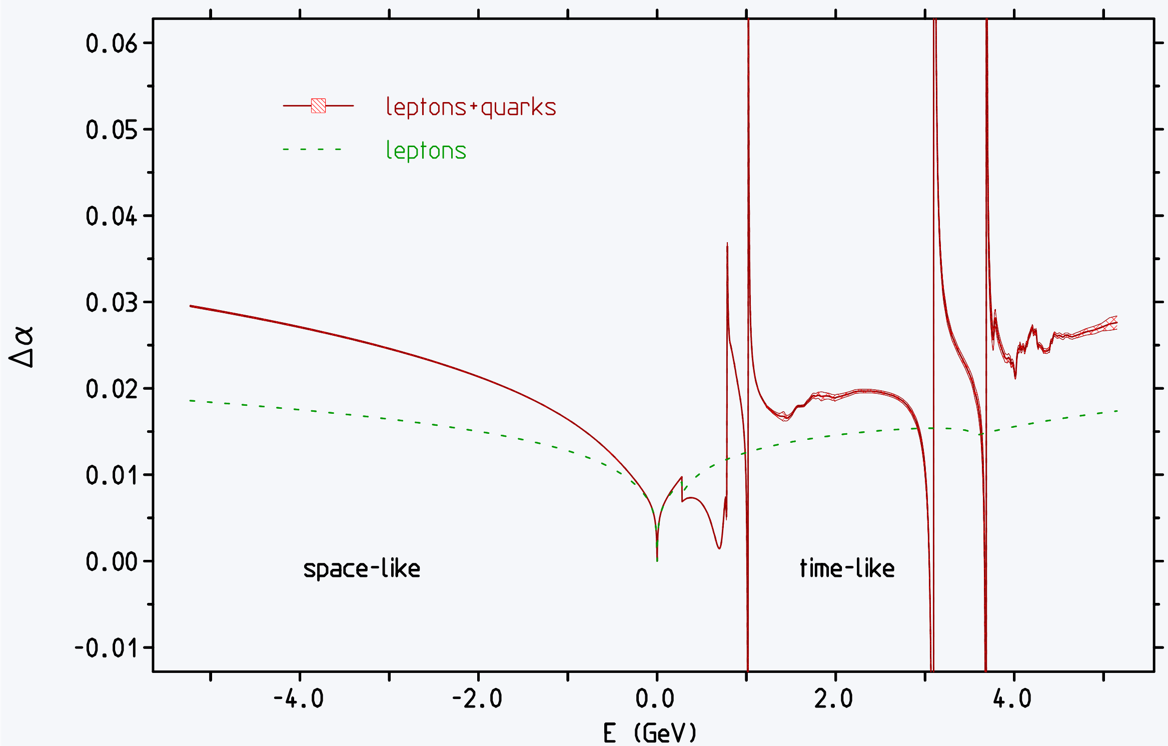
More on `rhad` package: now already included in alphaQEDc26 see INSTALLandRUN!

Original instruction (now outdated):
Download the Harlander-Steinhauser program rhad from http://www.rhad.de/
expanding the tar file creates directory ./rhad-1.01/ with all files.
Copy the interfaces rqcdHSn.f and rqcdHS3x.f into that directory and compile it like the "example" provided with rhad package
.. by Robert V. Harlander and Matthias Steinhauser,
.. Comp. Phys. Comm. 153 (2003) 244,
.. CERN-TH/2002-379, DESY 02-223, hep-ph/0212294.
.. Changes: [March 30, 2009] v1.01
.. - exact results at α^4 included
choose option lverbose = .false. in parameters.f
you may also want to switch off warnings on non-perturbative regions
just before format statement 2003 in rhad.f, I use iwarn=0 flag to switch it off

```
      if (iwarn.eq.1) then
         write(6,2003) dsqrt(scms)
      endif
```

also change in parameters.f iunit = 6 to something not used by the `alphaQEDc26` package

Sample Plots:



Shift of the effective fine structure constant $\Delta\alpha$ as a function of the energy scale in the time–like region $s > 0$ $(E = \sqrt{s})$ vs the space–like region $-s > 0$ $(E = -\sqrt{-s})$. The band indicates the uncertainties

alphaQEDr26 the real version

This version works without the $R(s)$ compilation and does not require the `rhad` package.

- untar file `alphaQEDc26.tar.gz`
- edit `Makefile.real`; adjust directories, load path [compiler?] etc.
- copy `Makefile.real` to `Makefile`
- make
- run ./alphaQEDr26 $\rightarrow$ writes results to files fort.1 - fort.4 and fort.11 - fort.14 (various real parts and detail results)

  which you may plot with your favorite plot program
- in user program use one of the function calls

Delta_alpha = dggvapx(s,error,errorsta,errorsys)

alpha_real=funalpqedrx(s,error,errorsta,errorsys,contributionflag)

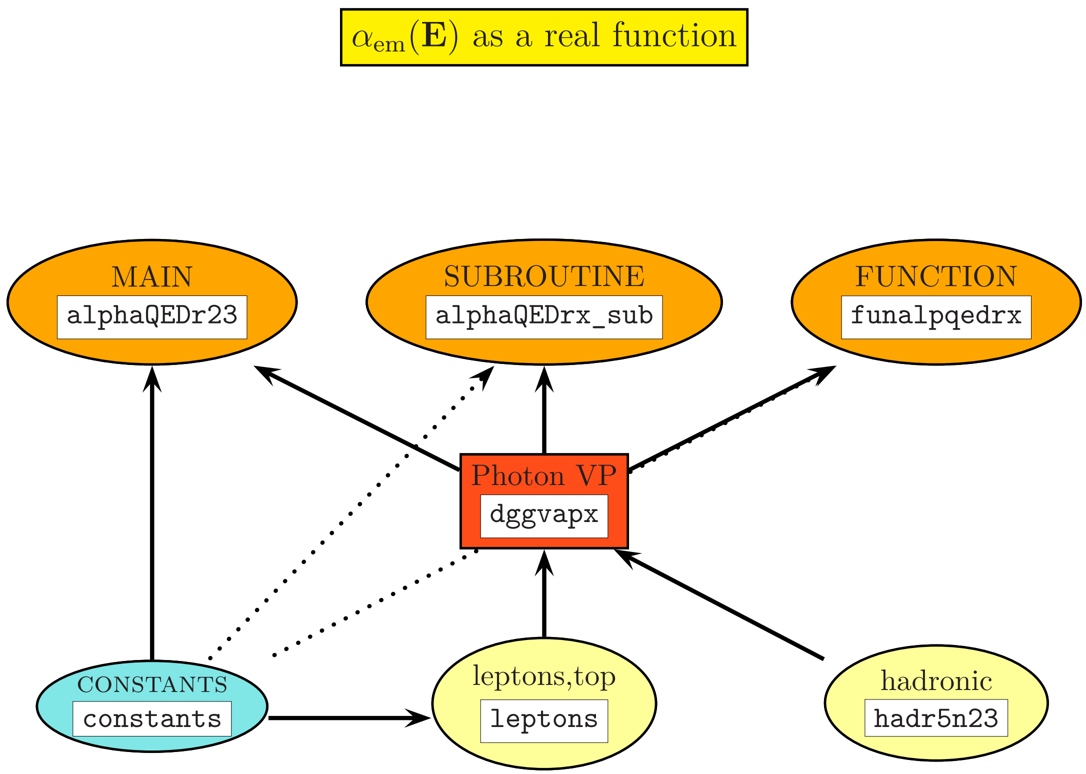
requires to include `funalpqedrx.h` as a header in your calling program. Check and edit, e.g., to change reference value of st2 (weak mixing parameter) required to define reference input $\alpha_2(0)$ in terms of $\alpha(0)$. Also applies to subroutine `alphaQEDrx_sub.f`.

Available variants: Main program `alphaQEDr26.f` and subroutine `alphaQEDrx_sub.f` to be called [Input: `s` ; $s = \pm E^2$; E in GeV]

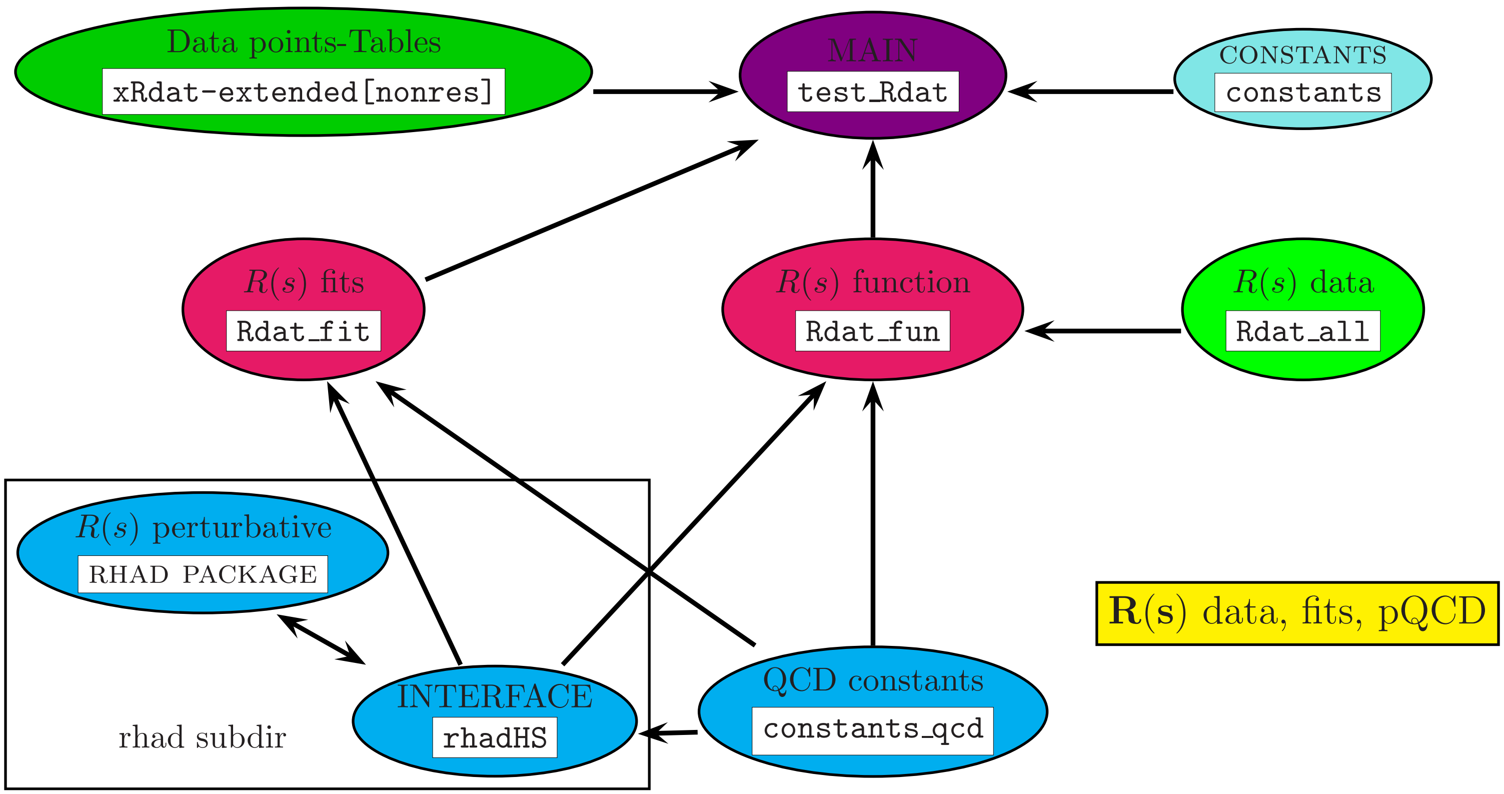
```
call alphaQEDrx(s,dvpt_new,dvpt_hig,dvpt_low,alphac,alphah
 & ,alphal,alphact,alphaht,alphalt,dalept,dahadr,daltop)
```

dvpt_new	$\Delta\alpha^{(5)}_{\rm had}(s)$ central value
dvpt_hig	$\Delta\alpha^{(5)}_{\rm had}(s)$ + total error
dvpt_low	$\Delta\alpha^{(5)}_{\rm had}(s)$ - total error
alphac	$\alpha(s)$ central value no top quark
alphah	high value
alphal	low value
alphact	$\alpha(s)$ central value incl. top quark
alphaht	high value
alphalt	low value
dalept	$\Delta\alpha$ leptonic
dahadr	$\Delta\alpha$ hadronic 5 quarks
daltop	$\Delta\alpha$ top quark

The program structure is displayed in the following diagram:



3. Test programs, Sample plots

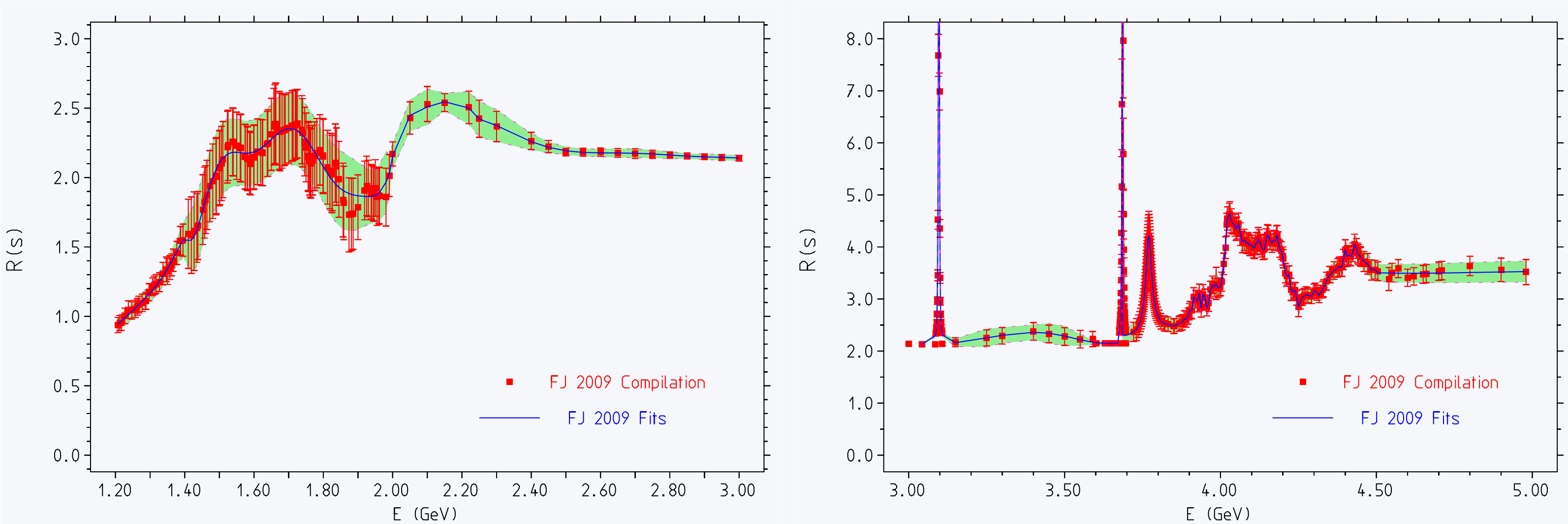


Sample program on subdirectory ./test/test_Rdat.f for extracting $R(s)$ data, fits and pQCD calculation (run ./test_Rdat_gnu.sh using gnuplot preset from testRdat_gnu.in):

To run test_Rdat:

- cd ./test
  Note: make -f Makefile_testRdat requires adlerhadr5x26.o to exist
  so first run gfortran -c adlerhadr5x26.f!
- edit `Makefile_testRdat`; adjust directories, load path [compiler?] etc.
- make -f Makefile_testRdat
- run ./test_Rdat → writes results to files
  fort.1 R-data , fort.2 R-fits , fort.3 R-pQCD
  which you may plot with your favorite plot program
- in order to obtain R-data without narrow resonances set iresonances=0
- fits without narrow resonances set IRESON=0
- data vs pQCD regions set by cuts EC to 9.5 GeV and above ECUT (> 11.5 GeV) to some max energy
- Note: `xRdat-extended.f` file yields energies point raster with fine binning in resonance regions [data points interpolated linearly]. Experimental energy points for off resonance data in `xRdat-nonres.f`

Sample results:



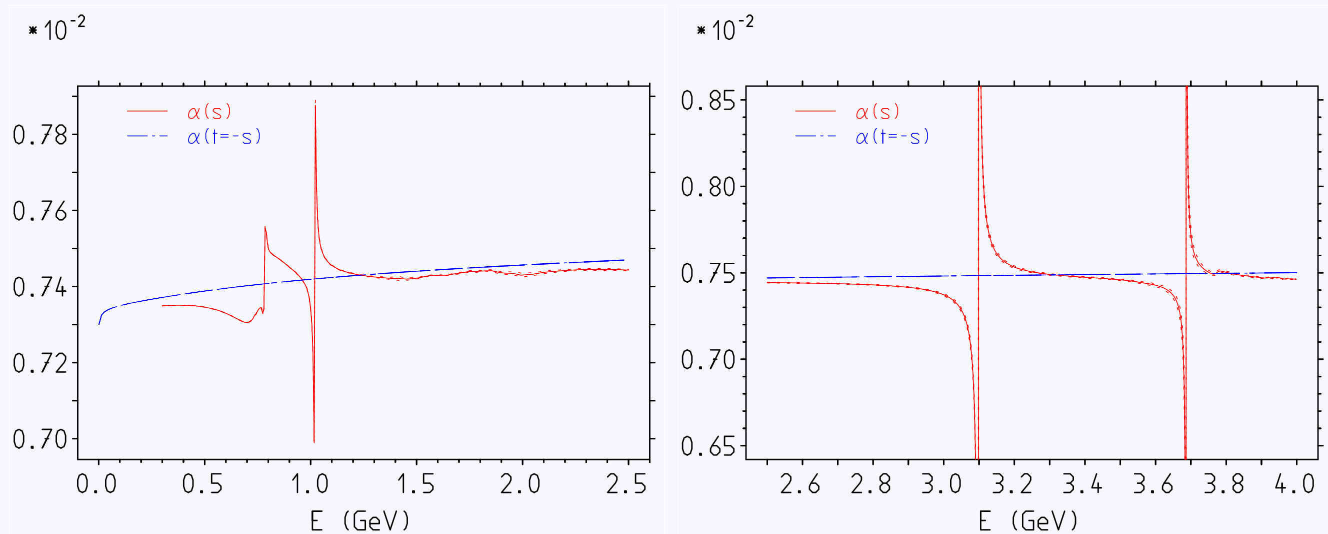
$R(s)\ e^+e^- \to$ hadrons data vs. Chebyshev polynomial fits

For gnuplot use

```
./test_Rdat_gnu.sh [emin emax iomegaphidat iresonances IRESON IUNMIX]
```

shows RN2, R3G, R33 and Rdat successively, for next window “alt-tab” key and RETURN.

The time-like vs space-like effective charge



Note that the smooth space-like effective charge agrees rather well with the non-resonant "background" above the Φ (kind of duality)

No proof that this cannot produce non-negligible shifts!

Time-like VP-subtraction cannot be implemented locally near OZI suppressed resonances: $J/\psi, \psi'$ and $\Upsilon_1, \Upsilon_2, \Upsilon_3$

4. My alphaQED versus alpha2SM programs

Corresponding $SU(2)_L$ coupling $\alpha_2 = g^2/4\pi$ ➠ alpha2SMr26 and alpha2SMc26

Corresponding functions:

α	α_2	comment
dggvapx	degvapx	use in space-like region
cggvapx	cegvapx	use in time-like region
funalpqedcx	funalp2smcx	
alphaQEDc26	alpha2SMc26	

Delta_alpha2 = cegvapx(s,cerror,cerrorsta,cerrorsys)

alpha2_complex=funalp2smcx(s,cerror,cerrorsta,cerrorsys,contributionflag)

provides in addition the corresponding imaginary parts. Requires to include funalp2smcx.h as a header in your calling program. Check and edit, e.g., to change reference value of st2 (weak mixing parameter) required to define reference input $\alpha_2(0)$ in terms of $\alpha(0)$.

Compiling: make prog=alpha2SMc26

❐ On flavor separation

Virtual hadronic effect in electroweak precision observables I have analyzed first in
F. Jegerlehner,"Hadronic Contributions to Electroweak Parameter Shifts: A Detailed Analysis,"
Z. Phys. C **32**, 195 (1986), doi:10.1007/BF01552495,
as implemented in improved form in the alphaQEDc26-package.

It allow us to calculate non-perturbative hadronic correction in $\gamma\gamma$, γZ, ZZ and WW self energies, as

$$\begin{aligned}
\Pi^{\gamma\gamma} &= e^2 \; \hat{\Pi}^{\gamma\gamma} \\
\Pi^{Z\gamma} &= \frac{eg}{c_\Theta} \; \hat{\Pi}_V^{3\gamma} - \frac{e^2 \, s_\Theta}{c_\Theta} \; \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{ZZ} &= \frac{g^2}{c_\Theta^2} \; \hat{\Pi}_{V-A}^{33} - 2 \, \frac{e^2}{c_\Theta^2} \; \hat{\Pi}_V^{3\gamma} + \frac{e^2 \, s_\Theta^2}{c_\Theta^2} \; \hat{\Pi}_V^{\gamma\gamma} \\
\Pi^{WW} &= g^2 \; \hat{\Pi}_{V-A}^{+-}
\end{aligned}$$

with $\hat{\Pi}(s) = \hat{\Pi}(0) + s\,\hat{\pi}(s)$. Leading hadronic contributions:

$$\begin{aligned}
\Delta\alpha^{(5)}_{\rm had}(s) &= -e^2 \left[\mathrm{Re}\,\hat{\pi}^{\gamma\gamma}(s) - \hat{\pi}^{\gamma\gamma}(0)\right] \\
\Delta\alpha^{(5)}_{2\,\rm had}(s) &= -\frac{e^2}{s_\Theta^2} \left[\mathrm{Re}\,\hat{\pi}^{3\gamma}(s) - \hat{\pi}^{3\gamma}(0)\right]
\end{aligned}$$

which exhibit the leading hadronic non-perturbative parts, i.e. the ones involving the photon field via mixing. $\Delta\alpha^{(5)}_{\rm had}(s)$ and $\Delta\alpha^{(5)}_{2\,\rm had}(s)$ via e^+e^--data and isospin arguments [(u,d), s flavor separation]:

$$\Pi^{3\gamma}_{ud} = \frac{1}{2}\,\Pi^{\gamma\gamma}_{ud} \qquad ; \qquad \Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

$$\Pi^{\gamma\gamma} = \Pi^{(\rho)} + \Pi^{(\omega)} + \Pi^{(\phi)} + \cdots \qquad \Rightarrow \qquad \Pi^{3\gamma} = \frac{1}{2}\,\Pi^{(\rho)} + \frac{3}{4}\,\Pi^{(\phi)} + \cdots$$

Flavor separation assuming OZI violating terms to be small $\Rightarrow$ perturbative rewighting $\Rightarrow$ disagrees with lattice QCD results!!!

Note that the “wrong” perturbative weighting

$$\Pi^{3\gamma}_{ud} = \frac{9}{20}\,\Pi^{\gamma\gamma}_{ud}\ ;\ \ \Pi^{3\gamma}_{s} = \frac{3}{4}\,\Pi^{\gamma\gamma}_{s}$$

has been proven to clearly mismatch lattice results, while the correction $\frac{9}{20} \Rightarrow \frac{10}{20}$ based on isospin symmetry relations is in good agreement. This also means the OZI suppressed contributions should be at the 10% level and not negligibly small.

Included in the alphaQEDc26-package are the $R^{\gamma\gamma}$, $R^{2\gamma}$, R^{33} data files as well as the R^{N2} for checking lattice QCD $n_f = 2$ data: RGG0322.f, R3G0322.f, R330322.f, RN20322.f

❒ Flavor separation by hand:

As direct low energy hadron production data is available only for the $\langle\gamma\gamma\rangle$ channel for the other channels one has to sort out individual flavor components from $R(s)$-data. Flavor separation is possible only in regions where exclusive channel cross sections are available. We perform this in the region 0.61 GeV to 2.1 GeV. Above this energy only inclusive $R(s)$ measurements are available. In the $SU(3)$ flavor symmetry limit approximation, below the charm threshold, the uds does not need any separation. What remains besides the narrow resonances is in the perturbative regime and can be rewighted by perturbative QCD (pQCD). For a more precise flavor separation the ud isospin components I=1 and I=0 and the s component have to by isolated as far as practicable.

We skip all final states involving photons like the subleading $\pi^0\gamma$, $\eta\gamma$ and $\eta'\gamma$ channels and separate as follows:

- as $ud,\ I = 0$ we include states with odd number of pions
- as $ud,\ I = 1$ we include states with even number of pions
- as $\bar{s}s$ we count all states with Kaons

States ηX with X some other hadrons are collected separately, and then split into $q = u, d$ and s components by appropriate mixing.

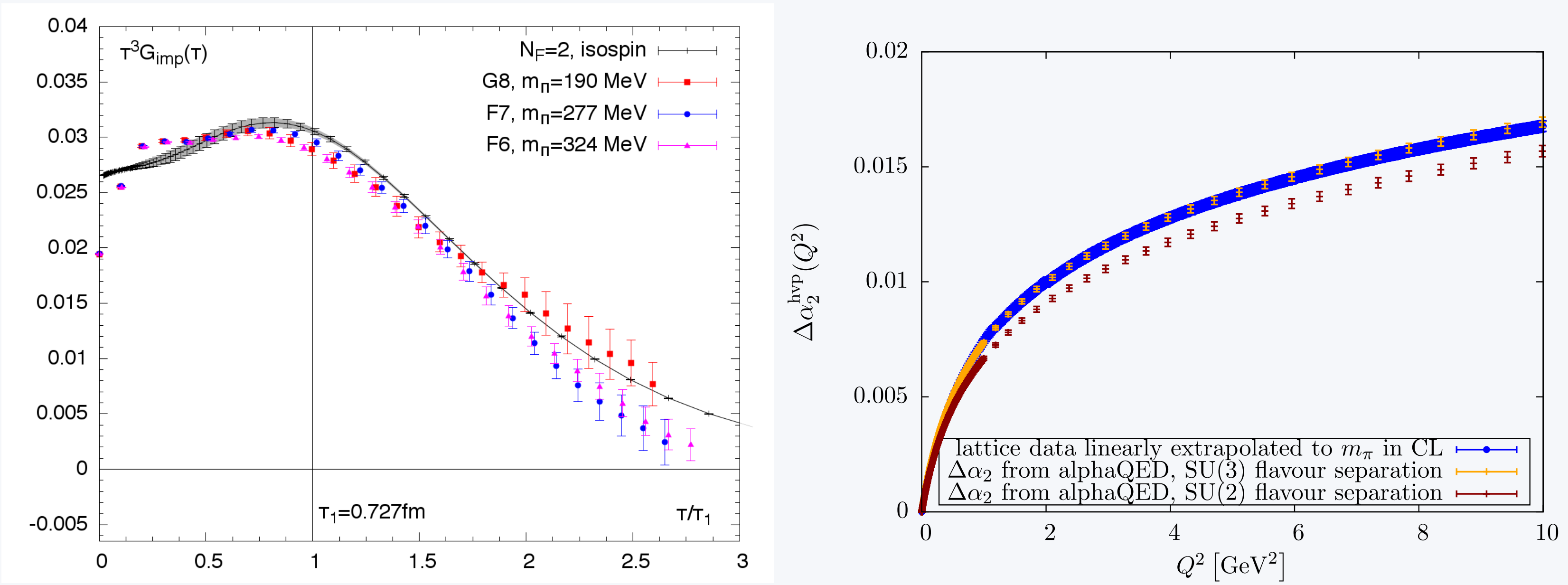
Above about 1.5 GeV there is an off-set of -2.2% (mainly due to SU(3) breaking by Kaon mass effects). The following tabular illustrates where the main difference comes from, here evaluated at $s = M_Z^2[\,500\ \mathrm{GeV}]$ (values in units $\sin^2\Theta_{\mathrm{eff}}\,\Delta g \times 10^4$)

	old $SU(3)$ scheme	new $SU(2)$ scheme
below c threshold	$\Delta g_{uds} = 35.183\,[35.171]$	$\Delta g_{usd} = 31.679\,[31.667]$
above c threshold	$\Delta g_{\mathrm{rem}} = 95.784[144.735]$	$\Delta g_{\mathrm{rem}} = 95.809[144.761]$

As expected the surprisingly large shift comes from the low energy region. The shift persists at higher energies since, with t_c the charm threshold,

$$\Delta g(q^2) = -\frac{g^2\,q^2}{12\pi^2}\int_{s_0}^{t_c}\frac{\mathrm{d}s}{s}\,\frac{R^{3\gamma}(s)}{s-q^2-\mathrm{i}\varepsilon} \overset{q^2\gg t_c}{\sim} \frac{\alpha_2}{3\pi}\int_{s_0}^{t_c}\frac{\mathrm{d}s}{s}\,R^{3\gamma}(s)$$

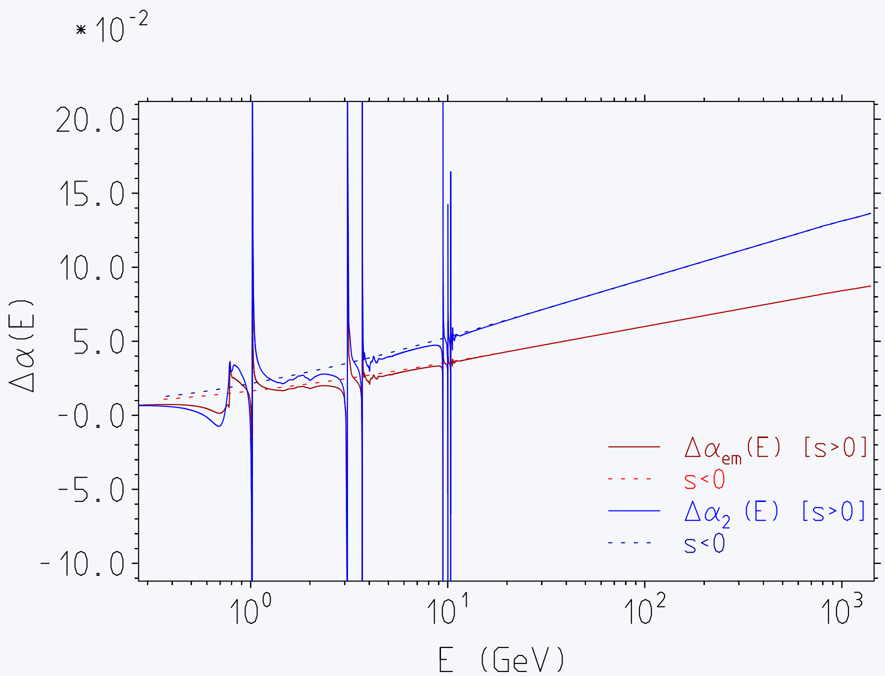
independent of q^2. It means that at large q^2 our effective coupling cannot be expected to agree with the perturbative result. This remains true without applying a cut-off of course. For more details see.



Testing flavor separation H. Meyer et al. [l], arXiv:1312.0035, K. Jansen et al. arXiv:1505.03283[r]

Note: gauge boson SE are potentially very sensitive to New Physics (oblique corrections)

➠ new physics may be obscured by non-perturbative hadronic effects; need to fix this!



$\Delta\alpha_{\rm em}(E)$ and $\Delta\alpha_2(E)$ as functions of energy E in the time-like and space-like domain. The smooth space-like correction (dashed line) agrees rather well with the non-resonant "background" above the ϕ-resonance (kind of duality). In resonance regions as expected "agreement" is observed in the mean, with huge local deviations.

5. The coupling α_2, M_W and $\sin^2\Theta_f$

How to measure α_2:

- charged current channel M_W ($g \equiv g_2$):

$$M_W^2 = \frac{g^2\,v^2}{4} = \frac{\pi\,\alpha_2}{\sqrt{2}\,G_\mu}$$

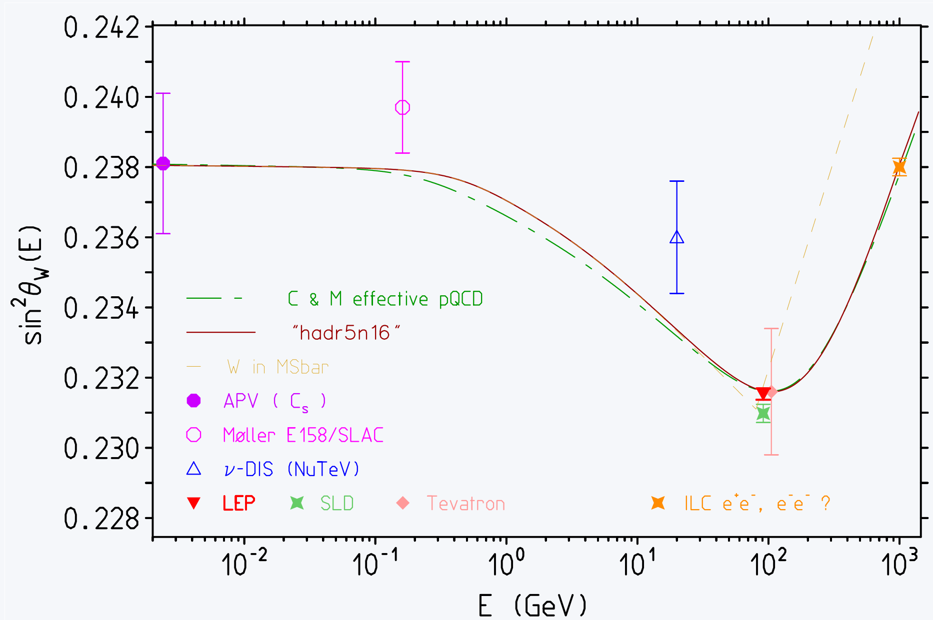
- neutral current channel $\sin^2\Theta_f$

In fact here running $\sin^2\Theta_f(E)$: LEP scale $\Longleftrightarrow$ low energy $\nu_e e$ scattering

$$\sin^2\Theta_e = \left\{\frac{1-\Delta\alpha_2}{1-\Delta\alpha} + \Delta_{\nu_\mu e,\mathrm{vertex+box}} + \Delta\kappa_{e,\mathrm{vertex}}\right\}\sin^2\Theta_{\nu_\mu e}$$

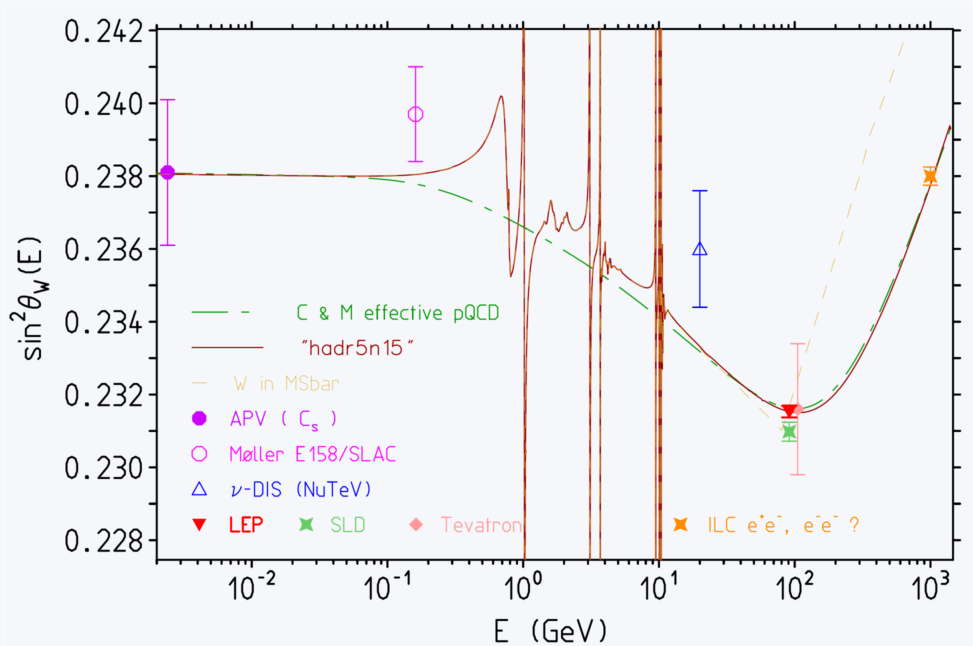
The first correction from the running coupling ratio is largely compensated by the ν_μ charge radius which dominates the second term. The ratio $\sin^2\Theta_{\nu_\mu e}/\sin^2\Theta_e$ is close to 1.002, independent of top and Higgs mass. Note that errors in the ratio $\frac{1-\Delta\alpha_2}{1-\Delta\alpha}$ can be taken to be 100% correlated and thus largely cancel.

Calculating $\sin^2\Theta_f$ use ACWMsin2theta, "make prog=ACWMsin2theta"



$\sin^2\Theta_W(Q)$ as a function of Q in the space-like region. Hadronic uncertainties are included but barely visible. Uncertainties from the input parameter $\sin^2\theta_W(0) = 0.23822(100)$ or $\sin^2\theta_W(M_Z^2) = 0.23153(16)$ are not shown. Future ILC/FCC measurements at 1 TeV would be sensitive to Z', H^{--} etc.

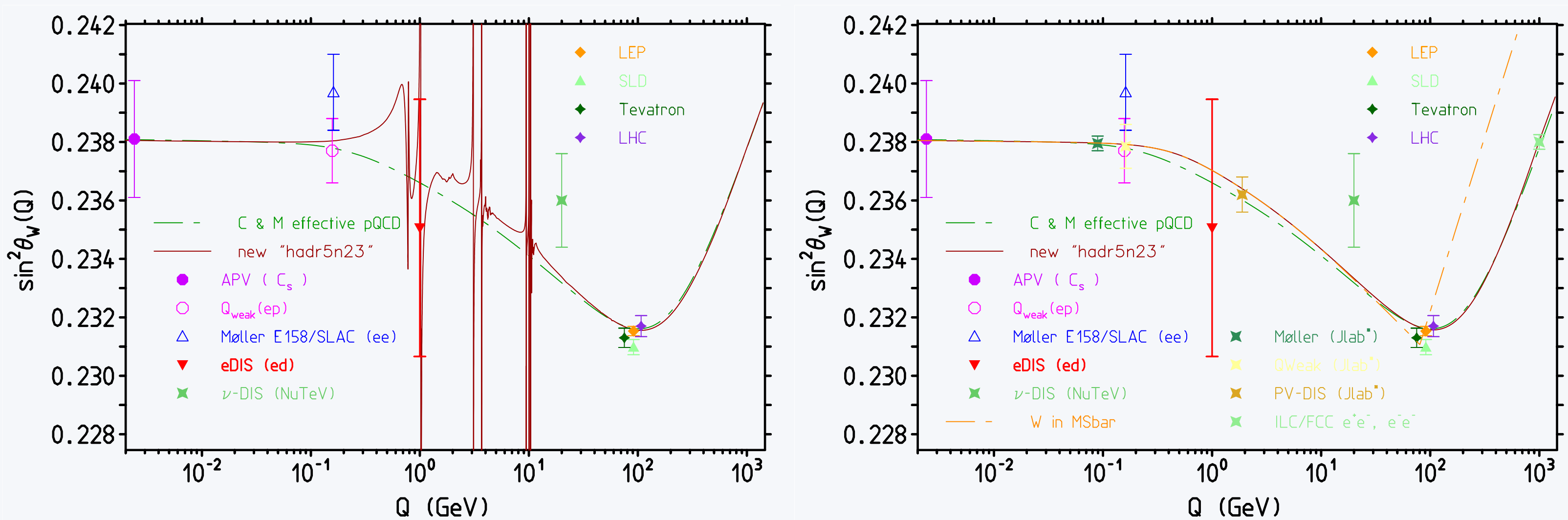
Except from the LEP and SLD points (which deviate by 1.8 σ), all existing measurements are of rather limited accuracy unfortunately!



$\sin^2\Theta_W(E)$ as a function of E in the time-like region. Note that $\sin^2\theta_W(0)/\sin^2\theta_W(M_Z^2) = 1.02876$ a 3% correction established at $6.5\ \sigma$.

$\sin^2\Theta_{\rm eff}$

exhibiting a specific dependence on the gauge boson SEs
is an excellent monitor for New Physics



2023 update for $\sin^2\Theta_{\rm eff}$ compared with data.
All predictions are adjusted to $\sin^2\Theta_{\rm eff}(M_Z)$ from LEP1.
The four last x points refer to planned measurements.

- Curves show various predictions:
  ❶ Czarnecki & Marciano using pQCD with effective quark masses together with bosonic ($W, Z-\gamma$) corrections (self-energy, vertex+box) determined by on-shell polarized Møller scattering,
  ❷ the same using hadronic VP effects based on new SU(2) flavor splitting of R data (deg entry in `hadr5n26.f`)

- Data points as selected by Erler & Freitas in their PDG Review

6. Integrating $R(s)$

The `intRdatx.f` program may be used to calculate integrals of the hadronic cross section data. Relevant files: intRdatx.f, intRdatx.inp, intRdatx_gnu.sh, Makefile_iRdx, intRdatx_clean.sh; Requires NAG library!

- edit `Makefile_iRdx`; adjust directories, load path [compiler?] etc.
- similarly customize `intRdatx_gnu.sh`
- run ./intRdatx_gnu.sh 0, which executes
  - make -f Makefile_iRdx
  - run ./intRdatx < intRdatx.inp
- Main results in files: dalhads.dat, intRdatx.tab

or see list of examples in intRdatx.inp and edit;
– copy marked block and paste in at the head of this parameter file

Example: in `intRdatx_gnu.sh` edit IUNMIX=0 and run ./intRdatx_gnu.sh 16 results in amuhad26.f and intRdatx.tab as follows:

```
c*****************************************************************************
c*                                                                           *
c*    --- intRdatx ---                              Thu Jun 18 01:28:50 2026  *
c*                                                                           *
c****************************# FJ@HU Berlin #*********************************

c  "Muon anomaly"              Update version (24/02/2026)

c ISCAN:  0, Ismooth: 0, IGS: 0, DSET: WAVE, iplot: 0 itestflavorspl: 0
c IKE,Emin,Emax,NP,N,EPSREL:  0, 0.0000E+00, 0.0000E+00,   1, 10, 1.00E-03,
c ALS(sta)(sys):  0.1184( 0.0007)( 0.0000), MZINP:  91.1880, MTOP:  172.40
c NF,IOR,ICHK: 5,4,2,ISU2: 0,NFL: 5,INAG: 1,inotail: 0, sin2W_ref: 1.00000
c CHPTCUT,EC,ECUT: 0.3180, 5.20, 11.50, sin2W_eff: 0.23153, s2W_ref/eff: 4.319
c iintRd,IUNMIX,iomegaphidat,iresonances,IRESON:1, 0, 2, 1, 1


 Result :   697.93     0.81     3.93     4.01


  intRdatx.f --- Timestamp:Thu Jun 18 01:28:50 2026

c  "Muon anomaly"              Update version (24/02/2026)


 E:     0.0000 iomegaphidat= 2 iintRdat= 1 IUNMIX= 0
```

```
 der contributions tab


   Chpt        & ( 0.28, 0.32) &      2.81 ( 0.06) ( 0.03)[ 0.06]&  2.3\% &
  $\\rho       & ( 0.32, 1.05) &    509.19 ( 0.58) ( 2.55)[ 2.62]&  0.5\% &
  $\\omega     & ( 0.42, 0.81) &     36.71 ( 0.29) ( 1.36)[ 1.39]&  3.8\% &
  $\\phi       & ( 1.00, 1.04) &     37.30 ( 0.23) ( 0.75)[ 0.79]&  2.1\% &
  $J/\\psi     & ( 3.10, 4.50) &      7.65 ( 0.41) ( 0.37)[ 0.55]&  7.2\% &
  $\\Upsilon   & ( 9.40,11.10) &      0.09 ( 0.00) ( 0.01)[ 0.01]&  8.2\% &
    had        & ( 1.05, 1.40) &     26.09 ( 0.06) ( 0.99)[ 0.99]&  3.8\% &
    had        & ( 1.40, 2.00) &     35.32 ( 0.07) ( 2.27)[ 2.27]&  6.4\% &
    had        & ( 2.00, 3.20) &     22.18 ( 0.07) ( 0.27)[ 0.28]&  1.3\% &
    had        & ( 3.20, 3.60) &      3.00 ( 0.01) ( 0.05)[ 0.05]&  1.6\% &
    had        & ( 3.60, 5.20) &      8.65 ( 0.04) ( 0.41)[ 0.42]&  4.8\% &
   pQCD        & ( 5.20, 9.46) &      6.15 ( 0.00) ( 0.00)[ 0.00]&  0.1\% &
    had        & ( 9.46,11.50) &      0.83 ( 0.00) ( 0.02)[ 0.02]&  2.3\% &
   pQCD        &(11.5,$\infty$)&      1.96 ( 0.00) ( 0.00)[ 0.00]&  0.0\% &
   data        & ( 0.28,11.50) &    689.82 ( 0.81) ( 3.93)[ 4.01]&  0.6\% &
   total       &( 0.3,$\infty$)&    697.93 ( 0.81) ( 3.93)[ 4.01]&  0.6\% &

 tab summary der contributions

der (errder):   697.93  0.81  3.93  (4.01)
test der:       697.93  (4.01)

Result :    697.931  (4.012)
```

Before running `intRdatx` run `intRdatx_clean.sh` to remove old data files (some get saved)!!!

Running intRdatx_gnu.sh using prefixed options:

`./intRdatx_gnu.sh x`

x=0,1,...,21; x=0 individual setting via `intRdatx.inp`

x=1,2,3,11 and 4,5,6,12 creates include-files for `hadr5n26.f`

Number of flavors may be set as a second parameter for space-like [Euclidean] HVP cases

time-like:		der	deg
x= 1 [4]	–	dalhadslow26_5.f	[including deghadslow26_5.f]
x= 3 [6]	–	dalhadshigh26_5.f	[including deghadshigh26_5.f]
space-like:		der	deg
x= 2 [5]	–	dalhadt26_5.f	[including deghadt26_5.f]
x=11[12]	–	dalhadthigh26_5.f	[including deghadthigh26_5.f]

⋆ ⚠ ⚠ ⚠ ⋆

These files are produced as *.f_new files in order not to overwrite the include files defining the core file `hadr5n26.f` on which the whole package is based. Unless you really want to replace them when R data or options for handling resonances are changed. This could be done by running `./intRdatx_upd.sh` followed by ./unnew.sh.

For Euclidean/space-like quantities, one also may specify the number of flavors to differ from $n_f = 5$ (either in intRdat.inp0 or) or providing nf=2,3,4,5 output via

```
./intRdatx_gnu.sh 19
```

and related Euclidean correlators via

```
./intRdatx_gnu.sh 20
```

nf=2,...,5 results are written to intRdatx_n2.dap, $\cdots$,intRdatx_n5.dap as plot-data.

Note that the 5-flavor data files ...26_5.f define the basic functions $\Delta\alpha^{(5)}_{\rm had}(s)$, $\alpha(s)$ and $\Delta\alpha^{(5)}_{2\,\rm had}(s)$, $\alpha_2(s)$ provided by the `alphaQED` and `alpha2SM` packages. They are created by running `./intRdatx_gnu.sh x [y z]` with the extra label .f_new, i.e., ...26_5.f_new etc., so that the master files are not accidentally overwritten. x refers to the channel 1,2,...; optionally one can overwrite the preset parameter iomegaphidat=2 [y] and IUNMIX=1 [z] by y=1,2,3,-1 and z=0,1,2,-1, treating resonance contributions in different ways.

intRdatx_gnu.sh channel flags:

flag		quantity	data files created
0	=	input set from intRdatx.inp	dalhads.dat, intRdatx.tab
1	=	hadr5n time-like low der	dalhadslow26_5.f
2	=	hadr5n space-like der	dalhadt26_5.f
3	=	hadr5n time-like high der	dalhadshigh26_5.f
4	=	hadr5n time-like low deg	deghadslow26_5.f
5	=	hadr5n space-like deg	deghadt26_5.f
6	=	hadr5n time-like high deg	deghadshigh26_5.f
7	=	hadr5n time-like low deg3	de3hadslow26_5.f
8	=	hadr5n space-like deg3	de3hadt26_5.f
9	=	hadr5n time-like high deg3	de3hadshigh26_5.f
10	=	amuhad(K3c) 1st integral scan	k3chadslow26.f input for 14,15
11	=	hadr5n space-like high der	dalhadthigh26_5.f
12	=	hadr5n space-like high deg	deghadthigh26_5.f
13	=	hadr5n space-like high de3	de3hadthigh26_5.f
14	=	amuhad(K3c) 2nd integral	k3chadres26.f
15	=	amuhad(K3c) 2nd integral error	k3chaderr26.f
16	=	amuhad LO	amuhad26.f
17	=	adlerslope	adlerslope26.f
18	=	franzslope	franziskaslope20.f
19		space-like der n_f=2,3,4,5 (intRdat19.inp)	intRdatx_n5,4,3,2.dap
20		Euclidean Correlators n_f=2,3,4,5 (ECintR.inp)	intRdatx_n5,4,3,2.dap
21		Adler function	adlerintrdatx26.dat

Options file: `intRdatx.inp`

CHPTCUT,EC,ECUT,ETAILCUT
ISCAN,Ismooth,IGS,DSET,iplot, ISU2 ,irennew, NFL ,INAG,inotail
IKE ,Emin,Emax,NP,N,EPSREL,XAXIS
ALS,EST,ESY,MZINP,MTOP,NF,IOR,ICHK
NNLO,IERK3C

Example:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0, 0 ,-1, 5 ,1,0
 0 ,1.0d0,1.d0,1,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0, 0
# Muon g-2 LO
```

./intRdatx_gnu.sh 0 ==> dalhads.dat

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',1, 2 ,-1, 5 ,1,0
 1 ,-120.0d0,-0.d0,10,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0, 0
# deg space-like -120.0 to 0.0
```

./intRdatx_gnu.sh 0 ==> intRdatx_3g.dap

more standard examples in `intRdatx.inp`

Channel 10 provides 1st integral for the double $R(s)$-integral for K3c NNLO amu kernel; channels 14 and 15 perform the second integrals. 14 for the central value, 15 for the error (see chapter 5.1.13 of my book "The Anomalous Magnetic Moment of the Muon").
Channel 10 read input data from intRdat.inp10 make sure it exist and exhibits the proper settings.

Channels 14,15 read input data from k3chadslow26.f (output from channel 10 reading intRdat.inp10) make sure it exist and exhibits the proper settings, the same for option 19 which reads input settings from intRdat19.inp and produces space-like correlator plot data for the various flavors: intRdatx_n2.dap, intRdatx_n3.dap, intRdatx_n4.dap, intRdatx_n5.dap

Many other integrals can be calculated by editing intRdatx.inp and running ./intRdatx_gnu.sh 0. The list of options:

```
' Kernel type IKE='
  0 (g-2) muon,
  1 (delta r),
  2 Adler function,
  3 (g-2) tau,
  4 (g-2) ele,
NNLO=0:                      NLLO=1:
  5 (g-2) muon HOa K2a,        5 (g-2) muon K3a,
  6 (g-2) muon HOb K2b,        6 (g-2) muon K3b,
```

```
 7 (g-2) ele  HOa K2a,     7 (g-2) muon K3LbL,
 8 Euclidean Correlators t^3*G_2(t)
 9 Euclidean Correlators t^4*G_3(t)
 10 Adler slope at s=0
 11 (g-2) muon K3c 1st integral
 12 (g-2) muon K3c 2nd integral
 13 (g-2) ele NNLO K3a

ISCAN=1,2,-1: use given X arrays (energy scan);
      1=time-like low,
      2=time-like high,
     -1=space-like
      0= choose
ISU2 =0[alpha],
      1[alpha_2,old], old= SU(3) flavor splitting (obsolete),
      2[alpha_2,new], new= separate u,d [SU(2)]+ s from data
      3[Pi33,old] obsolete,
      4[Pi33,new],
```

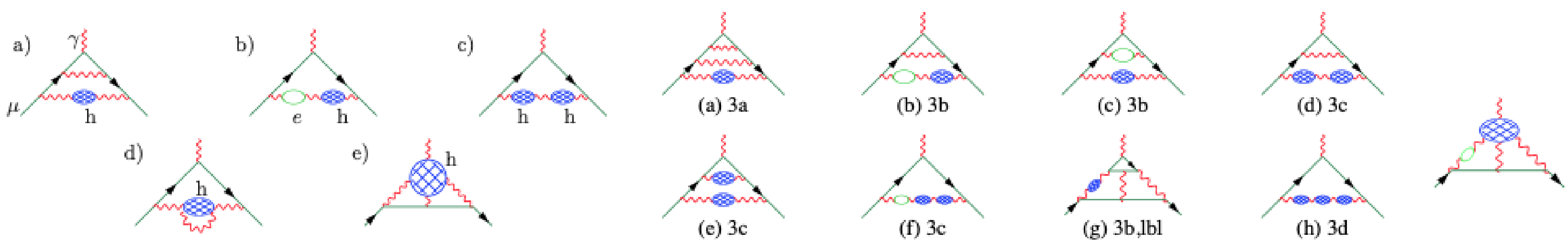
Flags:

variable	options	default/standard
1st line	CHPT and pQCD ranges	
CHPTCUT	CHPT tail from threshold to CHPTCUT	0.318d0
EC	pQCD window from EC to M_{Υ_1}	5.20d0
ECUT	pQCD above ECUT	11.500d0
ETAILCUT	upper limit dispersion integral $>$ 1.0d3	1.0d3
	cuts if set 0 = use default	
2nd line	options how to calculate?	
ISCAN	see list above	0
Ismooth	0=R data, 1=R fit	0
IGS	$\pi\pi$ channel: 0 data, 1 Gounaris-Sakurai parametrization	0
DSET	DSET=CMD2,WAVE,EPM2,TAU2,WAV1,EPM1,TAU1	'WAVE'
iplot	table/plot output: iplot=0/1	0
ISU2	see list above	0
irennew	VP subtraction (resonances)	-1
NFL	number of flavors NFL=2,3,4,5?	5
INAG	SIMPSON OR NAG? INAG=0,1	0
inotail	pQCD tail for NF=2 inotail=0,1,-1 (notail=)	0

3rd line	Kernel, energy range, ...	
IKE	see list	
Emin,Emax,NP		
momentum transfer: E>0 time-like , E<0 space-like		
if NP =1 : table for E=Emin (Emax redundant)		
if E=Emin < Emax and NP > 1 data file for graph		
N	N number of sub-intervals for SIMPSON-Rule integration	20
EPSREL	relative error demanded for NAG-Routine integrals	1.0d-2
XAXIS	linear vs logarithmic energy axis	'LIN'
4th line	**pQCD parameters**	
ALS	QCD coupling $\alpha_s(\mu)$	$\alpha_s(M_Z)$
EST	stat error	
ESY	syst error	
MZINP	reference energy scale μ	91.1880d0
MTOP	top quark mass	172.4d0
NF	NF=5(no top), 6(with top)	5
IOR	IOR=1,2,3,4 order of pQCD	4
ICHK	ICHK=0,1,2 pQCD: 0(m=0), 1 (95) [CK], 2 (00) [HS]	2
	[CK] Chetyrkin, Kühn, [HS] Harlander, Steinhauser (rhad)	

5th line	NNLO flags	
NNLO	switch to NNLO for muon (g-2)	0
IERK3C	IERK3C=1 integrate VP, IERK3C=3 integrate error	1
6th line	if run via intRdatx_gnu.sh overwrite ISCAN and ISU2 as selected by script	
ISCAN		0
ISU2		0

All higher order contributions to the Muon g-2 (see next Figure) can be calculated by editing intRdatx.inp and running ./intRdatx_gnu.sh 0.



Hadronic higher order contributions: involving NLO K2x and NNLO K3x vacuum polarization, and light-by-light scattering insertions. Hadronic “blob” must be free of external photon radiation effects! Hard to achieve via Data which are “all inclusive”, i.e. need disentangle QCD from QED.

Example (g-2) muon HOa K2a:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0, 0 ,-1, 5 ,1,0
 5 ,1.0d0,1.d0,1,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
 0 , 0
# Muon g-2 NLO K2a
```

Example (g-2) muon HOa K3a:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0, 0 ,-1, 5 ,1,0
 5 ,1.0d0,1.d0,1,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
 1 , 0
# Muon g-2 NNLO K3a
```

./intRdatx_gnu.sh 0 ==> dalhads.dat

New: October 2024 intRdatx.f now incorporates Muon g-2 NLO and NNLO with two or three hadronic blob insertions into the photon line of the LO QED diagrams: NLO K2c, NNLO K3d

These may be obtained setting IKE=18
and NNLO,IERK3C ==> ilep,IHOc

Options: ilep = 1,2,3 for electron, muon, tau
Options: higher order VP hadronic bubbles:

LO	–		iHOc= 0	for 1 insertion
NLO	diag c,	kernel K2c:	iHOc= 1	for 2 insertions
NLO	diag b,	part of kernel K2b:	iHOc= 2	one had+lep insertions
NNLO	diag h,	kernel K3d:	iHOc= 3	for 3 insertions
NNLO	diag f,	part of kernel K3c:	iHOc= 4	for 2 had+lep insertions
NNLO	diag ?,	part of kernel K3b:	iHOc= 5	for had+ 2 lep insertions

Except for K2c and K3d the others are already included in the kernels of classes NLO a,b and NNLO classes a,b,c covered by other IKE/NNLO/IERK3C channels of intRdatx (see next table)

Edit intRdatx.inp as follows:

Example (g-2) muon NNLO class K3d (diag h):

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0,0,-1,5,1,0
18,1.0d0,1.d0,1,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
2, 3
# Muon g-2 NNLO K3d
```

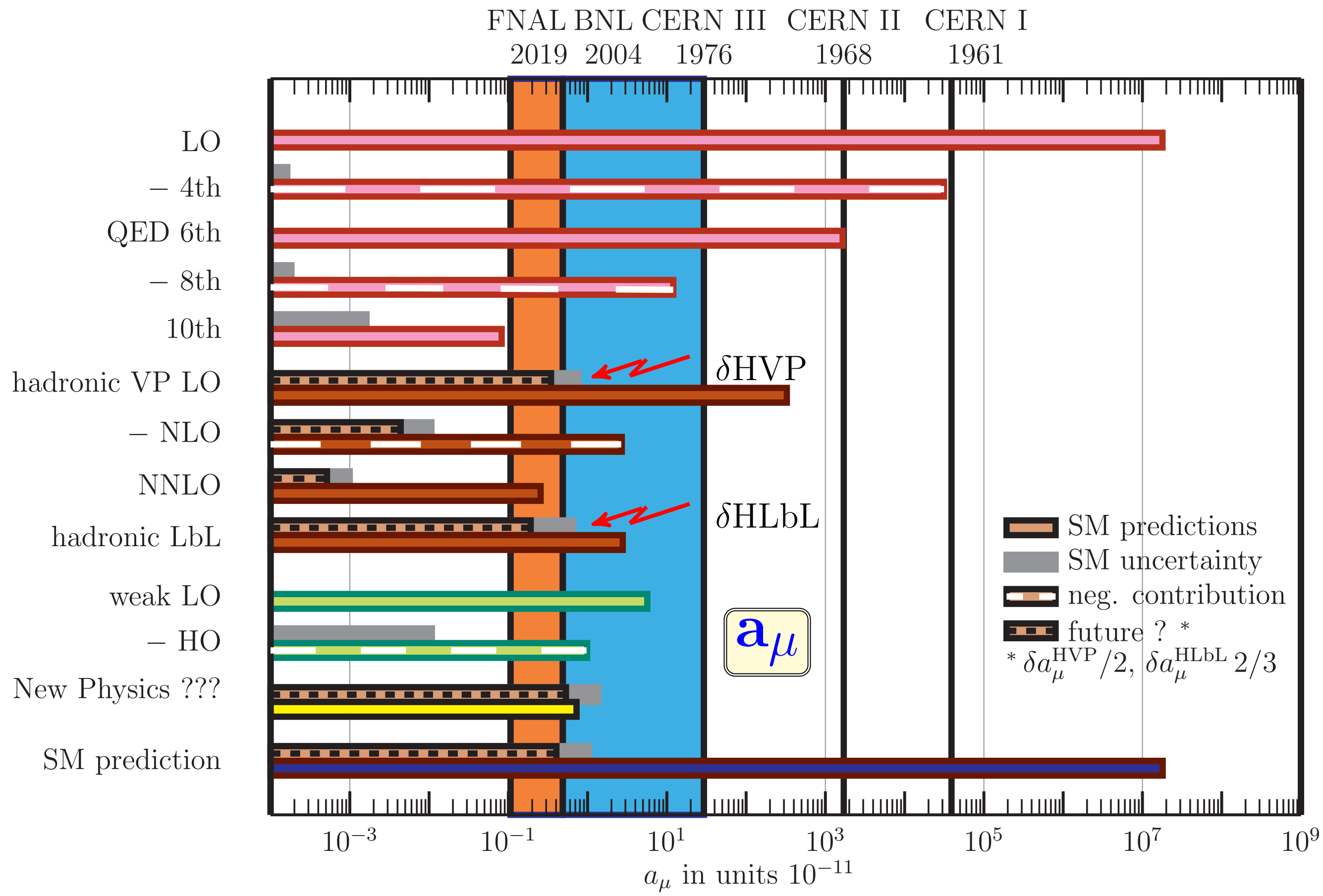
and run ./intRdatx_gnu.sh 0 output on dalhads.dat

Summary of a_μ and a_e contributions:

Hadronic contributions to a_μ times 10^{10}.

Order	Kernel	IKE/NNLO/IERK3C	IUNMIX=0	IUNMIX=1
LO	–	0/0/0	697.931(4.012)	704.450(3.973)
NLO	K2a	5/0/0	-20.892(0.133)	-21.106(0.132)
NLO	K2b	6/0/0	10.497(0.061)	10.602(0.060)
NLO	K2c	18/2/1	0.348(0.007)	0.356(0.006)
NNLO	K3a	5/1/0	0.7930(0.0057)	0.8008(0.056)
NNLO	K3b	6/1/0	-0.4086(0.0028)	-0.4128(0.0028)
NNLO	K3LbL	7/1/0	0.9124(0.0059)	0.9216(0.0059)
NNLO	K3c	12/0/1(3)	-0.0589(0.0004)	-0.0595(0.0004)
NNLO	K3d	18/2/3	5.44(0.17)$\cdot 10^{-4}$	5.62(0.14)$\cdot 10^{-4}$
$a_\mu^{(2)}$			-10.047(0.077)	-10.147(0.077)
$a_\mu^{(3)}$			1.238(0.001)	1.251(0.001)
$a_\mu^{\mathrm{had\,LO+NLO+NNLO}}$			689.122(3.951)	695.953(3.828)

Table entries may be obtained using the script ./intRdatx_amuHO.sh producing tables amuHO1tab and amuHO2.tab



Past and future muon $g-2$ experiments testing various contributions. New Physics ? = deviation $(a_\mu^{\rm exp}-a_\mu^{\rm the})/a_\mu^{\rm exp}$. Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

Hadronic contributions to a_e times 10^{12}.

Order	Kernel	IKE/NNLO/IK3	IUNMIX=0	IUNMIX=1
LO	–	4/0/0	1.8755(0.0110)	1.8933(0.0104)
NLO	K2a	7/0/0	-0.2245(0.0013)	-0.-2266(0.0013)
NLO	K2c	18/1/1	1.7$\cdot 10^{-8}$	1.6$\cdot 10^{-8}$
NNLO	K3a	13/0/0	0.0284(0.0002)	0.0287(0.0002)
NNLO	K3LbL	–	0.037(0.005)	[33]
$a_e^{(2)}$			-0.2245(0.0013)	-0.2266(0.0013)
$a_e^{(3)}$			0.0284(0.0002)	0.0287(0.0002)
$a_e^{\mathrm{had\ LO+NLO+NNLO}}$			1.6794(0.0093)	1.6953(0.0092)

Here the table entries may be obtained using the script ./intrdatx_aelHO.sh producing tables aelHO1.tab and aelHO2.tab

7. Euclidean Correlators

make -f Makefile_iRdx

./intRdatx_clean.sh

./intRdatx < intRdatEC.inp1

intRdatEC.inp1=intRdatx.inp preset as:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',1, 0 ,0, 3 ,1,0
8 ,0.0072d0,100.d0,400,20,1d-3, 'LOG'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0,0
# Euclidean Correlators t3̂*G_2(t)
```

Output data:intRdatx_3g.dap intRdatx_33.dap intRdatx_gg.dap intRdatx_n2.dap

Variants:	IKE	=	8,9	
	ISU2	=	0,2,4	i.e. $<\gamma\gamma>$,$<\gamma 3>$,$<33>$
	NFL	=	3,4,5,6	i.e. number of quark flavors
	NF	=	5,6	i.e. pQCD number of quark flavors
	XAXIS	=	'LIN','LOG'	

I also include a script file for EC's

./ECintR.sh shows a menu

./ECintR.sh 0 run with parameters set in intRdatx.inp

(same as ./intRdatx_gnu.sh 0 see list of examples in intRdatx.inp)

./ECintR_gnu.sh in addition displays the correlator with gnuplot

Some preset options:

./ECintR.sh ggxy where x=3,4,5 and y=lin,log

providing a set of standard parameters and producing

ECgg5lin.dap, ECgg4lin.dap, ECgg3lin.dap ECgg5log.dap, ECgg4log.dap, ECgg3log.dap

in the subdirectory ./dat/

the ggxlog versions also create fortran data tables ECggxoft26.f where x=2,3,4,5 includefiles for amuviaEC.f

Note: in order to get $\langle\gamma\gamma\rangle$ correlators for $N_f < 5$ one has to use the channel $\langle 3\gamma\rangle$, (based on flavor separation and reweighting with results written to `intRdatx_3g.dap`,) in conjunction with the flag‘ `itestflavorsplitting=1` = flavorseparation without reweighting. This option is automatically selected when ISU2=0 and NFL<5

data files may be plotted with gnuplot using ./ECintR_gnu.sh or:
gnuplot> plot ’ECgg5lin.dap’ skip 16 using 1:2
gnuplot> plot "ECgg5log.dap" skip 16 using 1:2:($2-$3):($2+$3) with yerrorbars ls 1 t "N_f=5"
or
cd dat
gnuplot ECintR_gnu.in

8. Integrating the Euclidean Correlators

$$a_\mu^{\mathrm{HVP-LO}} = 4\,\alpha^2\, m_\mu \int\limits_0^\infty \mathrm{d}t\, t^3\, G(t)\, \overline{K}(t)$$

with kernel

$$\overline{K}(t) = \frac{2}{m_\mu\, t^3} \int\limits_0^\infty \frac{\mathrm{d}Q}{Q}\, f(Q^2) \left[(Q/E_0)^2 - 4\, \sin^2\left(\frac{1}{2}\, Q/E_0\right) \right]_{E_0=1/t}$$

and

$$f(s) = \frac{1}{m_\mu^2}\, rZ(r)^3\, \frac{1-rZ(r)}{1+rZ(r)^2}\ ;\ \ Z(r) = \frac{\left(\sqrt{r^2+4r}-r\right)}{2r}\ ;\quad ;\ \ r = s/m_\mu^2\ .$$

Using EC data include file `ECgg5oft26.f`, created via "`./ECintR.sh gg5log`", the program `amuviaEC.f` calculates $a_\mu^{\mathrm{HVP-LO}}$:

make -f Makefile_amuviaEC

run ./amuviaEC

Output:
==> $695.25(1.10)(4.82) \times 10^{-10}$
from Euclidean time correlator for the HVP LO contribution (obtained as a 2-step integration) is in very good agreement with the result of the direct integration of $R(s)$
==> $694.96(0.77)(3.88) \times 10^{-10}$.
In order to create EC data file `./dat/ECgg5oft.f` use extended range option

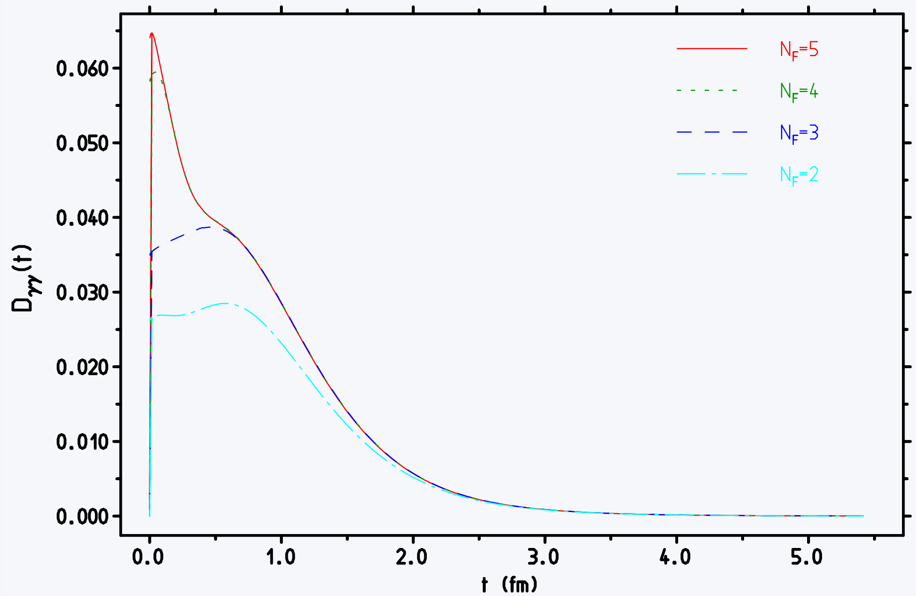
```
-8,0.0072d0,100.d0,400,20,1d-2,'LOG'
```

in order not to miss contributions from tails.

Note: `./dat/ECgg5oft.f` is a copy of output file `dalhads.dat` from running `ECintR.sh gg5log` (maybe dalhads.dat requires some editing of the array boundary e.g. removing last entry)

A shell script `compare.sh` allows one to compare all fortran files on the current directory with the corresponding files in the directory supplied by the user.

Flavor dependence of `amuviaEC` integrands



IUNMIX=0 ($\gamma - \rho^0$ mixing left in data)

n_f		$a_\mu^{\rm had}[n_f]$	stat	syst		$\Delta a_\mu^{\rm had}$
2	$[u,d\,]$	565.44	0.59	2.76	$[u,d\,]$	565.44
3	$[u,d,s\,]$	680.72	0.77	4.57	$[s\,]$	115.28
4	$[u,d,s,c\,]$	694.98	1.10	4.82	$[c\,]$	14.26
5	$[u,d,s,c,b\,]$	695.25	1.10	4.84	$[b\,]$	0.27

IUNMIX=1 ($\gamma - \rho^0$ mixing subtracted from data)

n_f		$a_\mu^{\rm had}[n_f]$	stat	syst		$\Delta a_\mu^{\rm had}$
2	$[u,d\,]$	568.55	0.65	2.83	$[u,d\,]$	568.55
3	$[u,d,s\,]$	690.78	0.82	4.59	$[s\,]$	122.23
4	$[u,d,s,c\,]$	703.90	1.15	4.84	$[c\,]$	13.12
5	$[u,d,s,c,b\,]$	705.14	1.15	4.84	$[b\,]$	1.24

9. 2023 extensions and updates

************* 2023 *** NEW, NEW, NEW *** 2023 ****************

see also CHANGES and INSTALLandRUN

In short:

basic datasets updated concern `Rdat_upd.f RGG0322.f, R3G0322.f, R330322.f, RN20322.f`

as well as hadr5n26.f including
`dalhadt26_5.f dalhadthigh26_5.f dalhadslow26_5.f dalhadshigh26_5.f` ,
which in turn are including
`deghadt26_5.f deghadthigh26_5.f deghadslow26_5.f deghadshigh26_5.f`,
These replace `dalhadshigh19.f dalhadslow19.f dalhadt19.f` of alphaQEDc19.

For comparison with lattice QCD hadronic VP correlator data in the Euclidean regime intRdatx.f was extended to allow calculation of hadronic α and α_2 shifts for different flavors NFL=3,4,5;

intRdatx_gnu.sh was adapted correspondingly. Run `intRdatx_gnu.sh 19` to get flavors 2 to 5 in one run (produces a gnuplot).

FORTRAN arrays representing Euclidean VP data can be created for different flavors as well by specifying NFL in `intRdat.inp0` and run ./intRdatx_gnu.sh 2 which creates `dalhadt26_NF.f` together with $\Delta\alpha_{\rm had}$ plot data output for the different flavors `intRdatx_n2,3,4,5.dap`

Similarly, ./intRdatx_gnu.sh 5 creates `deghadt26_NF.f` together with $\Delta\alpha_{2\,\rm had}$ plot data output for the different flavors `intRdatg3_n3,4,5.dap`

In details:

Euclidean time correlators for different numbers of flavors

using the updated version of intRdatx.f

adapting intRdatx.inp

input is

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0,0,-1,NFL,0,0
8,0.0072d0,100.d0,400,10,1d-2,'LOG'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0 0
# Euclidean Correlators t^3*G_2(t)
```

where NFL=2,3,4,5

running via intRdatx_gnu.sh: ./intRdatx_gnu.sh 0 each run produces dalhads.dat which is copied to fortran data tables ECgg2oft26.f, etc., where $2 \to 3,4,5$ as includefiles for amuviaEC.f

In order to get $a_{\mu\,\mathrm{had}}^{(5)}$ run amuviaEC.f

compile with Makefile_amuviaEC

amuviaEC.f now asks for number of flavors nf=2,3,4,5?

Calculating $\Delta\alpha^{\mathrm{had}}(-Q^2)$ for different numbers of flavors

using the updated version of intRdatx.f

$\Delta\alpha^{\mathrm{had}}(-Q^2)$ now available for flavor numbers NF=2,3,4,5

running via intRdatx_gnu.sh: `./intRdatx_gnu.sh 2`

writes results of space-like hadronic shifts to plot data files:

intRdatx_n2.dap, intRdatx_n3.dap, intRdatx_n4.dap, intRdatx_n5.dap

Crosscheck: der from hard5 may be tested using testhadr5x [make prog=testhadr5x and ./testhadr5x writes result for der on fort.1 and for deg on fort.2].

General advice: to make sure about the meaning of input parameter settings, check intRdatx.inp for the comments on what the entries in input data files mean!

Check with grep -f read intRdatx.f to see what parameters are to be set!

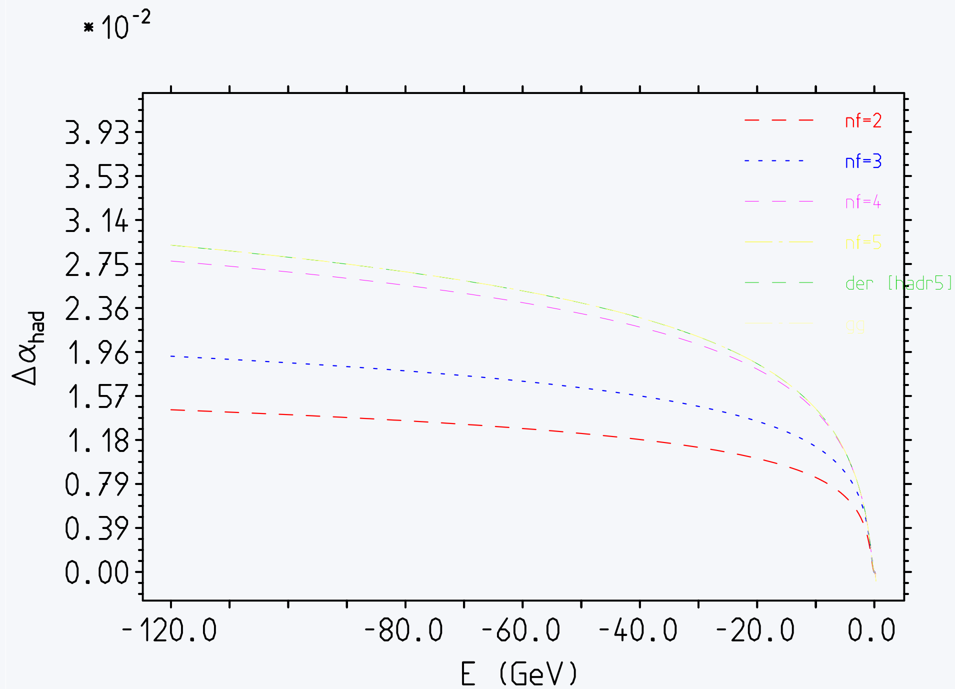
Note: output data for $\alpha(s)$ and $\Delta\alpha^{\rm had}(-Q^2)$ = der are obtained for ISU2=0 , $\alpha_2(s)$ and $\Delta\alpha_2^{\rm had}(-Q^2)$ = deg are obtained for ISU2=2 !

The following plot is obtained with ./intRdatx_gnu.sh 19

input is intRdat19.inp (need to set space-like energy range box emin,emax,n):

```
0.318d0,5.2d0,11.5d0,1.D3
 -3 ,0,0,'WAVE',0, 0 ,-1, 5 ,1,0
 1 , -120.d0,-0.0001d0,120 ,20,1d-3,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0,1
 -3 , 0
# -19- hadr5n space-like der -120 GeV -> 0.
```

produces plot data for the different flavors intRdatx_n2,3,4,5.dap



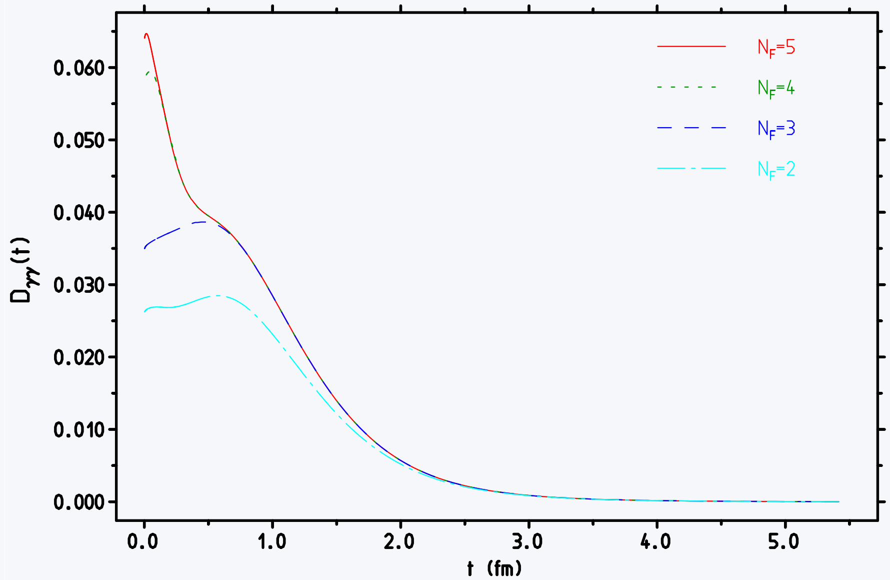
The renormalized Euclidean correlator $\langle\gamma\gamma\rangle(Q^2)$ denoted by $\Delta\alpha_{\rm had}(-E)$ for $n_f = 2, 3, 4, 5$ as a function of the space-like energy transfer denoted by $-E$.

The following plot is obtained with `./ECintR.sh 0`

input is ECintR.inp:

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0, 0 ,-1,3,1,0
 8 ,0.072d0,40.d0,300,20,1d-2,'LOG'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0,0
```

produces plot data for the different flavors intRdatx_n2,3,4,5.dap



The renormalized Euclidean time correlator $t^3 * G_2^{\gamma\gamma}(t)$ as a function of the Euclidean time $t = \mathrm{fm}/E$ where fm=0.1973269631 = fermi in GeV^{-1}.

gnuplot figures can directly by obtained with ./intRdatx_gnu.sh 19

and ./ECintR_gnu.sh 0 respectively.

Note that similarly testhadr5a tests the table entries of hadr5x26.f.

The full QCD $n_f = 6$ result, which besides $\Delta\alpha^{(5)}_{\rm had}$ includes the top-quark contributions, is available form alphaQEDc26.

Calculating $\Delta\alpha_2^{\rm had}(-Q^2)$ for different numbers of flavors

using the updated version of intRdatx.f.

Note: output data for $\alpha_2(s)$ and $\Delta\alpha_2^{\rm had}(-Q^2)$ = deg are obtained for ISU2=2 in the input files displayed in the previous section.

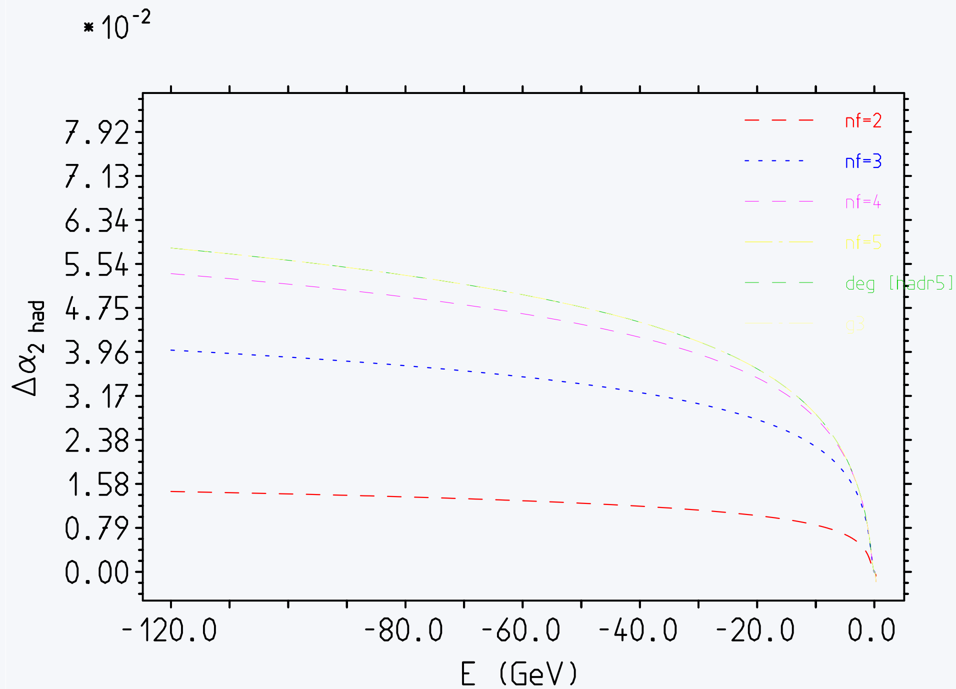
$\Delta\alpha_2^{\rm had}(-Q^2)$ now available for flavor numbers NF=2,3,4,5

running via intRdatx_gnu.sh: `./intRdatx_gnu.sh 5`

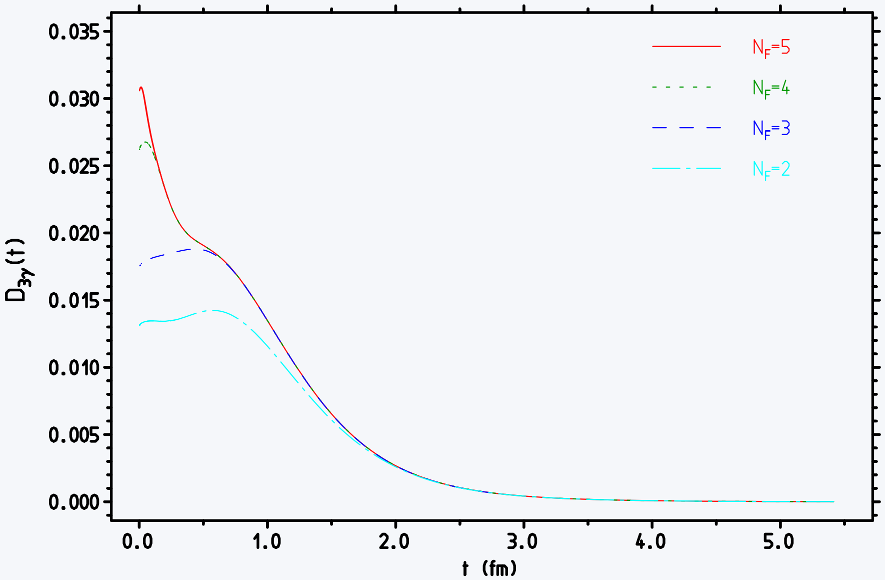
writes results of space-like hadronic shifts to plot data files:

intRdatg3_n3.dap, intRdatg3_n4.dap, intRdatg3_n5.dap

Crosscheck: get deg from hard5 using to testhadr5x



The renormalized Euclidean correlator $\langle 3\gamma\rangle(Q^2)$ denoted by $\Delta\alpha_{\rm had}(-E)$ for $n_f = 2, 3, 4, 5$ as a function of the space-like energy transfer denoted by $-E$.



The renormalized Euclidean time correlator $t^3 * G_2^{3\gamma}(t)$ as a function of the Euclidean time $t = \mathrm{fm}/E$ where fm=0.1973269631 = fermi in GeV^{-1}.

The full QCD $n_f = 6$ result, which besides $\Delta\alpha^{(5)}_{2\,\mathrm{had}}$ includes the top-quark contributions, is available form alphaQEDc26.

The standalone version of hadr5n26.f is hadr5x26.f which includes the include-files dalhad...26_5.f and deghad...26_5.f already.

Note: hadr5n26.f includes

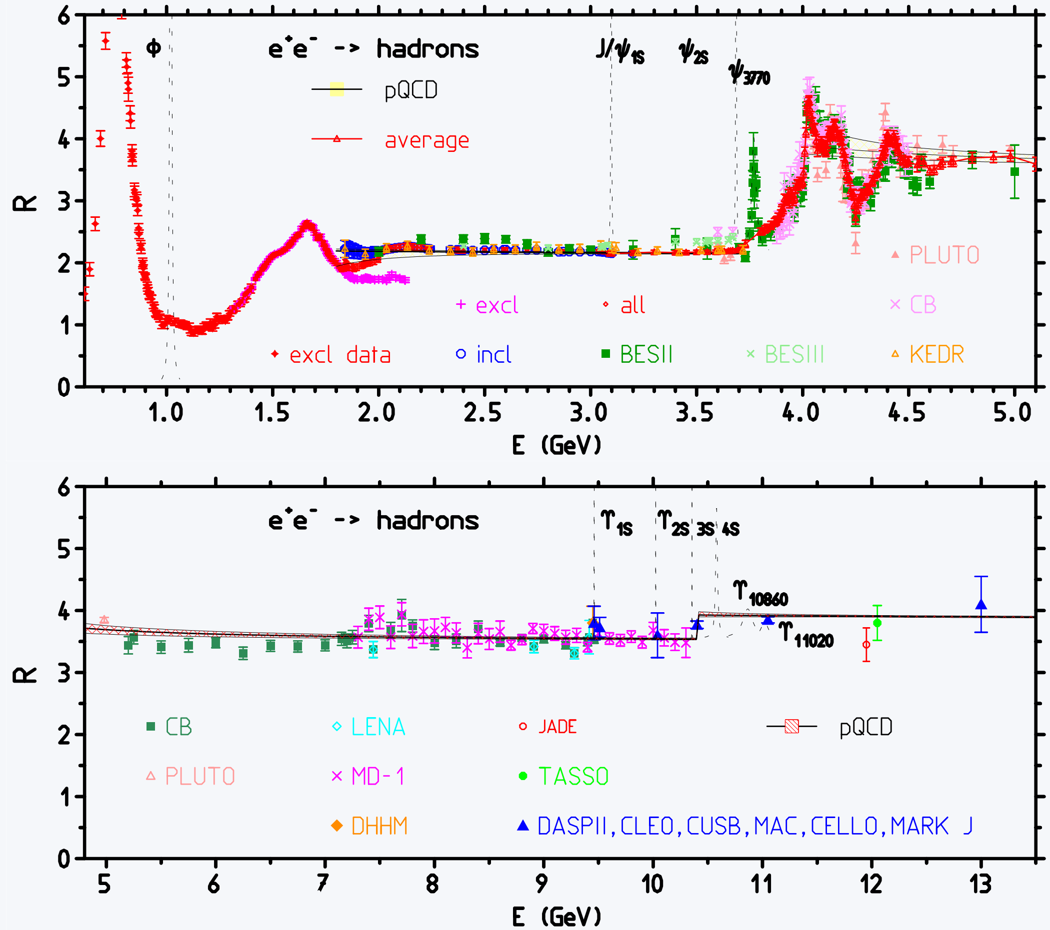
- dalhadslow26_5.f which includes deghadslow26_5.f
- dalhadshigh26_5.f which includes deghadshigh26_5.f
- dalhadt26_5.f which includes deghadt26_5.f and also dalhadthigh26_5.f, deghadthigh26_5.f

These includefiles have been generated with the help of

./intRdatx_upd.sh

This will change hadr5n26.f only if new data compilations for Rdat_upd.f included in Rdat_all.f and/or , R3G0321.f etc. become available.

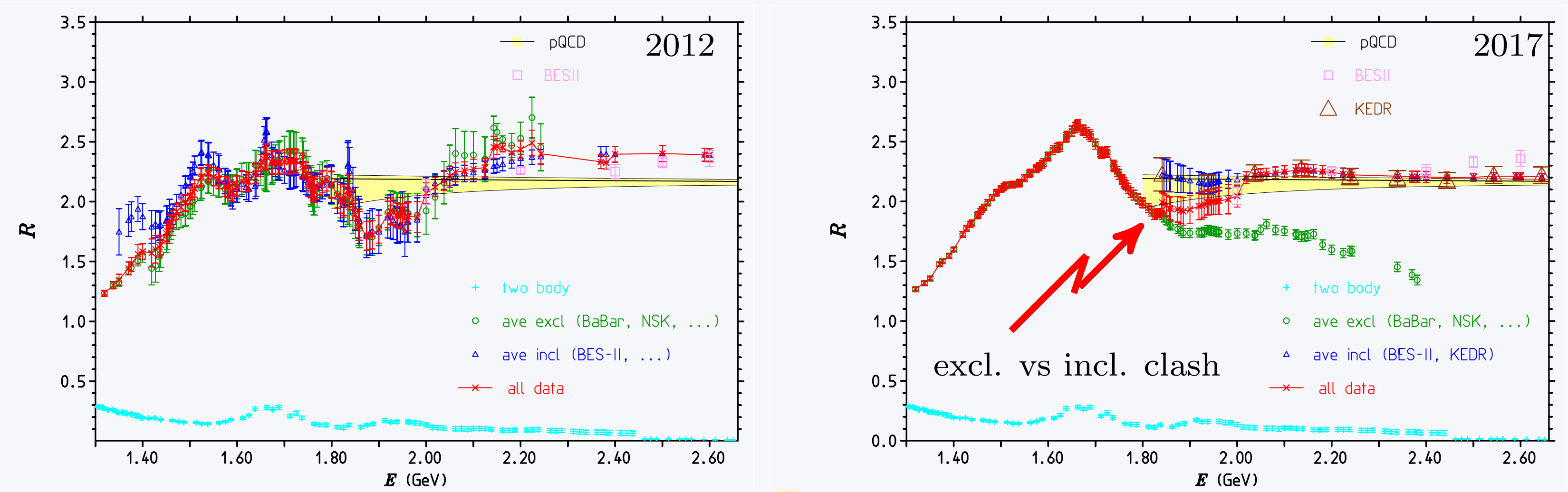
Updated R compilation



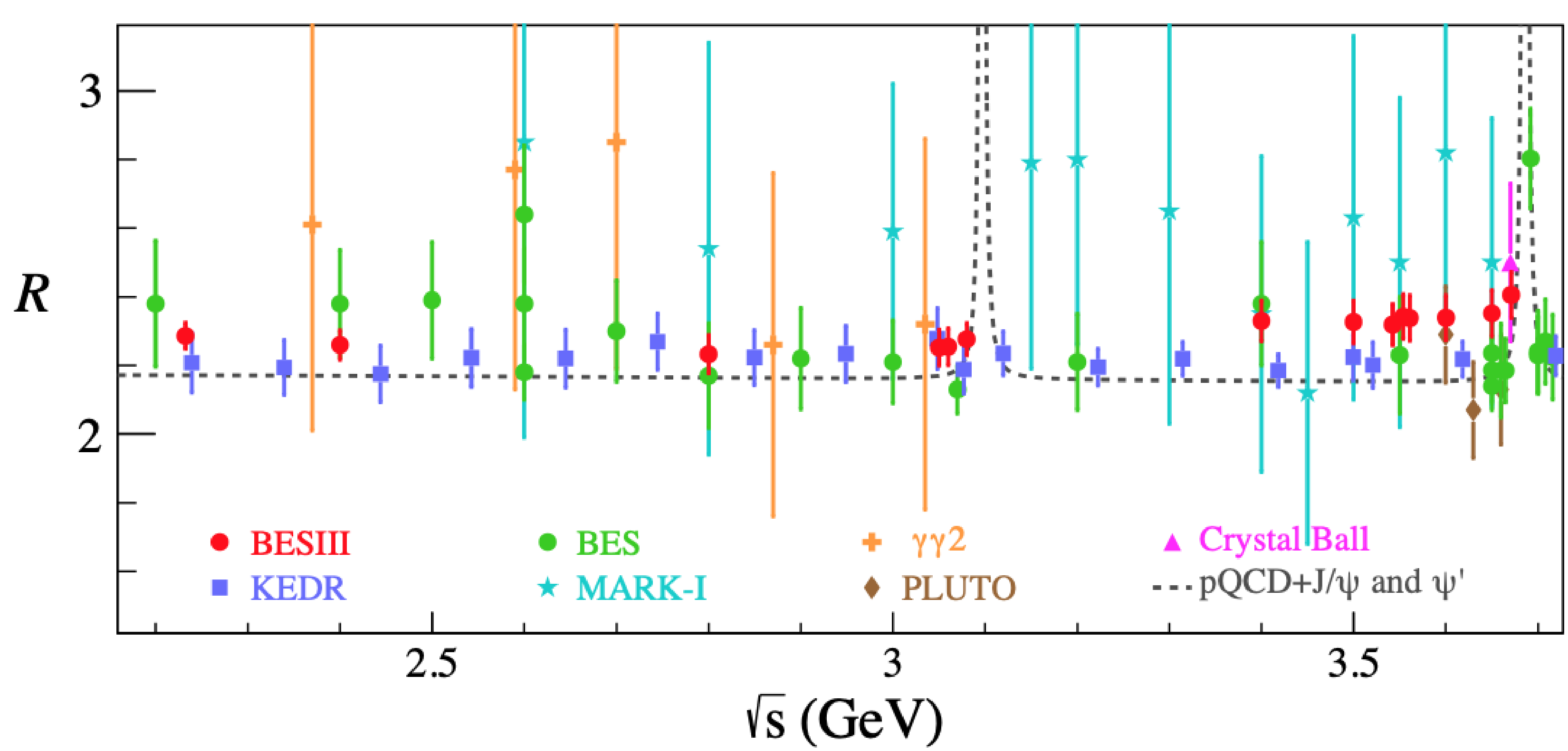
R-data versus pQCD

-> updated Rdat_all.f, Rdat_fun.f and Rdat_fit.f as well as hadr5n19.f -> hadr5n26.f

Region 1.2 to 2 GeV data updated; test-ground exclusive vs inclusive R measurements (more than 30 channels!) VEPP-2000 CMD-3, SND (NSK) scan, BaBar, BES III radiative return! still contributes 50% of uncertainty. Newly, all



Illustrating progress by BaBar and NSK exclusive channel data vs new inclusive data by KEDR. Why point at 1.84 GeV so high?

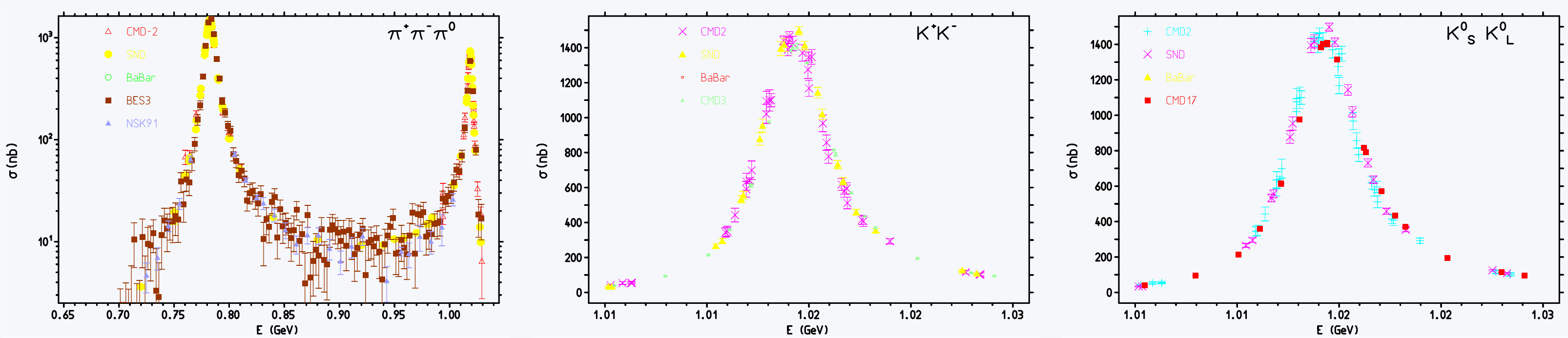


New inclusive R-data by BESII vs. KEDR and older results. Copied from BESIII arXiv:2112.11728 [hep-ex]

Accordingly:
all R-data files Rdat_upd.f included in Rdat_all.f and , R3G0322.f, RN20322.f, R330322.f RGG0322.f and all fit-parameter files had to be updated. This concerns all includefiles *.h entering the Rxxx_fit.f functions. The fit-functions are needed in preforming Principal Value Integrals in the time-like regions! In an appropriate interval around the location of the pole data are replaced by the fit-function (i.e. smoothed data).

Comment: As expected because of the resonant 4π channels there is a resonance enhancement (the $\rho(1700)$) overshooting pQCD predictions near 1.7 GeV. According to quark hadron duality, a compensating undershooting must follow at increasing energy before the pQCD level is expected to be reached. The exclusive data beautifully show a valley after the resonance enhancement as it should be. Therefore the lowest KEDR inclusive point seems to lie way to high, while the summed exclusive result misses reaching the pQCD level when approaching the 2 GeV region, where reliable inclusive measurements from BES II and KEDR exist. Taking a weighted average between exclusive and inclusive data seems to give a reasonable shape for $R(s)$ in the 2 GeV region.

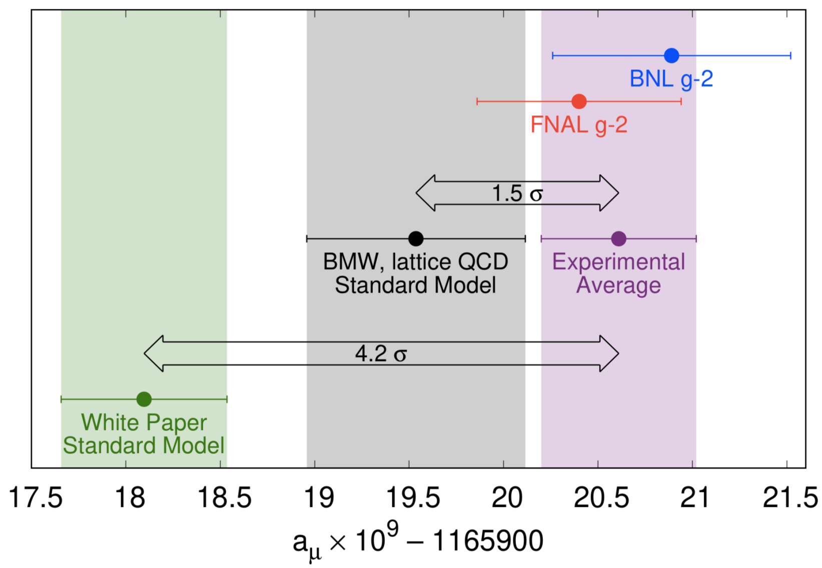
Update ω and ϕ data:



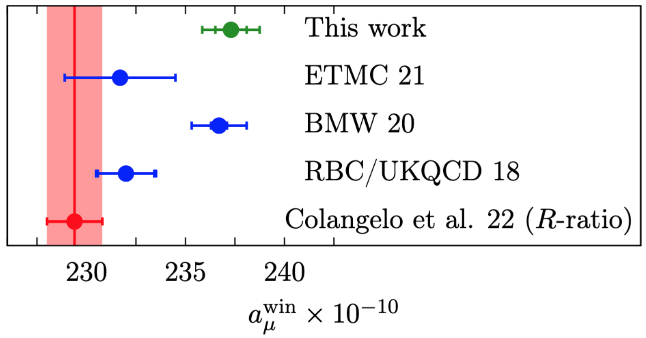
Cross-sections used im actual compilation: the channel $\pi^+\pi^-\pi^0$ is dominant in the ω-resonance range while K^+K^- and $K^0\bar{K}^0$ dominate the ϕ-resonance.

HVP in Muon $g-2$

New LQCD result $a_\mu^{\rm win} = 237.30(1.46) \times 10^{-10}$ from Mainz confirms BMW result!



Talk Harvey Meyer



Muon g-2 plot incl. lattice QCD results. Left: BMW, right: Mainz plots

White paper HVP gives 4.2 σ;
BMW lattice HVP gives 1.6 σ;
Mainz lattice HVP 1.6 σ;
RBC/UKQCD 2.1 σ;

$$a_\mu^{\rm HVP-LO}[\rm THE] = 693.1(4.0) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[\rm BMW] = 707.5(5.5) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[\rm Mainz] = 707.8(5.5) \times 10^{-10}$$
$$a_\mu^{\rm HVP-LO}[\rm RBC] = 703.2(5.5) \times 10^{-10}$$

Theory leading uncertainty: HVP (and HLbL); DR+Data vs lattice QCD

Theory: all of SM counts

$$a_\mu^{\rm SM} = (g_\mu/2-1) \;=\; a_\mu^{\rm (QED+EW+HVP_{LO}+HVP_{NLO}+HVP_{NNLO}+HLbL_{LO}+HLbL_{NLO})}$$

			QED	+HVP	+EW	
$a_\mu^{\rm the}$	=	0.00	1165	91	810	(43)
$a_\mu^{\rm exp}$	=	0.00	1165	92	061	(41)

agree to 8 digits! Would need 9 digits SM precision?

$$\delta a_\mu = a_\mu^{\rm exp} - a_\mu^{\rm SM} = 25.1 \pm 5.9 \times 10^{-10} = \Delta a_\mu^{\rm BSM}???$$

Gap $\Delta a_\mu^{\rm BSM}$ is 1.6 times EW contribution! Big effect missing!

SM theory incl. τ-decay spectra:

[19] Davier et al., arXiv:0906.5443: $a_\mu^{\rm HVP-LO}[\tau[ee]] = 705.3 \pm 4.5\,[689.8 \pm 5.2] \times 10^{-10}$

[20] [17, 18] Masjuan,Miranda&Roig, arXiv:2310.14102: $a_\mu^{\rm HVP-LO}[\tau] = 704.1^{+4.1}_{-4.0} \times 10^{-10}$

$\Leftrightarrow$ $a_\mu^{\rm had}[{\rm BMW, Mainz}] = 707.5 \pm 5.5 \times 10^{-10}$

If “BSM = missing HVP” need $a_\mu^{\rm HVP-LO} = 718.2 \times 10^{-10}$

Also remember:

Belle 2008: τ data-driven result

$$a_\mu^{\pi\pi}[2m_\pi, 1.8\ \text{GeV}] = (523.5 \pm 3.9) \times 10^{-10}\ (\tau:\ \text{Belle})$$

compared to e^+e^- data-driven result

$$a_\mu^{\pi\pi}[2m_\pi, 1.8\ \text{GeV}] = (504.6 \pm 10.1.1) \times 10^{-10}\ (e^+e^-:\ \text{CMD2, SND})$$

difference 18.9×10^{-10}, if added to alphaQED result
$a_\mu^{\text{HVP-LO}}[e^+e^-] = (694.79 \pm 4.18)\ \times 10^{-10}$ we get 713.7×10^{-10}

CMD-3 2023: e^+e^- for the range 0.327 to 1.2 GeV, relative to White paper
$a_\mu^{\text{had;LO}}(2\pi, \text{CMD3}) = (526.0 \pm 4.2)\ \times 10^{-10}$, compared to $(506.0 \pm 3.4)\ \times 10^{-10}|_{\text{WP}}$
difference 20.0×10^{-10}, if added to
$a_\mu^{\text{HVP-LO}}[e^+e^-] = (693.1 \pm 4.0) \times 10^{-10}|_{\text{WP20}}$ we get $(713.1 \pm 4.7) \times 10^{-10}$

τ data driven results seem to support recent LQCD results!

New! The new puzzle:

$\delta a_\mu^{\mathrm{HVP-LO}}[\mathrm{LQCD-DR}] = 14.4(6.8)\times 10^{-10}$ vs $a_\mu^{\mathrm{EW}} = 15.36 \pm 0.11 \times 10^{-10}$

Commonly forgotten: mixing of ρ^0, ω, ϕ with the photon [$\rho^0 - \gamma$ mixing] i.e. effect concerning relation

$\langle A(x)\, A(0)\rangle$ $\leftrightarrow$ $\langle j(x)\, j(0)\rangle$

photon propagator, e^+e^- data — current correlator, LQCD data

e^+e^- measurement $\Leftrightarrow$ LQCD calculation

- how to disentangle QED from QCD in e^+e^--data ?
- $\rho^0 - \gamma$ absent in CC $\tau \to \nu_\tau \pi\pi$ data,
  but QED-QCD interference part incl. in $e^+e^- \to \pi^+\pi^-$ data,
- for getting had blob in e^+e^- the $\gamma - \rho^0$ mixing has to be removed!

- for the I=1 part of $a_\mu^{\rm had}[\pi\pi]$ results in

$$\delta a_\mu^{\rm had}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10} \,,$$

as a correction applied for the range [0.63,0.96] GeV. The correction is not too large, but at the level of $1\,\sigma$ and thus non-negligible. Davier et al. got $\delta a_\mu^{\rm had}[\tau - ee] \simeq 15.5 \pm 6.9 \times 10^{-10}$ later reduced to $9.2 \pm 6.3 \times 10^{-10}$ a factor 2 larger, what data tell us, moves in the right direction!

To be clarified by QED supplemented LQCD!

Need to compute photon-propagator to compare with current-correlator!

What is needed in hadronic blob expansion of a_μ is the current-correlator (what LQCD provides) not the partially undressed photon-propagator.

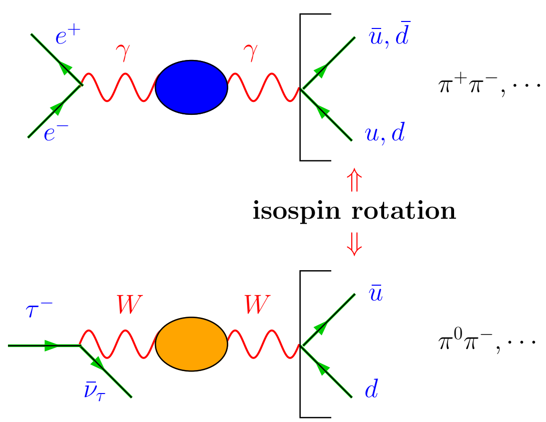
The role of τ decay spectra

What about τ-decay spectra ALEPH, CLEO, Belle: completely different set-up. Not mediated via photon propagator $\rho^\pm$ no mixing like ρ^0 mixing with γ, ω and ϕ. Corrected from QED corrections no VP subtraction! A pure I=1 Breit-Wigner shape. Strong IB, pion masses and meson mixing (in $\pi\pi$ channel the ω) to be added. On the e^+e^- side $\rho^0 - \gamma$ mixing (a QCD-QED interference) to be removed!

Additional data besides e^+e^- ones providing improvements:

❶ τ-decay spectra: good idea, use isospin symmetry to include existing high quality τ-data

(applying isospin breaking corrections) Alemany, Davier, Höcker 1996



bare $\pi^+\pi^-$ spectrum after photon VP, ISR and FSR subtraction

CVC $\leftrightarrow$ isospin rotation symmetry

bare $\pi^0\pi^-$ spectrum after EW radiative and phase space correction

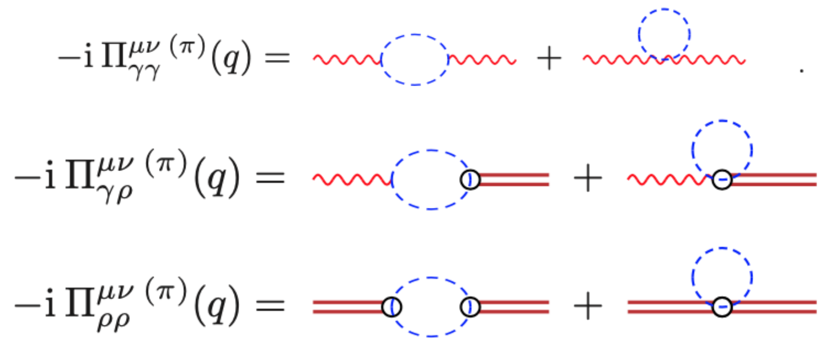
Standard IB corrected data: large discrepancy [$\sim$ 10%] persists! τ vs. e^+e^- puzzle! [manifest since 2002 Davier et al, resolved 2011 taking into account $\rho^0-\gamma$ interference]

Why is it still a good idea to include τ CC spectra? Can shed light on $e^+e^- \to \pi^+\pi^-$ data clashes (e.g. BaBar vs KLOE)!

Taking into account $\rho-\gamma$ interference resolves τ (charged channel) vs. e^+e^- (neutral channel) puzzle, [8] F.J.& R. Szafron [JS11], M. Benayoun et al.. However, not accepted by WP as a possible effect, which is analogous to $Z-\gamma$ interference established at LEP in the 90's.

$\rho - \gamma$ interference
(absent in charged channel)
often mimicked by large shifts
in M_ρ and Γ_ρ
ρ^0 is mixing with γ:
propagators are obtained by
inverting the symmetric 2×2
self-energy matrix

$$\hat{D}^{-1} = \begin{pmatrix} q^2 + \Pi_{\gamma\gamma}(q^2) & \Pi_{\gamma\rho}(q^2) \\ \Pi_{\gamma\rho}(q^2) & q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2) \end{pmatrix}.$$



Irreducible self-energy contribution at one-loop

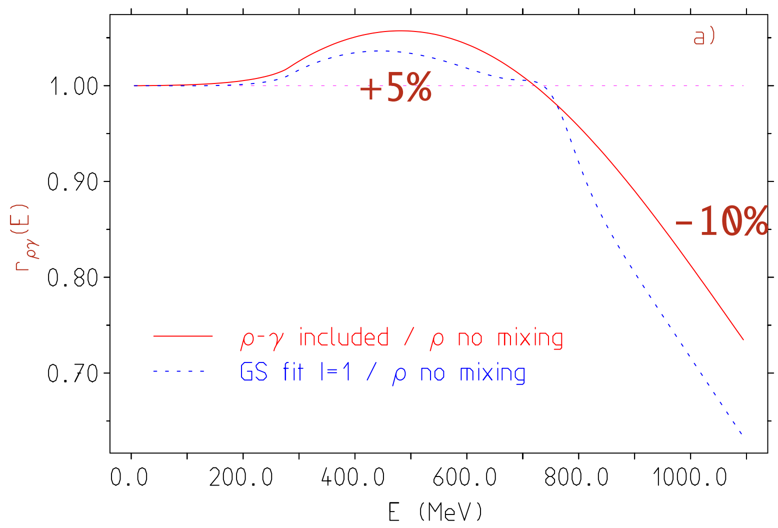
Taking $q^2 - M_\rho^2 + \Pi_{\rho\rho}(q^2)$ term only $\Rightarrow$Gounaris-Sakurai; $\Pi_{\gamma\rho}(q^2)$ $\gamma - \rho$ interference

Effect well known from LEP Z resonance physics: $Z-\gamma$ mixing affects Z lineshape.

Self-energies: pion loops to photon-ρ vacuum polarization (VMD II+sQED)

$$\Pi_{\gamma\gamma} \;=\; \frac{e^2}{48\pi^2}\,f(q^2)\;,\;\; \Pi_{\gamma\rho}=\frac{e g_{\rho\pi\pi}}{48\pi^2}\,f(q^2)\quad \text{and}\quad \Pi_{\rho\rho}=\frac{g^2_{\rho\pi\pi}}{48\pi^2}\,f(q^2)\;,$$

No unknown adjustable parameters: e, $g_{\rho\pi\pi}$ and M_ρ $\Rightarrow\Gamma_\rho$, mixing etc are predictions!



Usually IB correction applied to τ spectra:

- ❒ τ require to be corrected for missing $\rho-\gamma$ mixing!
- ❒ results obtained from e^+e^- data is what goes into a_μ

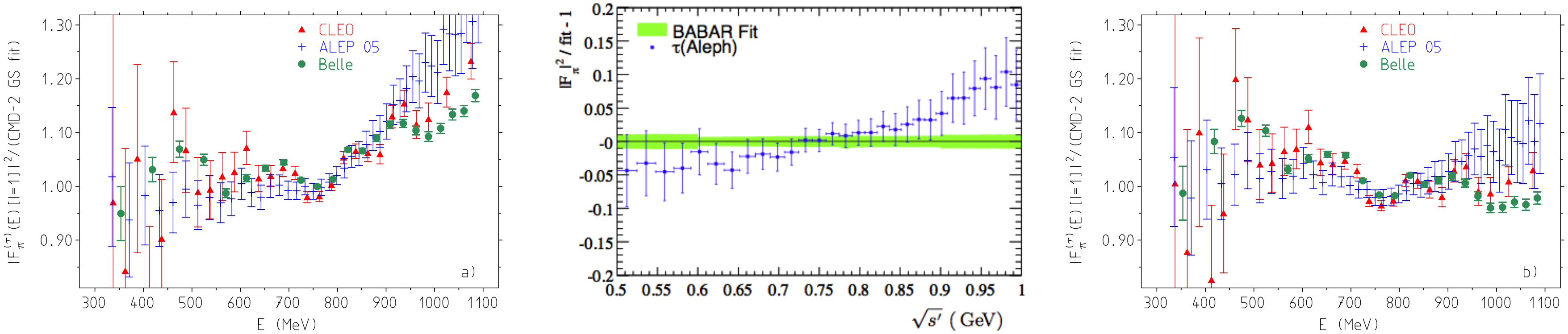
New! In fact, lattice QCD results reveal:

- ● to get required pure QCD 1PI "blob" $v_h(s)$ $\Rightarrow \rho-\gamma$ mixing should be removed from usual "bare" $R_\gamma(s)$ data

Correct implementation: $v_0(s) \Rightarrow v_h(s)$

$$v_h(s) = R_{\rm IB}(s)\, v_-(s)$$ from τ-spectra

$$v_h(s) = r^{-1}_{\rho\gamma}(s)\, v_0(s)$$ from e^+e^--data



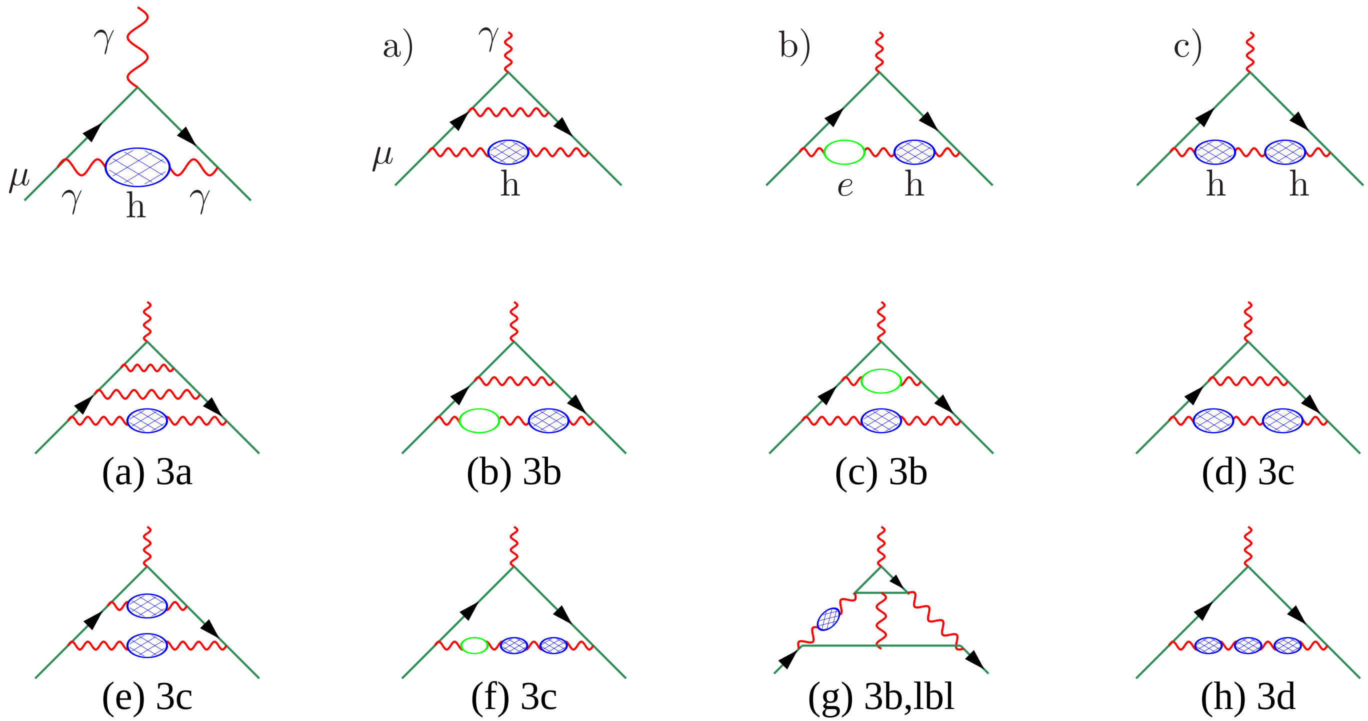
$|F_\pi(E)|^2$ in units of e^+e^- $I = 1$ (CMD-2 GS fit): Left: τ data uncorrected for $\rho - \gamma$ mixing [8]. Center: ALEPH τ vs BaBar e^+e^- [Davier et al. 2009]. Right: after correcting for mixing. $\pi\pi$ [+5% − 10%].

After LQCD results: apply correction in opposite direction

Shift: $\delta a_\mu^{\rm had}[\rho\gamma] \simeq (5.1 \pm 0.5) \times 10^{-10}$,
thus $693.46(3.94) \times 10^{-10} \Longrightarrow 698.56(3.97) \times 10^{-10}$! closer to LQCD (BMW&Mainz)

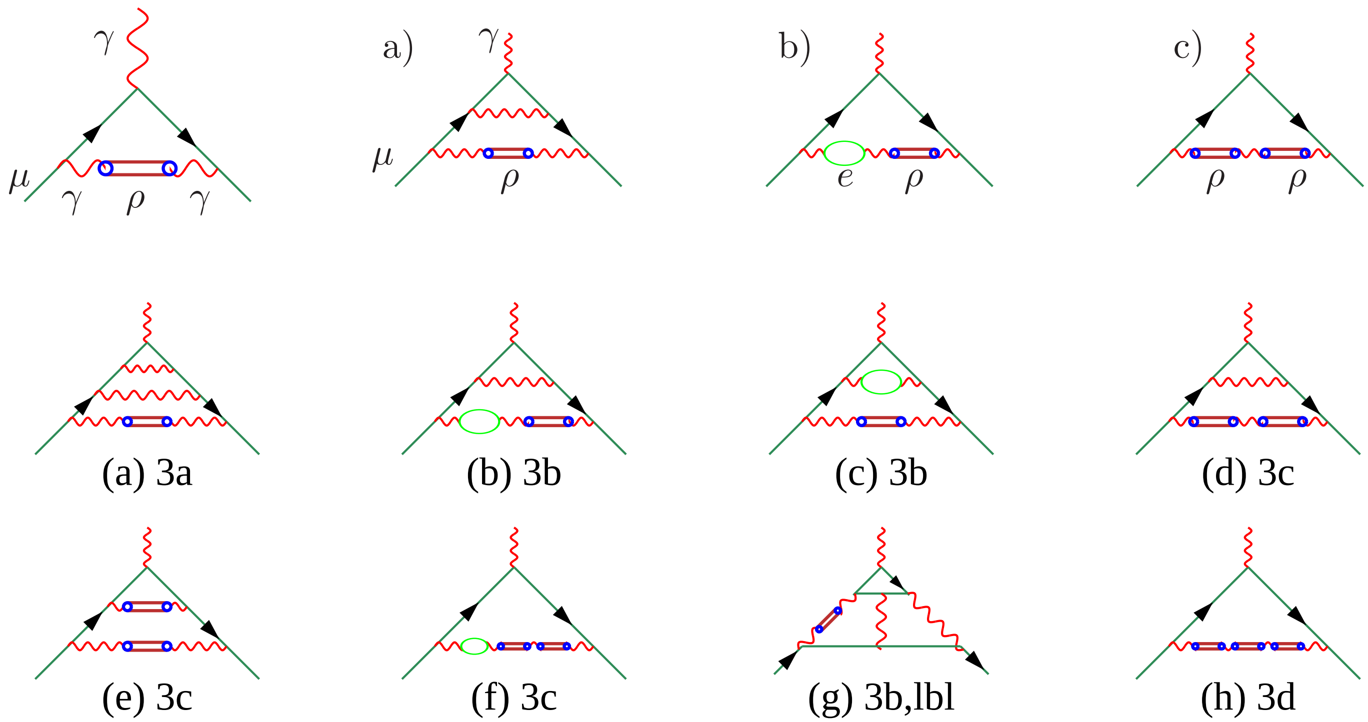
❒ **Is there a specific ρ contribution in the HVP blob expansion?**

❒ a_μ HVP bubble expansion in powers of α



Hadronic VP contributions: involving LO, NLO, and NNLO vacuum polarization. Hadronic VP "blob" must be free of external photon radiation effects! Hard to achieve via data that is "all-inclusive", i.e., need to disentangle QCD from QED.

❒ Note HVP is 75% ρ-resonance, i.e., bubble expansion is dominated by ρ-exchange insertions.

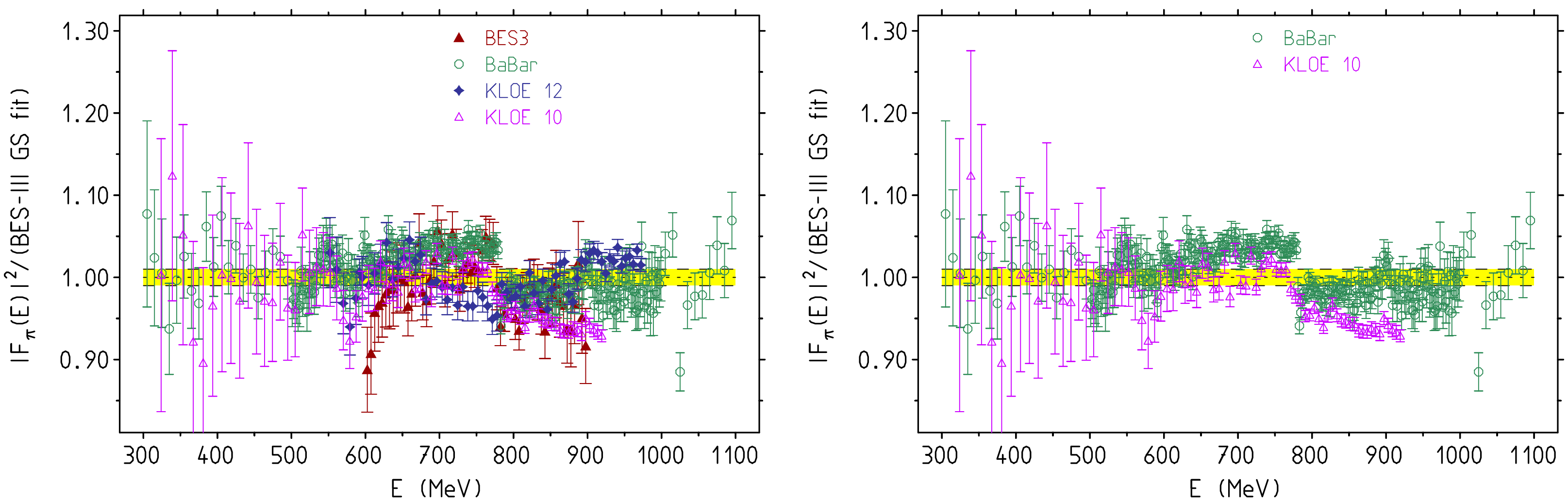


The "vector-meson-dominance" phenomenon (large N_c QCD feature) implies that the HVP in the Muon $g-2$ is dominated by ρ-exchange (h$\approx\rho$).

In the dispersive approach based on time-like $e^+e^- \to \text{hadrons}$ data the ρ-resonance yields the bulk of $a_\mu^{\rm had}$ (see e.g. E. de Rafael, PLB 322, 239 (1994)). Integrals may be performed in the Minkowski space by integrating $R_\gamma(s)$ data or in the Euclidean set up by integrating the VP function $\Pi'_\gamma(t=-s)$. The Wick rotation involved does not change the values of the integrals. This also applies to individual contributions like the contribution from the ρ. In the time-like region, the ρ contribution is given by a prominent peak in the cross-section, but the same result we obtain in the space-like approach where the time-like resonance appears completely smoothed out as if there were no ρ. If we undress the time-like $\pi\pi$-data from $\rho^0-\gamma$ mixing the ρ-resonance gets slightly distorted, by up to -5% below the M_ρ and up to +10% above M_ρ but still below 1GeV above which the VMD=II+sQED modeling breaks down, but fortunately there also the $\pi\pi$ cross-section gets subdominant. The shift looks to represent the observed discrepancy between the e^+e^--data and the τ-decay spectra after commonly accepted IB corrections have been applied. Overall, we obtain a non-negligible extra contribution to the $a_\mu^{\rm had}$ integral. As said before, the effect remains the same when working in the Euclidean with $\Pi'_\gamma(t=-s)$.

❐ Still unclear $\pi\pi$ below 1 GeV

Experimental input for HVP: NSK, KLOE, BaBar, BESIII, CLEO-c, VEPP-2000



Recent BES-III vs BaBar and KLOE

KLOE vs. BaBar in conflict: e.g. Resonance Lagrangian Approach global fit:

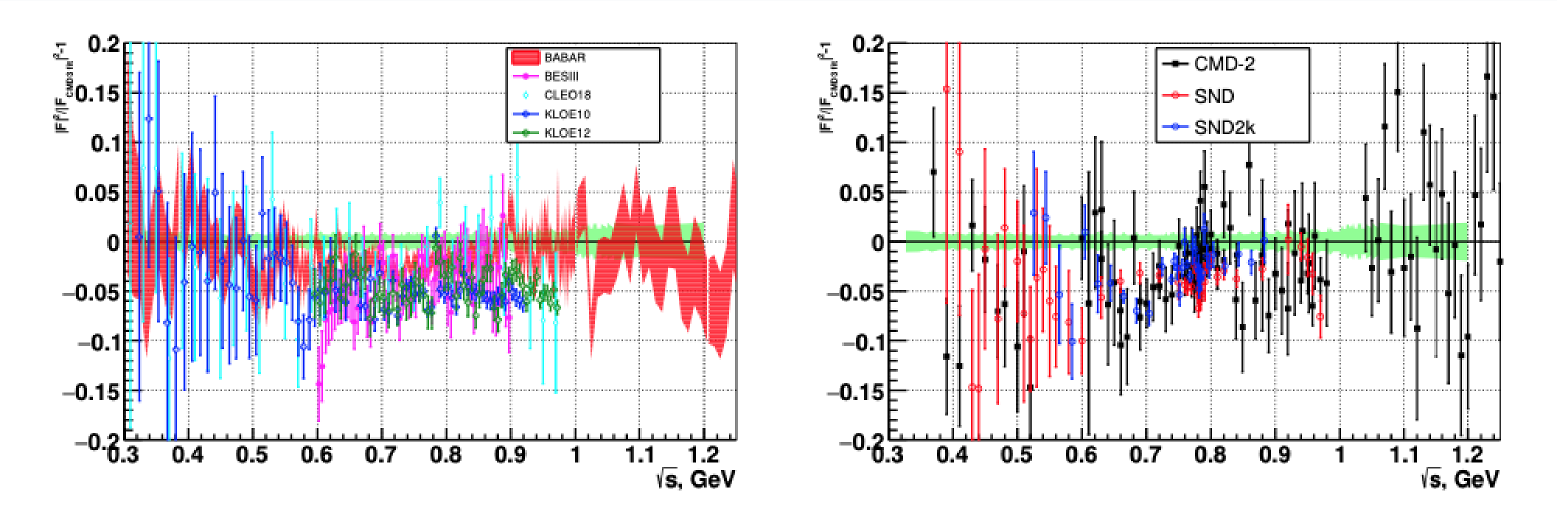
$a_\mu^{\mathrm{HVP-LO}}(KLOE) = (687.31 \pm 2.93) \times 10^{-10}$ 90% fit NSK, BESSIII,CLEO-c +KLOE

$a_\mu^{\mathrm{HVP-LO}}(BaBar) = (692.36 \pm 2.95) \times 10^{-10}$ 40% fit NSK, BESSIII,CLEO-c +BaBar

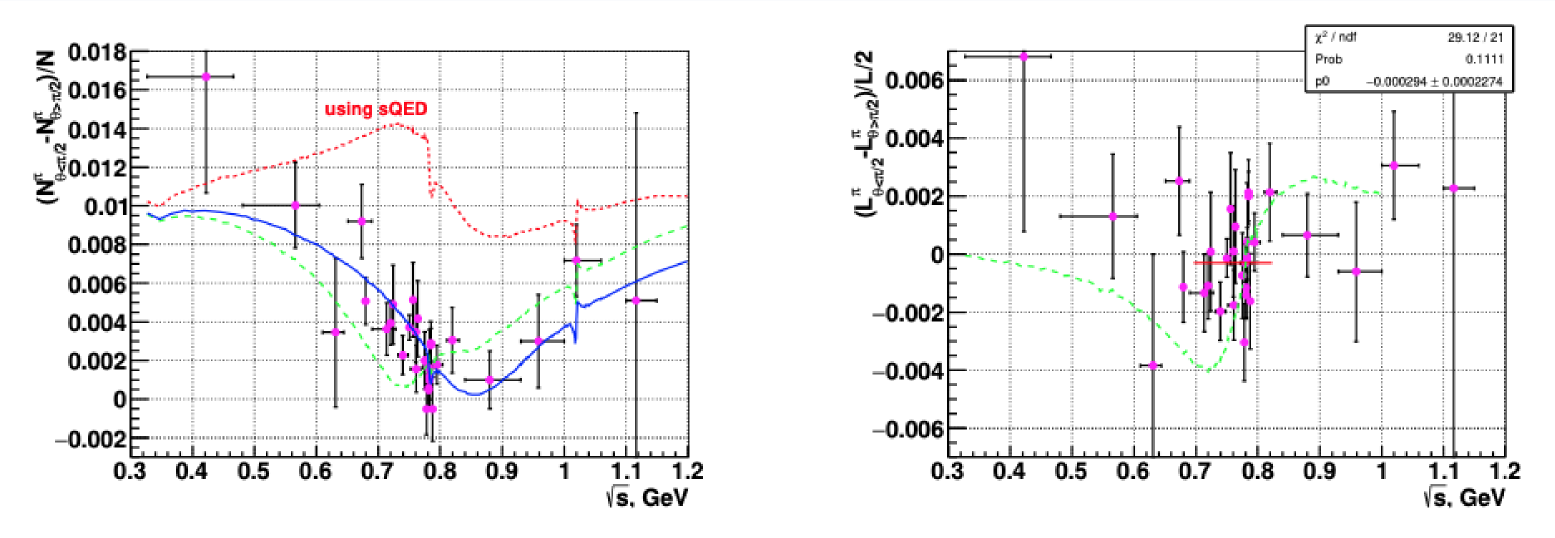
with BaBar close to WP $693.1(4.0) \times 10^{-10}$

M. Benayoun, L. Del Buono, F. J. 2021 arXiv:2105.13018 differ by 1.7 σ.
Should be understood!

New CMD-3 measurement arXiv:2302.08834 [hep-ex] report large deviation from all previous $\pi\pi$ channel data



Left: The relative difference of the ISR measurements to the CMD-3 fit. Right: The relative difference of the previous energy scan measurements to the CMD-3 fit.



$a_\mu^{\rm had}[\pi\pi, 0.6-0.88] = 367.42(0.28)(1.56)[1.58]$ my average

$a_\mu^{\rm had}[\pi\pi, 0.6-0.88] = 379.35(0.30)(2.95)[2.97]$ CMD-3 2023

$\delta a_\mu^{\rm had}[\pi\pi, 0.6-0.88] = 11.93(0.28)(1.56)[1.58]$ my average

$a_\mu^{\rm had} = 706.72(0.30)(2.95)[2.97]$ CMD-3 2023 which agrees with recent lattice QCD results!

Compares to LQCD results:

White paper HVP gives 4.2 σ; $a_\mu^{\mathrm{HVP-LO}}[\mathrm{THE}] = 693.1(4.0) \times 10^{-10}$

BMW lattice HVP gives 1.6 σ; $a_\mu^{\mathrm{HVP-LO}}[\mathrm{BMW}] = 707.5(5.5) \times 10^{-10}$

Mainz lattice HVP 1.6 σ; $a_\mu^{\mathrm{HVP-LO}}[\mathrm{Mainz}] = 707.8(5.5) \times 10^{-10}$

RBC/UKQCD 2.1 σ; $a_\mu^{\mathrm{HVP-LO}}[\mathrm{RBC}] = 703.2(5.5) \times 10^{-10}$

10. A last update and summary on lepton moments

A key quantity for the high-precision test of the SM is the anomalous magnetic moment of the muon a_μ, one of the most precisely measured quantities in particle physics, which also can be predicted with comparably high accuracy and at the same time represents a susceptible window to physics beyond the SM. After the improved measurement $a_\mu^{\rm exp} \equiv (g_\mu - 2)/2 = 116\,592\,091(54) \times 10^{-11}(0.54\ \ {\rm ppm})$ [21] (CERN 1977 had achieved a 7 ppm accuracy) at Brookhaven National Laboratory (BNL) in the years 1999 to 2004 a $4.3\ \sigma$ deviation between experiment and theory [24] fueled the hope that the SM was in trouble, and the deviation pointed to possible new physics. At that time, the minimal supersymmetric expansion of the SM (MSSM) was the most promising way to close the gap between the experiment and the SM prediction. However, the search for new particles at the LHC in 2012 already showed that no convincing new physics scenario could produce such an effect. Recently, the muon $g-2$ experiment at the Fermi National Accelerator Laboratory (FNAL) was able to further improve the accuracy with the result $a_\mu^{\rm exp} = 116\,592\,055(24) \times 10^{-11}$ in full agreement with the BNL result. The world average $a_\mu^{\rm exp} = 116\,592\,059(22) \times 10^{-11}\ \ (0.19\ \ {\rm ppm})$ [22, 23] confronted with the White Paper result [11], appeared to increase the discrepancy to about 5.2 σ. These results relied on the data-driven dispersive calculations of non-perturbative hadronic vacuum polarization (HVP) given by $a_\mu^{\rm LO-HVP} = 697.93(0.81)(3.93)[4.01] \times 10^{-10}$ with alphaQEDc26 package. However, progress in the ab initio lattice QCD calculations [25] has yielded a much larger HVP $a_\mu^{\rm LO-HVP} = 714.1(2.2)(2.5)[3.3] \times 10^{-10}$, which, together with other SM contributions [11] gives the result $a_\mu^{\rm the} = 116\,592\,019(38) \times 10^{-11}$, which differs with $a_\mu^{\rm exp} - a_\mu^{\rm the} = 40(44) \times 10^{-11}$ from the experiment, i.e., only 0.9 standard deviations. This result validates the standard model up to the 0.37 ppm level, as highlighted in [25]. The various contributions and their uncertainties of the SM predictions are shown in Fig. 1. See also my most recent update [2].

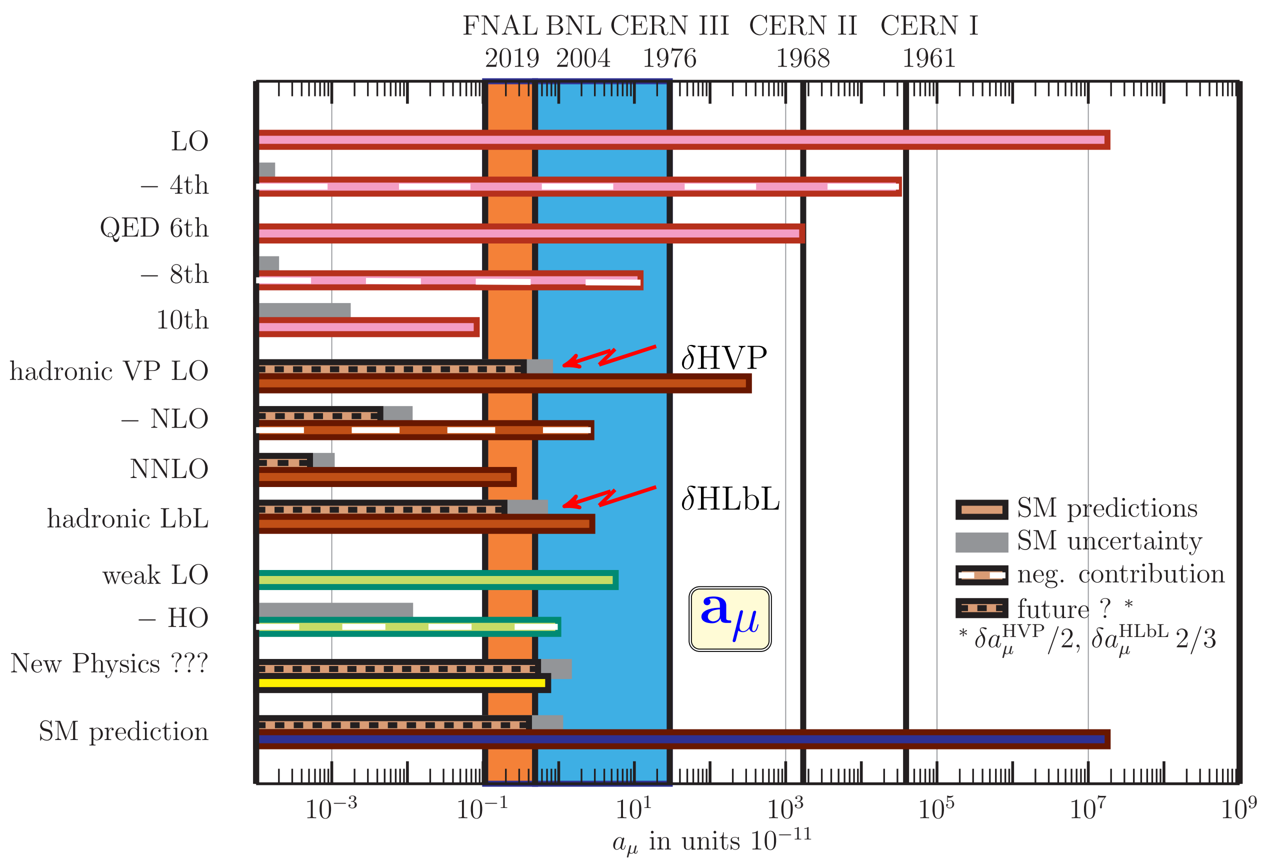


Figure 1: Past and future muon $g-2$ experiments testing various contributions. New Physics ? = deviation $(a_\mu^{\rm exp}-a_\mu^{\rm the})/a_\mu^{\rm exp}$. Limiting theory precision: hadronic vacuum polarization (HVP) and hadronic light-by-light (HLbL)

For the anomalous magnetic moment of the electron $a_e = (g_e - 2)/2$, the agreement between theory and experiment provides a similarly impressive test of the SM prediction. On the experimental side, we have the remarkably accurate recent measurement by the Gabrielse team $g_e/2 = 1.001\,159\,652\,180\,59\,(13)(0.13\ \text{ppt})$ [27]. The accurate prediction of a_e heavily depends on an accurate electromagnetic fine structure constant. Here the substantial progress in atomic interferometry with ^{87}Rubidium atoms with the result $\alpha^{-1}(\text{Rb20}) = 137.035999206(11)(81\ \text{ppt})$ [26] has provided a significant step towards the improved prediction $a_e^{\text{the}} = 1.15965218059(9)$[4]. Confronted with the experimental value, we obtain $a_e^{\text{exp}} - a_e^{\text{the}}\ 0.18(16) \times 10^{-11}$, 1.1 standard deviations and an accuracy of 8.1×10^{-11}. Update Dec 2024: a correction of the tenth-order QED contribution to a_e [28, 29] lowers the value of a_e by 5.4×10^{-14} such that $a_e^{\text{the}} = 1.15965218024(9)$ and $a_e^{\text{exp}} - a_e^{\text{the}}\ 0.35(16) \times 10^{-11}$ a 2.2 standard deviations. Accidentally, the lattice QCD shift of $+5.2 \times 10^{-14}$ of the hadronic contribution and the correction of the tenth-order QED contribution cancel each other. In future atomic interferometry experiments (the AION project based on Strontium and Yttrium) are expected to reduce the uncertainty of the electromagnetic fine structure

[4] a_e^{LO-HVP} rescaled by a_μ^{LO-HVP} lattice vs. dispersive result, i.e., $a_e^{LO-HVP} = 1.871(11) \times 10^{-12}$ to $a_e^{LO-HVP} = 1.923(09) \times 10^{-12}$.

constant such that an a_e prediction with an accuracy of 2×10^{-14} would be possible. The various contributions and their uncertainties of the SM predictions are shown in Fig. 2. See also my most recent update [2].

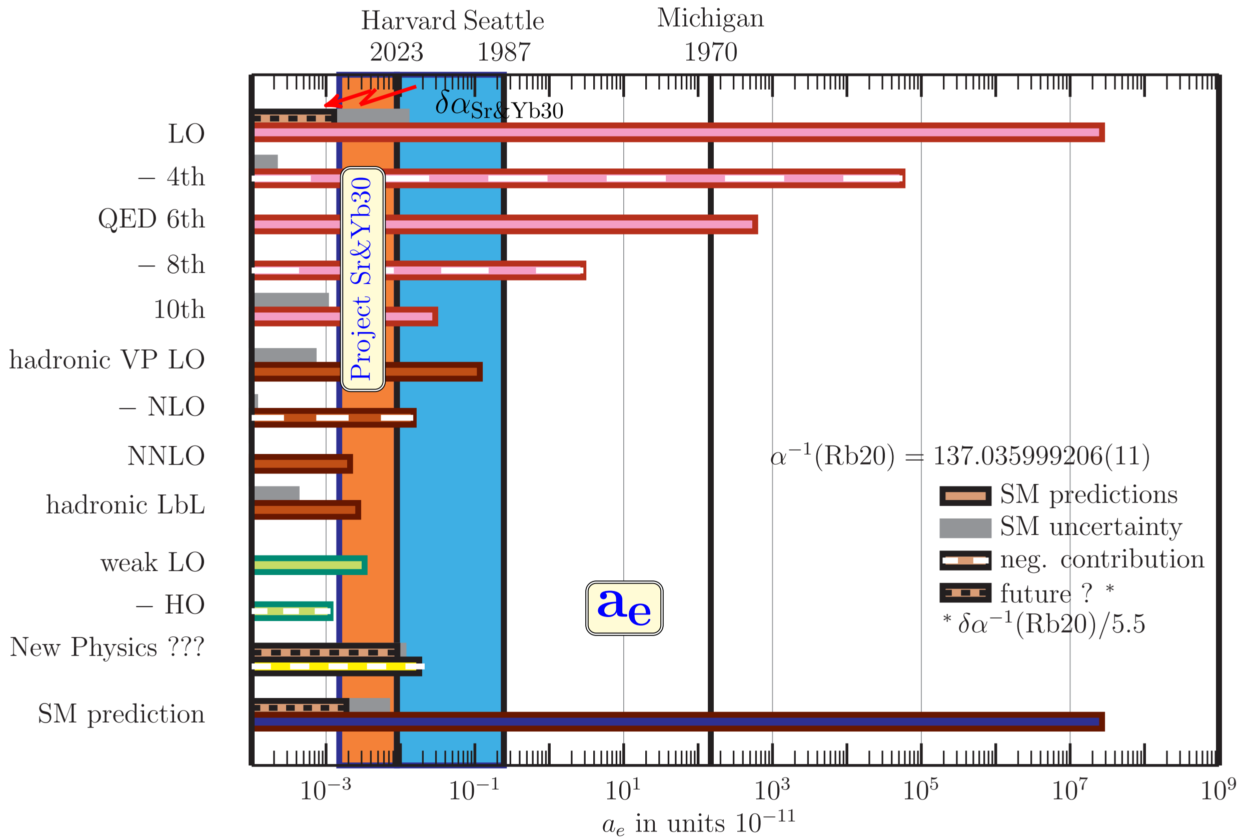
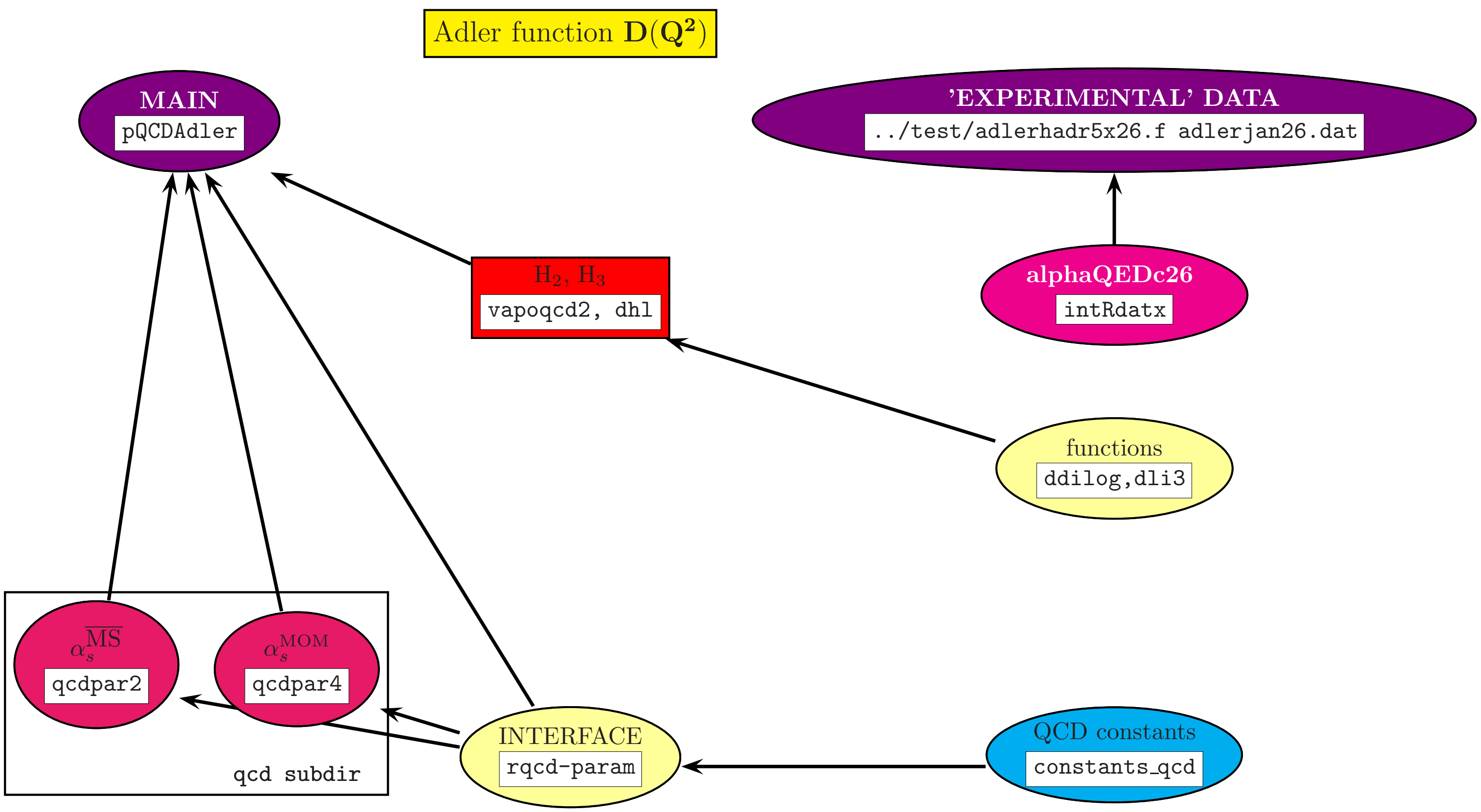


Figure 2: Status and sensitivity of the a_e experiments testing various contributions. The error is dominated by the uncertainty of α(Rb20) from atomic interferometry. "New Physics" ? = deviation $(a_e^{\rm exp} - a_e^{\rm the})/a_e^{\rm exp}$, i.e., essentially absent presently. The blue band illustrates the improvement by the Harvard experiment. The orange band shows the possible progress by the Sr&Yb30 atomic interferometry project.

11. The Adler Function

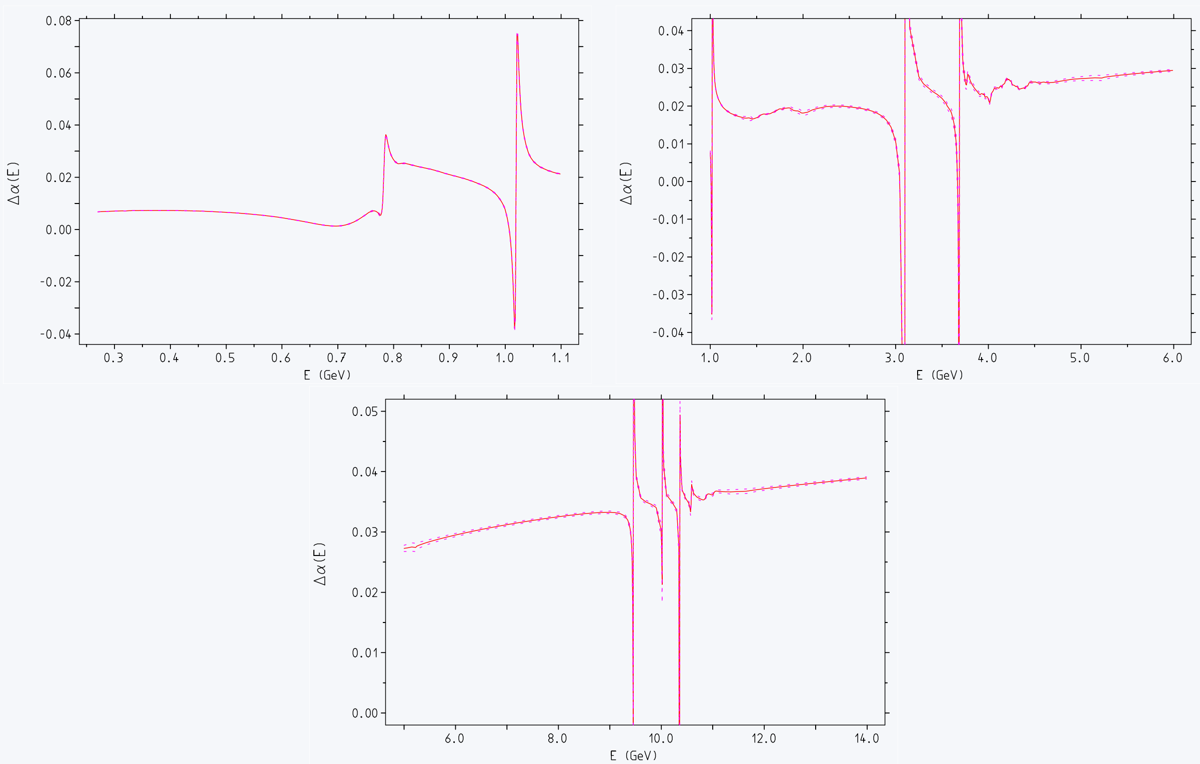


The FORTRAN Adler function [30, 31] package pQCDAdler ⇒ pQCDAdler.tar is now included as part of the package alphaQEDc26 in the subdirectory ./adler/, with the QCD coupling constants in MSbar and BF-MOM schemes (see [32]) generated and stored in the directory ./adler/qcd/. The Adler function data obtained with the alphaQEDc26 package (./adler/adlerxxx26.dat) can be generated in the directory ./test using testadlerhadr5x_gnu.sh which is compiling and executing testadlerhadr5x.f. The corresponding scripts are ./adler/pQCDAdler.sh and ./adler/qcd/qcdparx.sh. Further information can be found at ⇒ pQCDAdler.pdf. The experimental Adler function data set appended to adlerhadr5x26.f was generated in the main directory by ./intRdatx_gnu.sh 0: where intRdatx.inp as

```
0.318d0,5.2d0,11.5d0,1.D3
0,0,0,'WAVE',0,0,-1,5,1,0
2,-100.D0,-1.D-6,610,20,1d-2,'LIN'
0.1184,0.0007,0.000,91.1880,172.4,5,4,2
0, 0
# Adler function; ./intRdatx_gnu.sh 0 ==> dalhads.dat, adlerintrdatx26.dat
cp dalhads.dat ./test/adlerhadr5x26.inc (data table)
cp adlerintrdatx26.dat ./adler/adlerfeb26.dap (plot file)
```

Appendix

Sample Plots:



Hadron Data Tables Summary

Tabulated data for Re $\Delta\alpha(s)$ and Im $\Delta\alpha(s)$:

Script intRdatx_upd.sh yields integrated R-data:

dalhadt26_5.f deghadt26_5.f dalhadthigh26_5.f deghadthigh26_5.f
dalhadslow26_5.f deghadslow26_5.f dalhadshigh26_5.f deghadshigh26_5.f,

which are include-files for hadr5n26.f, while hadr5x26.f contain these tables in a single file.
All results incorporate the correction for eliminating th bulk of the $\gamma - \rho$ mixing effect.
For comparison Tables (standard corrections only) wihout this correction are included as "5x26a" versions.
For visualization use ./test/testhadr5x_gnu.sh running testhadr5x.f and testchadr5x.f, which also provide examples of how to use the various versions of data tables.

Overview

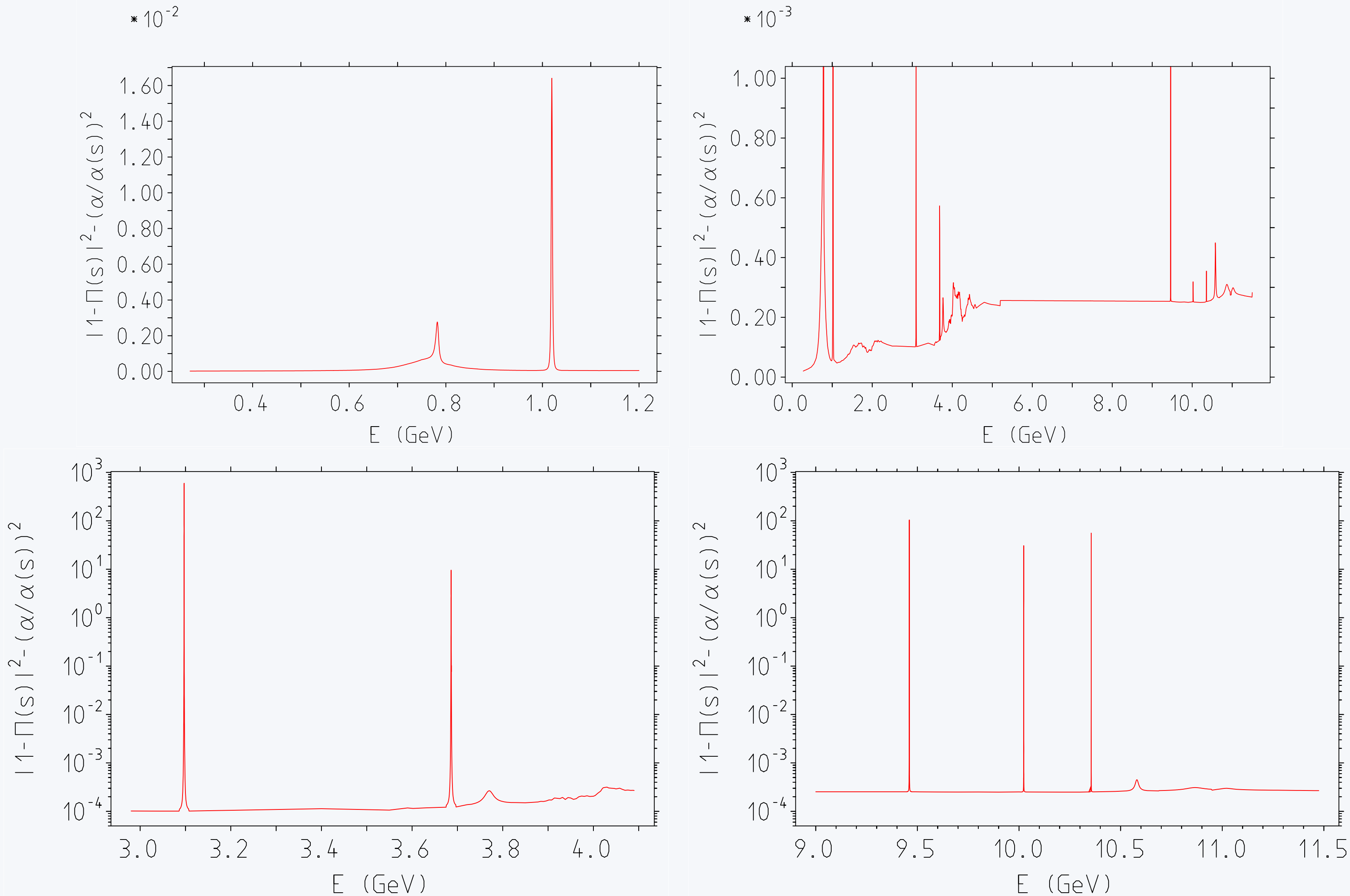
file	subroutines (single, double precision)
hadr5n26.f	hadr5x26, dhadr5x26
hadr5x26.f	hadr5x,dhadr5x
chadr5x26.f calls hadr5x26	chadr5x
complexifytables.f calls chadr5x	makes hadr5x26_complex.f
hadr5x26_complex.f	chadr5x26,dchadr5x26

Complex vs. real α VP correction

In our analysis we have subtracted vacuum polarization (VP) effects by replacing the real running $\alpha(s)$ by α, i.e. $R(s)$ is corrected by $(\alpha/\alpha(s))^2 = |1 + \mathrm{Re}\,\Pi'(s)|^2$ ($\Pi'(0)$ subtracted). More precisely, one actually has to subtract $|1 + \Pi'(s)|^2 = (\alpha/|\alpha_c(s)|)^2$ where $\alpha_c(s)$ is the complex generalization of its real counterpart. This is what the Novosibirsk CMD-2 Collaboration has been using in recent analyzes. The corresponding code has been made public recently and is available from Fedor Ignatov's Web page ✼⋙. In the following figure we plot the correction

$$\boxed{1 - |1 + \Pi'(s)|^2/(\alpha/\alpha(s))^2}$$

as a function of energy. Typically, corrections are below the one per mill level, except at resonances where corrections are the larger the smaller the widths:



* 10^-2
|1-Π(s)|^2-(α/α(s))^2
1.60
1.40
1.20
1.00
0.80
0.60
0.40
0.20
0.00
0.4
0.6
0.8
1.0
1.2
E (GeV)
* 10^-3
|1-Π(s)|^2-(α/α(s))^2
1.00
0.80
0.60
0.40
0.20
0.00
0.0
2.0
4.0
6.0
8.0
10.0
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
3.0
3.2
3.4
3.6
3.8
4.0
E (GeV)
|1-Π(s)|^2-(α/α(s))^2
10^3
10^2
10^1
10^0
10^-1
10^-2
10^-3
10^-4
9.0
9.5
10.0
10.5
11.0
11.5
E (GeV)

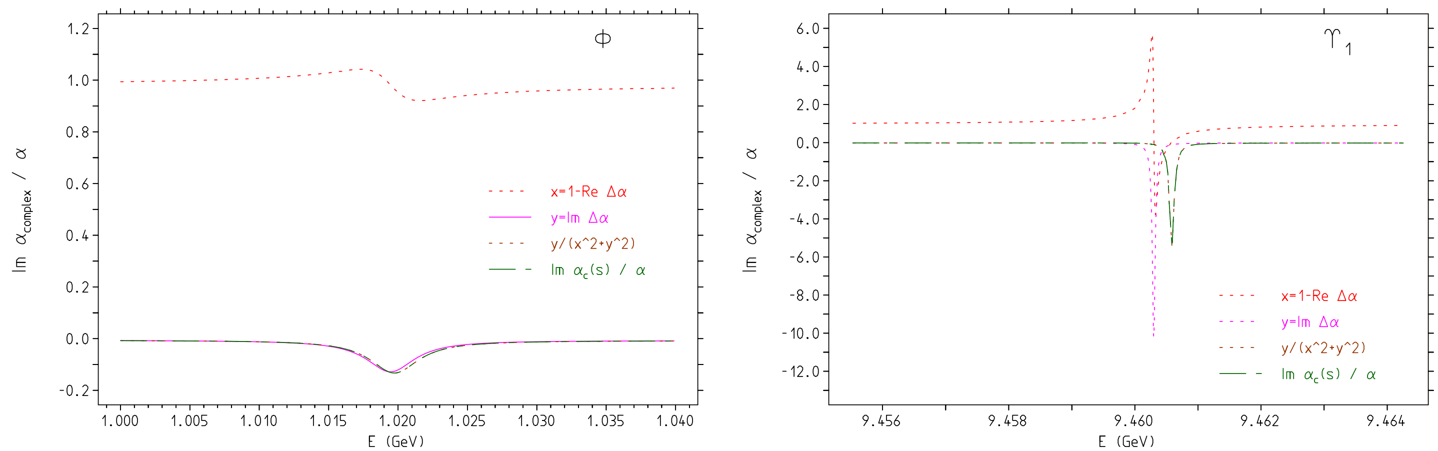
Note: imaginary parts from narrow resonances, $\mathrm{Im}\,\Pi'(s)) = \frac{\alpha}{3}\,R(s) = \frac{3}{\alpha}\,\frac{\Gamma_{ee}}{\Gamma}$ at peak, are sharp spikes and are obtained correctly only by appropriately high resolution scans. For example,

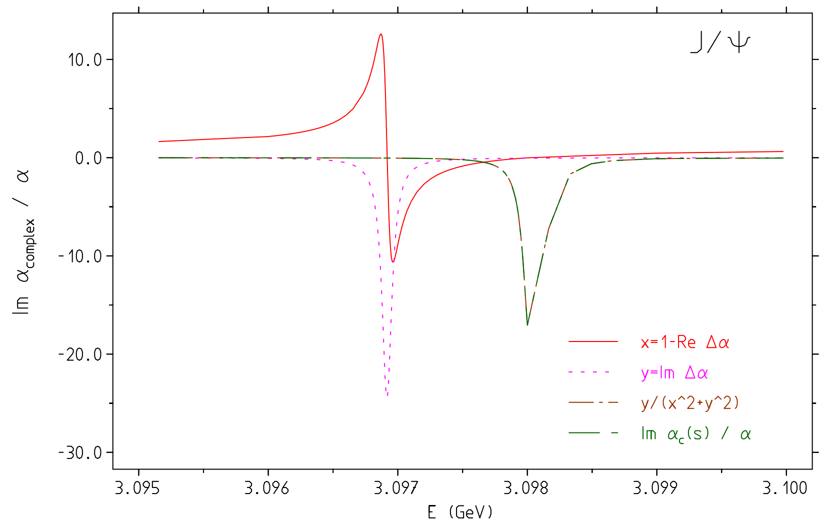
$$\boxed{|1+\Pi'(s)|^2 - (\alpha/\alpha(s))^2 = (\mathrm{Im}\,\Pi'(s))^2}$$

at $\sqrt{s} = M_R$ is given by

1.23×10^{-3} [ρ], 2.76×10^{-3} [ω], 1.56×10^{-2} [ϕ], 594.81 [J/ψ], 9.58 [ψ_2], 2.66×10^{-4} [ψ_3], 104.26 [Υ_1], 30.51[Υ_2], 55.58 [Υ_3]

Except for the ρ and ψ_{4-6} narrow resonances are taken into account as Breit-Wigner resonances, starting with physical parameters as listed by the PDG. They thus have to be undressed (VP subtraction) by renormalizing it with $(\alpha/\alpha(s))^2$. We actually apply the complex running coupling, because the real version has Landau poles at the resonances $J/\psi, \psi_2$ and $\Upsilon_1, \Upsilon_2, \Upsilon_2$. The tables in hadr5n26.f have been obtained by integrating the VP subtracted data collected in Rdat_all.f and processed via Rdat_fun.f without the narrow resonances (options iresonances=0 and IRESON=0). The resonance contributions are integrated separately with appropriate renormalization (undressing = VP subtraction). If resonances are switched on in Rdat_fun.f and Rdat_fit.f renormalization has to be applied in the Breit-Wigner function FUNCTION BW(S,M,G,P) attached in Rdat_fun.f. There at present we apply real coupling subtraction only, which does not include the imaginary part which we attempt to calculate at the same time. Renormalization effects are dramatic for the very narrow resonances in particular for J/ψ and Υ_1.





For very narrow resonances the Breit-Wigner "bump" appears shifted dramatically (about 6 $\times\Gamma_{J/\psi}$ for the J/ψ) in the imaginary part of $\alpha_c(s)$. The real version of $\alpha(s)$ would have a Landau pole. For the ϕ corrections are in the "perturbative regime", i.e. $|\mathrm{Re}\,\Delta\alpha|, |\mathrm{Im}\,\Delta\alpha| \ll 1$.

How to extract $\mathrm{Im}\,\Pi'_\gamma$ when the Dyson series does not converge?

One is measuring $\mathrm{Im}\,\Pi'_\gamma(s)|\alpha(s)/\alpha(0)|^2$ if one would measure $|\alpha(s)/\alpha(0)|^2$ from $e^+e^- \to \mu^+\mu^-$ for example. Thus one could get $\mathrm{Im}\,\Pi'_\gamma(s)$ by normalizing it to $e^+e^- \to \mu^+\mu^-$, or better to $\sigma(e^+e^- \to \mu^+\mu^-)/\sigma(e^+e^- \to \mu^+\mu^-)_0$ where $\sigma(e^+e^- \to \mu^+\mu^-)_0$ is the Monte Carlo cross section with VP switched off. Thus,

$$R_{\text{had}}(s) = \sigma(e^+e^- \to \text{hadrons})/\sigma(e^+e^- \to \mu^+\mu^-) \cdot \sigma(e^+e^- \to \mu^+\mu^-)_0$$

If $\sigma(e^+e^- \to \mu^+\mu^-)$ is not known precise enough one could use $\mathrm{Re}\,\Pi'_\gamma(s) \approx Re\Pi'_\gamma(s)$ as an additional input:

Let $y = \mathrm{Im}\,\Pi'_\gamma$; $x = \mathrm{Re}\,\Pi'_{\gamma,\text{ren}}$. Measured is $Z = y/|(1-\Delta\alpha)|^2 = y/((1+x)^2+y^2)$, which yields $y = \frac{1}{2Z}\left(1 \pm \sqrt{1-4Z^2\,(1+x)^2}\right)$. Note that x space-like is always small (perturbative) while x and y time-like are non-perturbative and in some cases huge, such that the Dyson series diverges.

New ω and ϕ handling:

ω region: 0.742 - 0.813 GeV set to [0.757 - 0.81]
ϕ region: 1.000 - 1.040 GeV

options background only omitting 3π, K^+K^- and $K^0\overline{K^0}$ data in this ranges

iomegaphidat = -1 evaluated background 0 background + BW+PDG 1 all data 2 background + omega and phi data (3pi,KKc,KKn) energy ranges switches for fits

MOM=0.777035d0 MFI=1.019461d3 CHPTCUTLOC=0.320d0 EFITOMBG=0.78710d0 EFITMIN=0.81d0 EFITMAX=1.4349d0 ERXYDAT=2.125d0 ERXYDAT=2.150d0 ECHARMTH=th(4) ! 4.2 GeV

omega resonance contribution in rfitrholow, rfitrhohig and epemfit07081

auxiliary functions: frome omega data, frphi phi data

deg [3g] stuff to intRdatx.tab requires to set lprintdeg-.true. [8] List of Makefiles:

Makefile, Makefile.real, Makefile.complex, Makefile_iRdx, Makefile_amuviaEC, Makefile_Rdat, make_testhadr5x,

./rhad/

Makefile, Makefile_rqcdHS

./test/

For some reason make -f Makefile_testRdat requires adlerhadr5x26.o to exist so first run 'gfortran -c adlerhadr5x26.f'

Makefile_testRdat

Main fortran programs:

alphaQEDc26.f, alphaQEDr26.f, alpha2SMc26.f, ACWMsin2theta.f, ACWMsin2theta_lin.f, hadr5x26.f, intRdatx.f, test_Rdat.f, testhadr5x.f, amuviaEC.f

Main shell scripts:

alphaQEDc26_gnu.sh alpha2SMc26_gnu.sh testhadr5x_gnu.sh ECintR_gnu.sh intRdatx_clean.sh

./rhad/

test_rqcdHS.sh

./test/

test_Rdat_gnu.sh

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