PSI-PR-94-21
June, 1994

Search for Anomalous Gauge Boson Couplings

F. Jegerlehner [1]
Paul Scherrer Institute,
CH-5232 Villigen PSI, Switzerland

Abstract

We review some aspects of the physics of gauge boson interactions. Perspectives for determining anomalous triple gauge boson couplings in the LEP II process $e^+e^- \to W^+W^-$ are discussed. We point out that recent data from LEP and SLC seem to severely constrain contributions from gauge-boson loops. This provides indirect support for the Standard Model gauge boson structure. Prospects for disentangling anomalous gauge interactions at future colliders are also considered.

[1] Talk given at the 1994 Zeuthen Workshop on Elementary Particle Theory "Physics at LEP 200 and Beyond", April 10 - 15, 1994, Teupitz/Brandenburg, Germany

Search for Anomalous Gauge Boson Couplings

F. Jegerlehner
Paul Scherrer Institute,
CH-5232 Villigen PSI, Switzerland

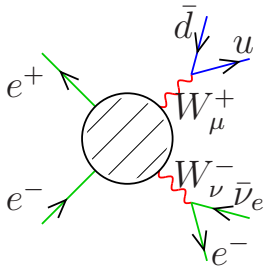
March 24, 2024

1 MOTIVATION

The best place to search for anomalous gauge boson couplings at the tree level is W–pair production in e^+e^-–annihilation [1, 2]. The process actually observed is

$$e^+e^- \to W^+W^- \to 4 \text{ fermions}$$

which, a few W-widths above the W–pair threshold, is dominated by diagrams of the form



with both W's near resonance. Note that all possible new physics is in the subprocess $e^+e^- \to W^+W^-$. The W–decays proceed via the well-established pure V-A charged current interaction, and the W–decay distributions just serve as polarization analysers for the W's. The shape of the total cross section in the narrow width approximation is shown in Fig. 1.

The threshold region is important for a precise determination of the W–mass, which is relevant for precision tests of the electroweak Standard Model (SM). The gauge field dynamics comes into play at higher energies. In the SM the gauge cancellations, typical for Yang–Mills interactions, led to a peak of the cross section at about 200 GeV. The region around the peak is the best place to test the gauge field structure of the SM and to discover deviations, which are expected to be small, in the gauge boson self–interactions.

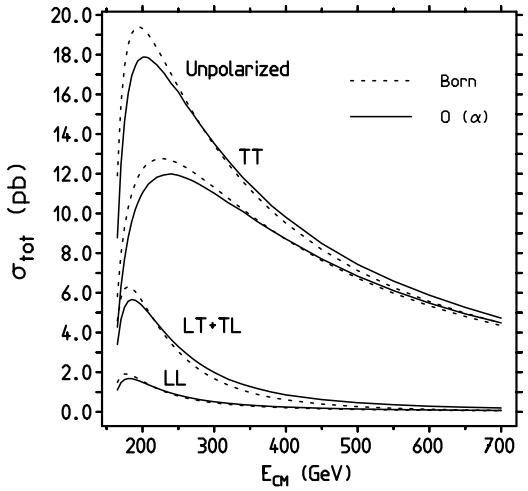


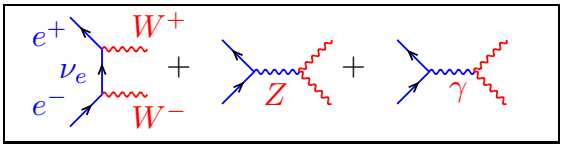
Figure 1: Total cross sections for $e^+e^- \to W^+W^-$.

2 W-PAIR PRODUCTION AND GAUGE FIELD DYNAMICS

We consider in more detail the idealized process

$$e^+(p_+, \bar{\sigma}) + e^-(p_-, \sigma) \to W^+(q_+, \bar{\lambda}) + W^-(q_-, \lambda)$$

in the approximation of stable W's. There are three diagrams



contributing to the process at the tree level. The first diagram only exhibits known interaction vertices, while the last two are new to the extent that no *direct* experimental test has been possible so far.
The differential cross section for definite helicities is given by

$$\frac{d\sigma(\pm;\lambda,\bar{\lambda})}{d(\cos\theta)} = \frac{\beta_W}{32\pi s} \mid M_0(\pm;\lambda,\bar{\lambda}) \mid^2 \qquad (1)$$

where $s = (2E_b)^2$, E_b is the beam energy, and $\cos\theta$ is the angle between the three-momentum vectors of the e^- and the W^- in the c.m. system. $\beta_W = \sqrt{1-4M_W^2/s}$ is the W velocity and $\gamma^2 = s/(4M_W^2)$ the Lorentz factor. The helicity matrix elements have the form

$$\begin{aligned} M_0(-;\lambda,\bar{\lambda}) &= S^{(-)}_{1(0)}\mathbf{M}_{S_1}(-;\lambda,\bar{\lambda}) + T^{(-)}_{1(0)}\mathbf{M}_{T_1}(-;\lambda,\bar{\lambda}) \\ M_0(+;\lambda,\bar{\lambda}) &= S^{(+)}_{1(0)}\mathbf{M}_{S_1}(+;\lambda,\bar{\lambda}) \ . \end{aligned} \qquad (2)$$

The gauge cancellation may be observed directly by writing the invariant amplitudes in the form:

$$\begin{aligned} S^{(-)}_{1(0)} &= -\frac{e^2}{s}(1^\gamma_V - \frac{1^Z_V}{1-M_Z^2/s}) - \frac{g^2}{2s}\frac{1^Z_A}{1-M_Z^2/s} \\ S^{(+)}_{1(0)} &= -\frac{e^2}{s}(1^\gamma_V - \frac{1^Z_V}{1-M_Z^2/s}) \qquad (3) \\ T^{(-)}_{1(0)} &= \frac{g^2}{s}\frac{1}{\frac{1+\beta_W^2}{2} - \beta_W\cos\theta} \end{aligned}$$

where the labels of $1^\gamma_V \equiv 1$ are just indicating the origin of the term, V and A denoting, respectively, vector and axial–vector current, and γ and Z denoting the s-channel exchange particle. While the gauge cancellation for the vector happens between the γ and Z directly in the invariant amplitudes the axial part of the Z exchange contribution only cancels in the sum (2) against the $T^{(-)}_{1(0)}$ amplitude. Here it is crucial that the ratio

$$\frac{\mathbf{M}_{T_1}(h_e;\lambda,\bar{\lambda})}{\mathbf{M}_{S_1}(h_e;\lambda,\bar{\lambda})} = \frac{1-\cos\theta}{2} \quad \text{for} \ \ \gamma\to\infty \ (\beta_W \to 1)$$

is equal for all final states with $|\Delta\lambda| < 2$. This allows to fix the axial Z coupling relative to the established t-channel ν exchange contribution. For a more detailed discussion see Ref. [3] and references therein. The gauge cancellation at work are illustrated in Fig. 2.

The gauge cancellation dictated by the gauge invariance of the theory implies that

- individual Feynman diagrams have no physical meaning
- the observable total is the result of a strongly destructive interference.

This is a new situation.

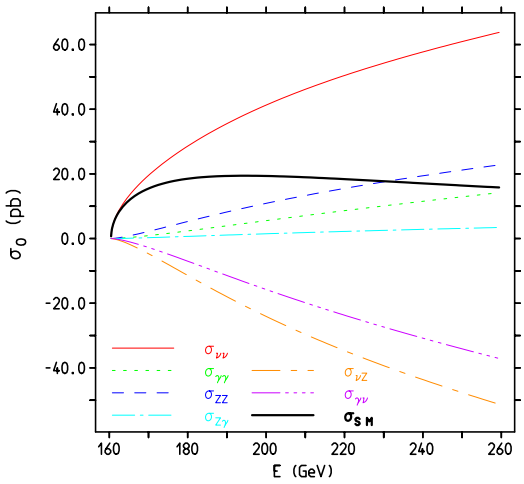


Figure 2: Gauge cancellation in $e^+e^- \to W^+W^-$.

Up to the present, the phenomenology of electroweak processes (from Fermi (1934) until LEP II will be running) was the history of fixing the structure of the $SU(2)_L \otimes U(1)_Y$ fermion current couplings, determined in "low energy" four fermion processes like μ–decay, neutrino scattering and e^+e^-–annihilation, either at low momentum transfer or near some resonance (e.g., the Z at LEP I). Couplings (form–factors) measured in such circumstances are dominated by single one–particle exchange processes and consequently provide simple interpretations for

the measurements. In W–pair production in e^+e^-–annihilation the situation is much more complex. It takes place in the continuum of states and one only observes interferences, so that form–factors have *no direct observable meaning*. This not only makes the interpretation of the results unclear, it makes precise predictions more difficult. Consequently, if deviations from the SM would be found the physics origin could remain obscure for quite some time.

2.1 Origin of non-abelian gauge symmetry (perturbative unitarity bounds)

A cross section is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\beta}{32\pi s}|M|^2$$

where $|M|^2$ is proportional to a transition probability. It thus must be a bounded function of the energy for the sake of unitarity.

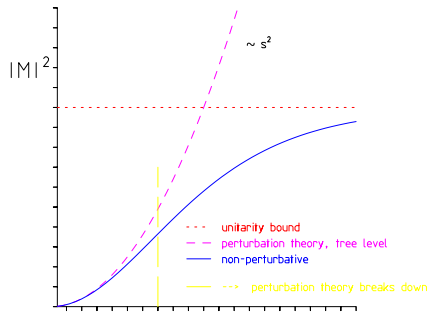
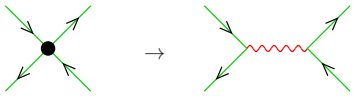


Figure 3: Unitarity and perturbative predictions of non-renormalizable theories.

It is well known that in a non–renormalizable theory one gets into trouble with perturbative unitarity. For example, in the Fermi theory, where $\simeq G_\mu s$ implies $|M|^2 \propto s^2$, one is led to a breakdown of perturbation theory at the Fermi scale $v = (\sqrt{2}G_\mu)^{-1/2} \simeq 250$ GeV. But, surprisingly, this did not mean that we had to go to non-perturbative methods. It meant that we had to change the theory. Since this problem does not show up in a renormalizable theory it was natural to try to extend the low-energy theory to render it renormalizable. This path led to the construction of the "minimal renormalizable extension" of the phenomenological four fermion interactions and it directly led to the electroweak SM. The first step was introducing the massive intermediate weak gauge bosons



which renders the four fermion sector "renormalizable" at tree and up to the one–loop level [4]. More generally [5], one requires $|M|^2$ to grow with energy at most as E^{4-N} for $E \to \infty$, where N is the number of external particles, so that perturbative unitarity is retained at tree level (necessary condition for renormalizability). The starting point is the general form of the fermion current. Let the spinor $\Psi_\alpha = (e, \nu_e, \mu, \nu_\mu, \ldots)$ include the known fermion fields. The current has the form

$$J^{\mu a} = \bar{\Psi}_\alpha \gamma^\mu \{L^a_{\alpha\beta} P_- + R^a_{\alpha\beta} P_+\} \Psi_\beta$$

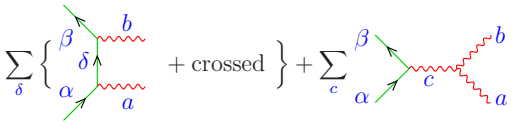
where $P_\pm = \frac{1}{2}(1 \pm \gamma_5)$ are the right and left handed projections, respectively. The intermediate vector boson Lagrangian then reads

$$\mathcal{L}_1 = J^{\mu a} W_{\mu a}$$

which leads to s^2–terms in the pair production of intermediate vector bosons in fermion–antifermion annihilation. This can be cured by adding new interactions, the triple gauge vertices (TGV's):

$$\mathcal{L}_2 = D_{abc} W^c_\mu W^b_\nu \partial^\mu W^{\nu a} \quad .$$

The condition of compensation of the s^2-terms in



leads to the algebraic relationships

$$\begin{aligned} \left[L^a, L^b\right] &= i D_{abc} L^c \\ \left[R^a, R^b\right] &= i D_{abc} R^c \end{aligned}$$

telling us that there is symmetry needed, the coupling structure of a Lie–group $G_L \otimes G_R$. Next, inspection of $WW \to WW$ scattering again exhibits s^2-terms. They can be cancelled by new quartic interactions

$$\mathcal{L}_3 = \frac{1}{4} E_{abcd} W^a_\mu W^{\mu b} W^c_\nu W^{\nu d} \quad .$$

The absence of s^2-terms implies that D_{abc} is antisymmetric and satisfies the Jacobi identity and

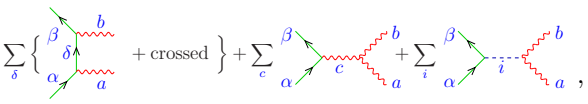
$$2E_{abcd} = D_{ade} D_{cbe} + D_{ace} D_{dbe} \quad .$$

Thus one finds that, provided

- all couplings have dimension ≤ 0 (otherwise they are not renormalizable anyway), and
- s^2–terms are absent,

we obtain a theory with *Yang–Mills couplings* in the fermion and gauge boson sector. This works for all channels at the tree level.

There are still unacceptable terms growing linearly with s. Since spin 1/2 and spin 1 fields have been taken into consideration in a general way there are no further compensations possible without adding new types of fields. Adding particles of spin bigger than 1 in any case would led to a non-renormalizable theory. Hence we are left with only one possibility: to introduce spin 0 particles, *the Higgs boson*. After including a scalar exchange as



the linear s-terms can also be eliminated. In fact the condition of absence of linear s-terms requires the "Goldstone solution with locally gauged fields," which means the Higgs mechanism.

This way to look at "spontaneously broken gauge theories" naturally leads us to the effective field theory scenario. It is reasonable to assume that the SM is a low-energy asymptotic (effective) theory of some unknown underlying theory which could show up at some larger scale Λ. Because the SM works so well this new scale is expected to be substantially higher than the present "high energy" scale, which is set by the Higgs vacuum expectation value $v \simeq 246$ GeV. The particle content of the SM represents the light particles of the underlying theory at the scale Λ. The Higgs could be an exception because of the "naturalness" problem [6]. The Higgs mass is not protected (required to vanish) by the "nearby" unbroken $SU(2)_L \otimes U(1)_Y$ symmetry (i. e. the limit $v \to 0$), which requires all other fields to be massless. The low-energy effective theory obtained by an asymptotic expansion in $\sqrt{s}/\Lambda \leq v/\Lambda$ exhibits corrections to the SM described by operators of increasing dimensions which are suppressed by increasing powers of Λ. Anomalous couplings naturally emerge here as next-to-leading order perturbations, which scale away at low energies but become more and more important as we go to higher energies.

2.2 General form of amplitudes and higher order corrections

The expected rates at LEP II should be sufficient to determine a number of quantities with about 1% accuracy, and therefore at least full one-loop corrections must be taken into account [7]. At this level the SM is CP invariant which implies:

$$M(\sigma, \bar{\sigma}; \lambda, \bar{\lambda}) = M(-\bar{\sigma}, -\sigma; -\bar{\lambda}, -\lambda) \quad ,$$

so that there remain 2×6 physical amplitudes, which agrees with the number of independent cross sections: two initial states $e^-_L e^+_R$ and $e^-_R e^+_L$ (assuming $m_e \simeq 0$) and for each, six final states consisting of two massive spin 1 bosons.

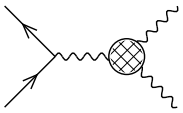
If CP is violated by some anomalous couplings (in the SM this would be the case at $O(\alpha^2)$ only, far beyond the sensitivity of LEP II) this could be clearly tested by observing deviations from unity ($\neq 1$) of the ratios $\sigma(h_e; --)/\sigma(h_e; ++)$, $\sigma(h_e; -0)/\sigma(h_e; 0+)$ and $\sigma(h_e; 0-)/\sigma(h_e; +0)$. Once this CP test is passed one can reduce the number of free anomalous couplings by imposing CP invariance. In the following we will label states by L (longitudinal) for $J_z = 0$ and by T (transverse) for $J_z \neq 0$:

$(\lambda,\bar\lambda)$		$\Delta\lambda$
(00)	LL	0
$(0-),(+0)$	LT+TL	+1
$(-0),(0+)$	LT+TL	−1
$(--),(++)$	TT	0
$(+-)$	TT	+2
$(-+)$	TT	−2

The CP invariant generalization of (2) may be written in terms of four s–channel amplitudes $S_i^{(\pm)}$ (i=1,2,3,4), a t–channel amplitude $T_1^{(\pm)}$, and a new amplitude $X_6^{(\pm)}$ which is induced by box diagram contributions. Thus (2) changes to

$$M(\pm;\lambda,\bar\lambda) = \sum_{i=1}^{4} S_i^{(\pm)} \mathbf{M}_{S_i}(\pm;\lambda,\bar\lambda) + T_1^{(\pm)} \mathbf{M}_{T_1}(\pm;\lambda,\bar\lambda) + X_6^{(\pm)} \mathbf{M}_{X_6}(\pm;\lambda,\bar\lambda) \ . \qquad (4)$$

The contributions from *triple gauge couplings* to the $e^+e^- \to W^+W^-$ amplitudes, described by diagrams of the type



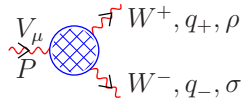
have the form

$$\begin{aligned}\delta S_i^{(\mp)} \;=\; & \sqrt{2} G_\mu \{(a \mp b)\, A_{i(r)}^{ZWW}(s)\, \chi_Z(s) \\ & - A_{i(r)}^{\gamma WW}(s)\, \chi_\gamma(s)\} \\ & + \text{ self energies} \\ & + \text{ other vertex corrections} \\ & + \text{ box diagrams.}\end{aligned}$$

a and b are the coefficients of the Zee vertex $\gamma^\mu\,(a - b\gamma_5)$, $\chi_Z(s) = 4M_W^2/(s - M_Z^2 + iM_Z\Gamma_Z)$ is the Z–exchange pole factor and $\chi_\gamma(s) = 4M_W^2 \sin^2\Theta_W/s$ the γ–exchange pole factor. For a detailed discussion of electroweak radiative corrections we refer to Beenakker's contribution [8].

3 ANOMALOUS TRIPLE GAUGE VERTICES

The triple gauge amplitudes



may be written in the form:

$$\begin{aligned}&\text{“}VWW\text{”} = \\ &\quad i\, C_V \,\{(r^\mu g^{\rho\sigma} + 2(g^{\mu\rho}P^\sigma - g^{\mu\sigma}P^\rho))\, A_1^{VWW}(s) \\ &\qquad + (g^{\mu\rho}P^\sigma - g^{\mu\sigma}P^\rho)\, A_2^{VWW}(s) \\ &\qquad + r^\mu \frac{P^\rho P^\sigma}{P^2}\, A_3^{VWW}(s) \\ &\qquad + i\,\varepsilon^{\rho\sigma\mu\alpha} r_\alpha\, A_4^{VWW}(s)\} \qquad (5)\end{aligned}$$

with $r = q_+ - q_-$, $C_\gamma = e$ and $C_Z = g\cos\Theta_W$. Eq. (5) represents the most general CP conserving form of the VWW-vertex function for on-shell W's [9]. At tree level we have $A_1^V = 1$ and $A_i^V = 0$ for $i = 2,3,4$ in the SM.

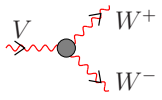
At present all data are well described by the SM, which suggests that possible deviations may be considered as perturbations of the SM:

$$\mathcal{L}_{\text{eff}} = \mathcal{L}_{SM} + \delta\mathcal{L}_{\text{eff}} \ . \qquad (6)$$

Here one assumes that new physics is the consequence of properties of nature at a much smaller length scale Λ^{-1}. We parametrise these by the CP-invariant effective interaction Lagrangian for the VWW interaction ($V = \gamma$ or Z)

$$\begin{aligned}\mathcal{L}_{\text{SM}}^V \;=\; & iC_V\{V^\nu(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu}) \\ & + V^{\mu\nu}W^+_\mu W^-_\nu\} \qquad (7) \\ \delta\mathcal{L}_{\text{eff}}^V \;=\; & iC_V\delta_V\{V^\nu(W^+_{\mu\nu}W^{-\mu} - W^-_{\mu\nu}W^{+\mu}) \\ & + V^{\mu\nu}W^+_\mu W^-_\nu\} \\ & + iC_V\delta\kappa_V V^{\mu\nu}W^+_\mu W^-_\nu \\ & + iC_V\frac{\lambda_V}{\Lambda^2}V^\mu{}_\nu W^{+\nu\rho}W^-_{\rho\mu} \\ & + C_V\frac{\xi_V}{\Lambda^2}(\partial^\mu\tilde V^{\nu\rho})\{W^+_\mu \overleftrightarrow{\partial}_\nu W^-_\rho \\ & + W^-_\mu \overleftrightarrow{\partial}_\nu W^+_\rho\}\ , \qquad (8)\end{aligned}$$

where $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ ($V = \gamma, Z$ or $W^\pm$) and $\tilde{V}_{\mu\nu} = \frac{1}{2}\varepsilon_{\mu\nu\rho\sigma}V^{\rho\sigma}$. Electromagnetic gauge invariance forces $\delta_\gamma = 0$. In standard terminology these parameters describe the static properties of the W:



$$\begin{array}{cc} \text{“electric” charge} & 1+\delta_V \\ \text{“magnetic” dipole} & \kappa_V = 1+\delta\kappa_V \\ \text{“electric” quadrupole} & \lambda_V \\ \text{anapole } (\not{P}) & \xi_V \end{array} \qquad (9)$$

for $V = \gamma$ or Z. We have normalized to unity the overall factors e and $e\cot\Theta_W$ for γ and Z, respectively. The anomalous couplings contribute to the SM amplitudes as follows:

$$\begin{array}{l} A_1^V \to A_1^V + \delta_V + \lambda_V s/(2\Lambda^2) \\ A_2^V \to A_2^V + \delta\kappa_V - \lambda_V(s/\Lambda^2 - 1) \\ A_3^V \to A_3^V - \lambda_V s/\Lambda^2 \\ A_4^V \to A_4^V - \xi_V s/\Lambda^2 \quad . \end{array} \qquad (10)$$

A selection of papers considering the effects of anomalous TGV's are Refs. [10, 11]. For an illustration of a compositeness scenario [12] see Fig. 4. For composite W's, like for the ρ meson in hadron physics, there is no reason why they should have $SU(2)_L$ gauge couplings. Obviously, they would have standard electric coupling and an anomalous magnetic moment $\mu = e/(2M_W)(1+\kappa)$. In the Standard Model we have $\kappa_{SM} = 1$. The absence of a $SU(2)_L$ coupling amounts to a change of the ZWW coupling to $-\tan^2\Theta_W$ times the Standard Model value, and an anomalous magnetic moment implies a non-vanishing S_2 amplitude in Eq. (4) at the tree level

$$S_2^{(\pm)} = (\kappa - 1)\, S_1^{(\pm)}.$$

Typically we observe that deviations from the Standard Model gauge couplings in general lead to substantially larger cross-sections, due to lack of gauge cancellations.

A comparison of SM radiative corrections with some anomalous effects was presented in van Oldenborgh's contribution [13]. For a much more profound discussion of anomalous effects we refer the reader to Bilenky's contribution [14].

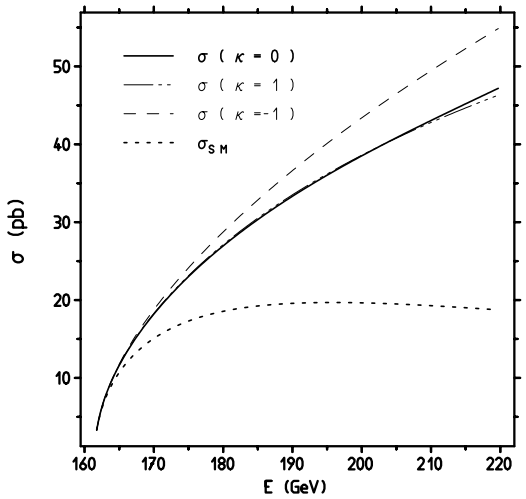


Figure 4: The effect of anomalous TGV's on the total cross section.

4 CONSTRAINTS FROM LEP I

Anomalous couplings, if they exist, spoil renormalizability and the calculation of higher order effects is not *a priori* well defined. Since SM higher order corrections are very well tested experimentally at lower energies, and in particular at LEP I and SLC, there is not too much room for surprises. There is a substantial improvement in the precision of electroweak measurements [15], showing for the first time the signal of the SM gauge-boson loops as illustrated in Fig. 5. The error bands reflect the theoretical uncertainties and include parameter errors $M_Z = 91.1886 \pm 0.0044$ GeV and $\alpha_s = 0.12 \pm 0.006$, and the variation of the Higgs mass between 60 GeV and 1 TeV. The central value $m_H = 300$ GeV has been used. Switching off the boson contributions leads to a clear discrepancy with the data as the direct lower bound on the top mass is now $m_t > 131$ GeV [16]. For a discussion of similar observations see Ref. [17].

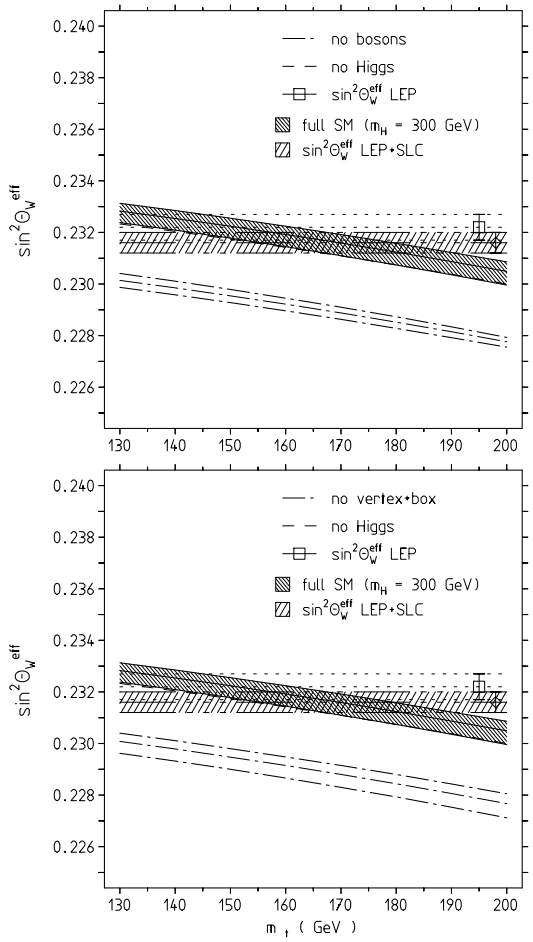
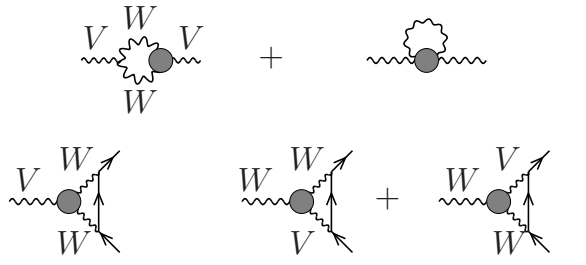


Figure 5: Testing the SM gauge-boson loops (a) and vertex+box loops (b)

Fig. 5 also illustrates that the effect is mainly due to the vertex+box corrections and not to the gauge-boson self-energies. While switching off the bosonic contributions is a finite and gauge invariant procedure, this is not the case for switching off the Higgs or the vertex+box corrections. If we switch off the Higgs we obtain, in the unitary gauge and the $\overline{MS}$ scheme at scale $\mu = M_W$, a curve which almost coincides with the lower limit of the SM prediction error band, i.e., no discrepancy can be produced. Switching off the vertex+box contributions, in the Feynman gauge and the $\overline{MS}$ scheme at scale $\mu = M_W$, one obtains essentially the same result as when the bosonic contributions are eliminated altogether. If the experimental and the theoretical errors are estimated correctly (remember the current discrepancy of the $\sin^2\Theta_W^{eff}$ values from the LEP I and SLC) this is a quite unexpected result. Of course, in view of this situation, it starts to get hard to imagine that LEP II will provide better constraints on anomalous couplings. However, as we shall argue, contributions from non–renormalizable interactions are hard to quantify and, as usual, a direct investigation is always preferable.

In order to set up a perturbation expansion for the low energy effective theory one has to proceed essentially as in chiral perturbation theory in QCD. As a first order perturbation the anomalous couplings contribute in gauge-boson self-energies and in gauge-boson fermion vertices:



.

Note that it makes no sense to consider anomalous quadratic contributions like before the linear corrections have been established. Indirect contributions may be obscured by other unknown physics contributions but large contributions from bosonic

loops are unlikely. Also, since the perturbed SM is non-renormalizable, predictions are not unique. They depend on the cut-off and the introduction of single additional couplings does not make much sense. What one has to expect in general is obvious from the general tensor structure (5), which translates into (8). Early work where loop–effects of anomalous couplings were investigated were based on this effective Lagrangian [18, 19, 20].

In a low energy expansion one would expect all kind of new interactions, not just anomalous gauge boson interactions. This was systematically investigated some time ago in Ref. [21]. Operators which contribute to measured observables at the tree level are severely constrained. Here we assume that these are absent. Operators which are not constraint at tree level by present phenomenology are called "blind" operators in Ref. [22]; they are the ones described by our perturbation Lagrangian (8).

It is important here to point out that $SU(2)_L \otimes U(1)_Y$ invariance does not restrict the allowed triple gauge couplings, contrary to statements which have appeared in the literature [22]. It is well-known [6, 23] that in the Higgs phase one may write all fields in a gauge invariant way up to the unbroken $U(1)_{em}$. In terms of the Higgs doublet field Φ_b, its Y-charge conjugate $\Phi_t = i\tau_2\Phi_b^*$, and the $SU(2)_L \otimes U(1)_Y$ covariant derivative D_μ we may write

$$Z_\mu^u = -\frac{i}{\sqrt{g^2+g'^2}}\,\frac{(\Phi_b^+ D_\mu \Phi_b - \text{h.c.})}{(\Phi_b^+\Phi_b)} \tag{11}$$

as a *neutral singlet field* and

$$W_\mu^{u+} = \frac{i}{\sqrt{2}g}\,\frac{(\Phi_t^+ D_\mu \Phi_b - (D_\mu\Phi_t)^+\Phi_b)}{(\Phi_b^+\Phi_b)} \tag{12}$$

as a *charged singlet field* modulo electromagnetic gauge transformations as required by the charge of the field. The photon is a $U(1)_{em}$ abelian field

$$A_\mu^u = \frac{gB_\mu + g'W_3^u}{\sqrt{g^2+g'^2}} = \frac{\sqrt{g^2+g'^2}}{g}B_\mu + \frac{g'}{g}Z_\mu^u\ .$$

We observe that the $SU(2)$ charge of the gauge fields is completely screened by the $SU(2)$ charge of the Higgs ghost fields φ and $\varphi^\pm$. The transformation properties of the fields now read:

$$Z_\mu^u \to Z_\mu^u,\ W_\mu^{u\pm} \to e^{\pm e\omega_A}W_\mu^{u\pm},\ A_\mu^u \to A_\mu^u + \partial_\mu\omega_A.$$

Due to the non-vanishing vacuum expectation value v of the Higgs field, the composite singlet field $\Phi_b^+\Phi_b = v^2/2\,(1+X)$ has a classical background term with quantum correction $X = 2H/v + (H^2+\varphi^2)/v^2 + 2\varphi^+\varphi^-/v^2$, and the perturbative expansion, $1/v^2 = \sqrt{2}G_\mu$ being the loop expansion parameter, is well defined. One easily checks that in the unitary gauge the singlet fields are identical to the usual U-gauge Z, W and A fields. In a renormalizable gauge they have the form of the standard R-gauge fields Z, W and A, plus a higher order composite ghost operator which does not contribute on the physical mass-shell and thus does not affect the S-matrix.

Any hermitean Lorentz invariant charge-neutral monomial of the fields Z^u, $W^{u\pm}$ and $F_{\mu\nu}^u = \partial_\mu A_\nu^u - \partial_\nu A_\mu^u$ and their derivatives $\partial_\rho Z_\mu^u$ and $D_\rho W_\mu^{u\pm} = (\partial_\rho \mp ieA_\rho^u)\,W_\mu^{u\pm}$, is $SU(2)_L \otimes U(1)_Y$ invariant and the couplings in front of them are free parameters in a corresponding effective Lagrangian. In this formalism the broken part of the gauge group does not give any restriction for the allowed couplings. What matters for distinguishing various interactions here is the high energy behaviour, i.e., one has to perform a systematic low energy expansion. We already know that the leading term is the one restricted by renormalizability, which fixes the couplings to those of the standard model [5]. The next-to-leading terms in this expansion are those which grow as $\ln(E^2/\Lambda^2)$, followed by E^2/Λ^2 terms etc. At each step it is important to take into account all operators of the light fields which can contribute. Here we have assumed that all fields of the SM, including the Higgs, are light, i.e., satisfy $m/\Lambda \ll 1$. This is the scenario of a linearly realized "spontaneous symmetry breaking" as assumed in the SM. By the naturalness argument [6] this scenario is considered to require a supersymmetric extension of the SM to protect the Higgs mass from "growing" too large in the low energy effective theory.

The crucial question is, which are the light fields? From the SM spectrum only the Higgs is in question. If the Higgs does not exist, and hence "spontaneous

symmetry breaking" is realized non-linearly,

$$\Phi_b^+ \Phi_b = \frac{v^2}{2} = \text{ constant }, \tag{13}$$

there is no renormalizable low energy effective theory and the new physics scale has to be identified with the Fermi scale $\Lambda = v \simeq 250$ GeV. If the Higgs is very heavy the perturbation theory breaks down and we would have to live with a strongly interacting Higgs sector. An expansion about the limit $m_H \to \infty$, is completely analogous to the chiral perturbation expansion in QCD [26]. In the linear case

$$\Phi_t = \frac{\rho + v}{\sqrt{2}} U(\theta) \chi_t \;\; ; \quad \Phi_b = \frac{\rho + v}{\sqrt{2}} U(\theta) \chi_b \tag{14}$$

where $\chi_t = \binom{1}{0}$ and $\chi_b = \binom{0}{1}$ are constant isodoublet vectors, ρ is the radial physical Higgs field and the $SU(2)$-matrix $U(\theta) = \exp(i \frac{\tau^i}{2} \theta_i)$ describes the Higgs ghosts $\varphi_i = v\theta_i$ (the "would–be" Goldstone bosons). The non-linear case follows from the linear one by setting $\rho = 0$.

One striking feature of the minimal SM with one Higgs doublet is the extra approximate "custodial" global $SU(2)$-symmetry, which becomes exact in the limit of vanishing $U(1)_Y$ coupling $g' \to 0$ (which implies $e \to 0$), and vanishing isospin-breaking mass differences in fermion doublets. It implies that for $g' \neq 0$ the ρ-parameter defined by the low-energy neutral- to charged-current ratio satisfies $\rho = 1$ at tree level. This is a relationship which has to be taken as *a basic property* of the low-energy effective theory. Therefore, non-renormalizable perturbations should be classified not only with respect to the dimension of operators, i. e., by performing a systematic expansion in E/Λ, but also with respect to their custodial symmetries. It should be noted that naive dimensional counting does not directly reflect the change in the high-energy behavior because again "gauge cancellation" may or may not be at work. In this sense we are led back to use a $SU(2)_L \otimes U(1)_Y$ invariant classification of the perturbations in order to separate in a sense the effective dimensions of operators. The gauge invariant operators up to dimension six have been systematically classified in Ref. [21]. The operators involving fermions have been shown to be tightly bound by phenomenology. We thus only consider bosonic operators here. In addition we assume that anomaluos contributions bilinear in the gauge boson fields, which contribute to the gauge boson propagators at tree level, are well constraint by LEP I in view of the discussion at the beginning of this Section (see Ref. [24]). The CP and P conserving operators contributing to the TGV's may be written as:

$$\delta \mathcal{L}_i \propto \frac{\lambda_i}{\Lambda^{d_{\mathcal{O}_i}-4}} \quad \mathcal{O}_i \tag{15}$$

with

$$\begin{aligned}
\mathcal{O}_{B\Phi} &= B^{\mu\nu} \left(D_\mu \Phi\right)^+ \left(D_\nu \Phi\right) \\
\mathcal{O}_{W\Phi} &= W_a^{\mu\nu} \left(D_\mu \Phi\right)^+ \frac{\tau^a}{2} \left(D_\nu \Phi\right) \\
\mathcal{O}'_{W\Phi} &= W_a^{\mu\nu} \Phi^+ \frac{\tau^a}{2} \Phi \left(D_\mu \Phi\right)^+ \left(D_\nu \Phi\right) \\
\mathcal{O}_W &= \epsilon_{abc} W_\mu^{a\nu} W_\nu^{b\lambda} W_\lambda^{c\mu} \\
\mathcal{O}'_W &= \epsilon_{abc} W_\mu^{a\nu} W_\nu^{b\lambda} B_\lambda{}^\mu \Phi^+ \frac{\tau^c}{2} \Phi \quad .
\end{aligned} \tag{16}$$

By appropriate choice (rescaling) of the coefficients we obtain back Eq. 8 with

$$\begin{aligned}
\delta_\gamma &= 0, \\
\delta_Z &= \frac{1}{c^2} \lambda_{W\Phi}, \\
\delta\kappa_\gamma &= \lambda_{B\Phi} + \lambda_{W\Phi} - \lambda'_{W\Phi}, \\
\delta\kappa_Z &= -\frac{s^2}{c^2} (\lambda_{B\Phi} + \lambda_{W\Phi}) - \lambda'_{W\Phi}, \\
\lambda_\gamma &= \lambda_W + \lambda'_W, \\
\lambda_Z &= \lambda_W + \frac{s^2}{c^2} \lambda'_W
\end{aligned} \tag{17}$$

where $s^2 = \sin \Theta_W^{eff}$ and $c^2 = 1 - s^2$. Notice that there is no basic principle which can prevent us from rescaling the effective couplings by *energy independent factors* like v/Λ or m_i/Λ where m_i is any of the light masses. If we do so there are many equivalent forms to write the effective Lagrangian and it is this chameleon-like appearance of effective Lagrangians which seems to have caused some confusing claims in the literature. But this is just the manifestation of the fact that in a non-renormalizable theory there

is no unique way to pin down predictions beyond the tree level unless there is some additional guiding principle, like for chiral perturbation theory in QCD. The fundamental difference with QCD is that the exact theory is known in the case of QCD, but it is not known in general if we talk about physics beyond the SM. In Fig. 6 we show an example for constraints on δ_Z, $\delta\kappa_\gamma$ and $\delta\kappa_Z$ from LEP I

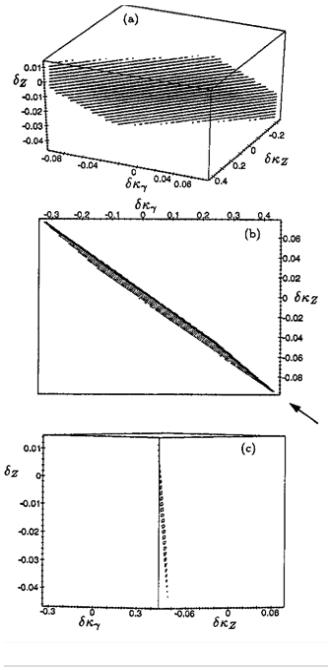


Figure 6: Phenomenological constraints on one–loop effects from anomalous TGV's (a). (b) and (c) show contours of constant δ_Z in ($\delta\kappa_\gamma$,$\delta\kappa_Z$) planes. The arrow in (b) indicates the direction of the view (c)

data in the subspace where the other anomalous couplings vanish. Notice that bounds on single couplings may not be very meaningful because of the strong cancellations in some directions in the parameter space. In fact the allowed region is a thin pancake as can be seen from Figs. 6(b) and (c). The large dependence of the indirect bounds on anomalous couplings on extra assumptions demonstrate that direct tests of the Yang-Mills gauge structure are mandatory. Direct bounds on anomalous couplings from LEP II will improve by about an order of magnitude as compared to the present situation. In the form of Eq. (8), typical single parameter sensitivities are about 0.05 at LEP II and would reach 0.005 at a 500 GeV e^+e^- linear collider, which is about the size of the "weak" radiative corrections.

5 HOW TO DISENTANGLE TRIPLE GAUGE COUPLINGS

The dependence of the cross section on the VWW form–factors shows up directly in the helicity amplitudes (see [3]):

$$\begin{aligned} M(-;\lambda,\bar\lambda) &= \sqrt{2}\beta_W\gamma^{2-|\lambda|-|\bar\lambda|}\, d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta) \\ &\left\{e^2\,(H_i^\gamma - \frac{H_i^Z}{1-M_Z^2/s}) + \frac{g^2}{2}\frac{H_i^Z}{1-M_Z^2/s}\right. \\ &\left. -\quad \frac{\hat M_{T_1}(-;\lambda,\bar\lambda)}{\beta_W}\,\frac{g^2}{\frac{1+\beta_W^2}{2}-\beta_W\cos\theta}\right\} \\ M(+;\lambda,\bar\lambda) &= \sqrt{2}\beta_W\gamma^{2-|\lambda|-|\bar\lambda|}\, d^{J_0}_{\Delta\sigma,\Delta\lambda}(\theta) \\ &\left\{e^2\,(H_i^\gamma - \frac{H_i^Z}{1-M_Z^2/s})\right\} \end{aligned} \tag{18}$$

with the correspondence

amplitude	state	
H_l^V	LL	
H_0^V	TT	; $\Delta\lambda = 0$
H_{-1}^V	TL, LT	; $\Delta\lambda = -1$
H_{+1}^V	TL, LT	; $\Delta\lambda = +1$

and we have

$$\begin{aligned} H_l^V &= (2+\gamma^{-2})A_1^V + 2A_2^V - \beta_W^2 A_3^V \\ H_0^V &= A_1^V \\ H_{-1}^V &= 2A_1^V + A_2^V + \beta_W A_4^V \\ H_{+1}^V &= 2A_1^V + A_2^V - \beta_W A_4^V \quad . \end{aligned}$$

The anomalous couplings contribute as follows

$$\begin{array}{lcl} H_l^V & = & 2\{(1+\frac{2M_W^2}{s})\delta_V + \delta\kappa_V + (1-\frac{M_W^2}{s})\lambda_V\} \\ H_0^V & = & \delta g_V + \frac{s}{2\Lambda^2}\lambda_V \\ H_\mp^V & = & 2\delta_V + \delta\kappa_V + \lambda_V \mp \beta_W \frac{s}{\Lambda^2}\xi_V \quad . \end{array} \tag{19}$$

The amplitudes H_i^V thus are the combinations of VWW form-factors which enter the helicity amplitudes, the squares of which are measurable. Of course in our analysis the remaining SM radiative corrections are included as well. Like at the Born level, the right-handed cross sections allow H_i^γ to be fixed relative to H_i^Z while H_i^Z is determined relative to the SM ν-exchange contribution. Near threshold, these terms are suppressed by β_W so that a separation of these couplings is possible for an intermediate energy window only. Indeed the mapping

$$H_i = M_{ik}A_k; \;\; \det M = \beta_W^3 \to 1 \text{ as } s \gg 4M_W^2 \tag{20}$$

is badly singular near threshold but regular at all larger energies. The explicit form of the form–factors in terms of the more directly observable amplitudes H_i is

$$\begin{array}{l} A_1^V = H_0^V \\ A_2^V = \frac{1}{2}(H_-^V + H_+^V) - 2H_0^V \\ A_3^V = \frac{1}{\beta_W^3}(H_-^V + H_+^V - (2-\frac{1}{\gamma^2})H_0^V - H_l^V) \\ A_4^V = \frac{1}{\beta_W}(H_-^V - H_+^V) \end{array} \tag{21}$$

and for the anomalous couplings we find

$$\begin{array}{l} \delta_V = A_1^V + \frac{1}{2}A_3^V \\ \delta\kappa_V = A_2^V + (\frac{\Lambda^2}{s} - 1)A_3^V \\ \lambda_V = -\frac{\Lambda^2}{s}A_3^V \\ \xi_V = -\frac{\Lambda^2}{s}A_4^V \quad . \end{array} \tag{22}$$

What can we learn from the cross section measurements about the triple gauge couplings? Assuming CP to be conserved the situation is as follows: We notice that the 2 dominating left-handed TT $\Delta\lambda = \pm 2$ polarized cross sections[1] and the 2 vanishing right-handed TT $\Delta\lambda = \pm 2$ polarized cross sections do not provide any information about the 8 amplitudes $A_i^{VWW}, \quad i = 1,2,3,4$. Nevertheless, the 8 remaining cross sections in principle suffice to determine all amplitudes. However, to disentangle the TGV's is a highly nontrivial task:

- near threshold: except for the dominating left-handed TT $\Delta\lambda = \pm 2$ cross sections all others are suppressed by β_W threshold factors. This is good for a "model independent" determination of the W-mass. Note that the amplitudes A_4^V and A_3^V are suppressed, respectively, by one and two additional velocity factors. Thus the energy should be increased as close as possible to the peak which is at about 200 GeV.

- at very high energy: besides the TT $\Delta\lambda = \pm 2$ cross sections, which are not affected by the TGV's, only the LL modes survive in the SM. This implies that the reduced number of possible independent measurements does not anymore allow us to disentangle the TGV's. Thus very high energy colliders may not be the best to disentangle in detail the TGV's.

- the e_R^+ cross sections are suppressed by about a factor 10^{-2} and hence high luminosity in the intermediate energy range 190 to 300 GeV may be more important for a study of Yang–Mills structure than higher energies.

At LEP II there will be no beam polarization. Then one is left with four different W–decay distributions which yield information about triple gauge couplings: LL, TL+LT $\Delta\lambda = +1$, TL+LT $\Delta\lambda = -1$ and TT $\Delta\lambda = 0$ which, respectively, yield 10%, 15%, 15% and 5% of σ_{tot} at LEP II energies. This provides four constraints on six CP conserving anomalous couplings left if we assume $\delta_\gamma = 0$ and $\xi_\gamma = 0$. The later conditions are implied in the static limit by electromagnetic gauge invariance. One should be aware of the fact that W–pair production does not test the static properties of W-couplings, however.

We finally mention that *transverse beam polarization*, the natural polarization at storage rings like LEP, would be an interesting tool for studying the interesting channel $W_L^+W_L^-$, which is the most sensitive to new physics. The transversal polarization asymmetry is completely dominated by this channel. Unfortunately it is small, about 1% only, but a very clean observable otherwise [3].

[1] Within the SM, they yield 50%, 94% and 99% of the total cross section at 200, 500 and 1000 GeV, respectively.

6 OUTLOOK

The best possibility is a detailed investigation of the $e^+e^- \to W^+W^-$ peak around 200 GeV up to about 300 GeV with a factor of 100 increased luminosity as compared to LEP II. However, if non-renormalizable perturbations of the SM couplings should be present, high energy e^+e^--colliders have a much higher sensitivity to such deviations.

Even if one has little reason to doubt the validity of the SM gauge boson interaction we think it is a real challenge to accurately determine these couplings. For a long time we were taught that Yang-Mills theories have nothing to do with the real world because gauge invariance required the gauge bosons to be massless and no such fields were seen. Mass generation by the Higgs mechanism resolved that problem. Then, however, for a long time, technical problems of quantizing "spontaneously broken gauge theories" while preserving renormalizability and unitarity, prevented real progress until 't Hooft's proof of renormalizability [27]. We are still lacking the experimental proof and it is a noble goal to establish this milestone in settling our understanding of electroweak phenomena. The question of the nature of electroweak symmetry breaking, the hunt for the Higgs, and the question at which scale deviations from the SM will show up remain the key questions for future high energy physics.

Acknowledgements

I wish to thank Tord Riemann for the invitation to this interesting workshop and for the very kind hospitality in Teupitz. I am very grateful to Paula Franzini and Geert Jan van Oldenborgh for help in preparing the manuscript.

References

[1] O.P. Sushkov, V.V. Flambaum, I.B. Khriplovick, Sov. J. Nucl. Phys. 20 (1975) 537; W. Alles, Ch. Boyer, A.J. Buras, Nucl. Phys. B119 (1977) 125; P. Méry, M. Perrottet, Nucl. Phys. B 175 (1980) 234; I.F. Ginzburg et al., Nucl. Phys. B 228 (1983) 285.

[2] G. Barbiellini et al., in "Physics at LEP," ed. by J. Ellis and R. D. Peccei, report CERN 86-02, Vol. 2, p. 1; M. Davier et al., ECFA Workshop on LEP 200, Aachen, report CERN 87-08, Vol. I, p. 120; K. Hagiwara, R.D. Peccei, D. Zeppenfeld, K. Hikasa, Nucl. Phys. B282 (1987) 253; and references therein.

[3] J. Fleischer, K. Kołodziej, F. Jegerlehner, Phys. Rev. D 49 (1994) 2174.

[4] M. Veltman, Nucl. Phys. B 7 (1968) 637.

[5] C.H. Llewellyn Smith, Phys. Lett. B 46 (1973) 233; J.S. Bell, Nucl. Phys. B 60 (1973) 427; J.M. Cornwall, D.N. Levin, G. Tiktopoulos, Phys. Rev. Lett. 30 (1973) 1268, 31(E) (1973) 572, Phys. Rev. D 10 (1974) 1145.

[6] G. 't Hooft, in "Recent Developments in Gauge Theories", G. 't Hooft et al. (eds.), Plenum Press, New York, 1980.

[7] M. Lemoine, M. Veltman, Nucl. Phys. B164 (1980) 445. M. Böhm, A. Denner, T. Sack, W. Beenakker, F. Berends, H. Kuijf, Nucl. Phys. B304 (1988) 463; J. Fleischer, F. Jegerlehner, M. Zrałek, Z. Phys. C42 (1989) 409; W. Beenakker, A. Denner, Preprint DESY 94-051, March 1994.

[8] W. Beenakker, these Proceedings.

[9] J.K.F. Gaemers, G.J. Gounaris, Z. Phys. C1 (1979) 259.

[10] C.L. Bilchak, J.D. Stroughair, Phys. Rev. D 30 (1984) 1881; M. Kuroda, F.M. Renard, D. Schildknecht, Phys. Lett. B 183 (1986) 366; P. Méry, M. Perrottet, F.M. Renard, Z. Phys. C 36 (1987) 249.

[11] M. Bilenky, J.L. Kneur, F.M. Renard, D. Schildknecht, Nucl. Phys. B 409 (1993) 22, Preprint Bielefeld Univ. BI-TP-93-47, October 1993; A.A. Pankov, N. Paver, Phys. Lett. B 324 (1994)

224; C.G. Papadopoulos, Preprint CERN-TH.7157/94.

[12] F. Jegerlehner, in "Radiative Corrections for e^+e^- Collisions", ed. J.H. Kühn, Springer, Heidelberg, 1989, p. 208.

[13] G.J. van Oldenborgh, these Proceedings.

[14] M. Bilenky, these Proceedings.

[15] A. Blondel, these Proceedings.

[16] S. Abachi et al., Phys. Rev. Lett. 72 (1994) 2138.

[17] S. Dittmaier, D. Schildknecht, K. Kołodziej, M. Kuroda, Bielefeld Univ. Preprint BI-TH 94/09, March 1994; P. Gambino, A. Sirlin, New York Univ. Preprint NYU-TH-94/04/01, April 1994; V.A. Novikov, L.B. Okun, A.N. Rozanov, M.I. Vysotsky, Preprint CERN-TH.7217/94, April 1994.

[18] S.J. Brodsky and J.D. Sullivan, Phys. Rev. 156 (1967) 1644; F. Herzog, Phys. Lett. 148B (1984) 355; (E) 155B (1985) 468; A. Grau and J.A. Grifols, Phys. Lett. 154B (1985) 283; J.C. Wallet, Phys. Rev. D 32 (1985) 813; P.Méry, S.E.Moubarik, M.Perrottet and F.M.Renard, Z. Phys. C 46 (1990) 229.

[19] M. Suzuki, Phys. Lett. B 153 (1985) 289; J.J. van der Bij, Phys. Rev. D 35 (1987) 1088, Phys. Lett. B 296 (1992) 239; H. Neufeld, J.D. Stroughair and D. Schildknecht, Phys. Lett. B 198 (1987) 563; J.A. Grifols, S. Peris and J. Solà, Int. J. Mod. Phys. A3 (1988) 225; G.L. Kane, J. Vidal and C.-P. Yuan, Phys. Rev. D 39 (1989) 2617.

[20] A. Grau and J.A. Grifols, Phys. Lett. 166B (1986) 233; J.A. Grifols, S. Peris and J. Solà, Phys. Lett. B 197 (1987) 437; R. Alcorta, J.A. Grifols and S. Peris, Mod. Phys. Lett. A2 (1987) 23.

[21] W. Buchmüller, D. Wyler, Nucl. Phys. B 268 (1986) 621.

[22] A. De Rújula, M.B. Gavela, P. Hernandez, E. Massó, Nucl. Phys. B 384 (1992) 3.

[23] F. Jegerlehner, J. Fleischer, Phys. Lett. B 151 (1985) 65; Acta Phys. Pol. B 17 (1986) 709; see also: D. London, C. P. Burgess, Phys. Rev. Lett. 69 (1992) 3428

[24] K. Hagiwara, S. Ishihara, R. Szalapski, D. Zeppenfeld, Phys. Lett. B 283 (1992) 353, Phys. Rev. D 48 (1993) 2182.

[25] P. Hernandez and F.J. Vegas, Phys. Lett. B 307 (1993) 116.

[26] A. Dobado, D. Espriu, M.J. Herrero, Phys. Lett. B 255 (1991) 405; D. Espriu, M.J. Herrero, Nucl. Phys. B 373 (1992) 117.

[27] G. 't Hooft, Nucl. Phys. B 33 (1971) 173, 35 (1971) 167; G. 't Hooft, M. Veltman, Nucl. Phys. B 50 (1972) 318.