Astronomy

Forces

Newton $F = G\frac{mM}{r^2}$

$F_R = m\frac{v^2}{r} = m\omega^2 r$ $v = \frac{2\pi r}{T}$ ← umlaufzeit

$F_L = q(\vec{E} + \vec{v}\times\vec{B})$ ← Lorentz force

$\vec{E}$ ↑ electric force

$pV = nkT$

inertia of sphere: $I = \frac{2}{5}mr^2$

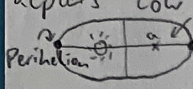
$v = \sqrt{\frac{GM}{r}}$ Bahngeschw. Lokale $v_e = \sqrt{\frac{2GM}{r}}$ escape velocity

$E_{kin} = (\gamma - 1)mc^2$

$\gamma = \frac{1}{\sqrt{1-(\frac{v}{c})^2}}$

$v = \omega \cdot r$

Keplers Law's



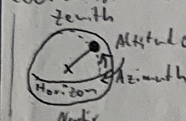
1) orbits of planets ellipse with sun in one of two foci

2) The connection line between planet/sun sweeps over equal areas in equal time

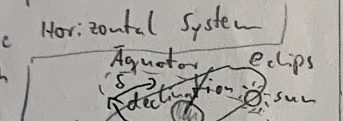
3) $T^2 \sim a^3$ $\frac{T_1^2}{a_1^3} = \frac{T_2^2}{a_2^3} = c$

$c_{sun} = 2{,}97 \cdot 10^{-19}\ \frac{s^2}{m^3}$ $\frac{T^2}{a^3} = \frac{4\pi^2}{G(M+m)} \approx \frac{4\pi^2}{GM}$

Coordination System

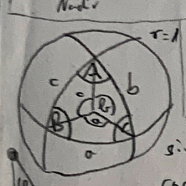


Horizontal System



24h ≙ 360°

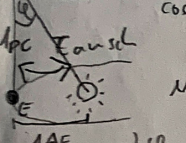
1h ≙ 15°



$\sin B \sin a = \sin A \sin b$

$\cos B \sin a = \cos A \sin b \cos c + \cos b \sin c$

$\cos a = \cos A \sin b \sin c + \cos b \cos c$



Parallaxe weist ↺

$\mu = \sqrt{\mu_\alpha^2 \cos^2\delta + \mu_\delta^2}$

$v_t = \mu \cdot r$ (v tangential)

red shift

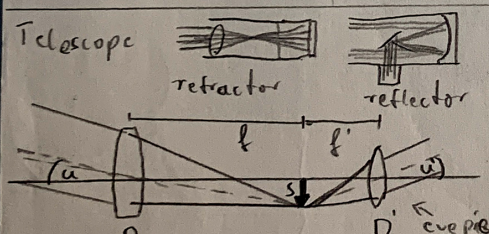
$\frac{\Delta\lambda}{\lambda_0} = \sqrt{\frac{c+v}{c-v}} - 1 \approx \frac{v}{c}$ ($c \gg v$) (v_r radial)

$v = \sqrt{v_r^2 + v_t^2}$ $z = \frac{\lambda_{obs} - \lambda_{Emit}}{\lambda_{Emit}} = \frac{f_{Emit} - f_{obs}}{f_{obs}}$

$z = \frac{H_0}{c} \cdot d$ $v = H_0 \cdot d$ $H_0 = 68\ \frac{km/s}{Mpc}$

$H_0^{-1} = T_0 = 14{,}4\,Gyr$ ≈ age of universe

Teleskope



$S = f\tan(u) \approx fu$ $\omega = \frac{u'}{u} = \frac{O}{D'} = \frac{f}{f'}$ magnification

$\Theta = 1{,}22\frac{\lambda}{D}$ resolution limit in rad!

deffraction at Grid $\sin\Theta = n\frac{\lambda}{d}$

Tscherenkow-Strahlung:

$\cos(\Theta) = \frac{1}{n\beta}$ | $\frac{dN}{dx\,d\lambda} = \frac{2\pi z^2 \alpha}{\lambda^2}\sin^2\Theta$

$v = c \cdot \beta$

Magnitude

$m = -2{,}5\log_{10}(f/f_x)$ f_x reference flux

$M = -2{,}5\log_{10}(L/L_x)$ $L_x = 78{,}7\,L_\odot$ | $2{,}53 \cdot 10^{-8}\ \frac{W}{m^2}$

$m - M = 5\log_{10}\left(\frac{r}{1Mpc}\right) + 25$ m-apparent magnitude

$m_{sun} = -26{,}6$

$M_{bol*} - M_{bol\odot} = -2{,}5\log_{10}\left(\frac{L_*}{L_\odot}\right)$ M-absolute magnitude $M_{sun} = 4{,}83$

F-flux at earth

$F = \frac{L}{4\pi r^2}\ \left[\frac{W}{m^2}\right]$

with Dust:

$m - M = 5\log\left(\frac{r}{10pc}\right) + A$

$\sim 2\ \frac{mag}{kpc}$ extinction

$\frac{1\,mag}{2\,mag} = \sqrt[5]{100} \approx 2{,}512$ | $m_1 - m_0 = \frac{-5\log_{10}\left(\frac{f_1}{f_0}\right)}{\log_{10}(100)}$

$6\,mag \cdot 100 = 1\,mag$

$E = h\cdot\nu$ | $E_n = -\frac{13{,}6\,eV}{n}$ | n-state of Hydrogen

$^1H + {}^1H \to {}^2H + e^+ + \nu_e$ Reaction in sun 99,5%

$^2H + {}^1H \to {}^3He + \gamma$ He chain

$B(\nu, T) = \frac{2h\nu^3}{c^2}\frac{1}{e^{h\nu/(k_BT)} - 1}$ Black Body Radiation

$\frac{d\nu}{d\lambda} = -\frac{c}{\lambda^2}$ $B_\lambda = -B_\nu\frac{d\nu}{d\lambda} = B_\nu\frac{c^2}{\lambda^2}$

$B_\lambda = \frac{2hc^2}{\lambda^5}\frac{1}{e^{hc/(\lambda k_BT)} - 1}$ $\lambda_{peak} = \frac{b}{T}$

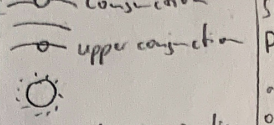
Black Body: $b = 0{,}0028978\,mK$

$P = \sigma \cdot A \cdot T^4 \Rightarrow L = \sigma \cdot T_{eff}^4$ $F = \frac{1}{4}\sigma T_{eff}^4$

Solar System

Rotation time sun ~25,4d

Sun has 99,87% Sunsystem Mass, 10^{11} stars in MW



Synodic Period

P_1, P_2: $\frac{2\pi}{P_1}, \frac{2\pi}{P_2}$ $P_1 > P_2$

after $P_{1,2}$ one Planet has one rotation more:

$P_{1,2} = \frac{2\pi}{P_1} = 2\pi + P_{1,2}\frac{2\pi}{P_2}$

$\frac{1}{P_{1,2}} = \frac{1}{P_1} - \frac{1}{P_2}$

In Atmosphere: $\bar{v} = \sqrt{\frac{3k_BT}{m}}$ $v_e = \sqrt{\frac{2GM}{R}}$

m-particle Mass velocity escape velocity

stable if: $\bar{v} \lesssim 0{,}2\,v_e$

Gass in ~~Star~~ Cloud compressed:

$\langle T \rangle = -\frac{1}{2}\langle U \rangle$

Virial satz: $\langle \vec{F}_i \cdot \vec{r}_i \rangle = -\frac{1}{2}\sum_i \langle \vec{F}_i \cdot \vec{r}_i \rangle$

time the systems feels force: $t_{sound} = \frac{R}{c_s}$

R-Radius, c_s-Speed of Sound $t_{sound} \simeq 5 \cdot 10^5\,yr$

free fall time: $t_{dyn} = \frac{1}{\sqrt{G\rho}} \simeq 2\,\mu s$ | $pV = nk_BT$

Jeans Length $= \lambda_J = \frac{c_s}{\sqrt{G\rho}}$ $M_{Jeans} = \sqrt{\frac{5}{\pi}}\sqrt{\frac{1}{\rho}\left(\frac{kT}{G\mu}\right)^3}$

$\lambda_J = \sqrt{\frac{15k_BT}{4\pi G m_p \mu \rho}}$ mass proton, mass particle

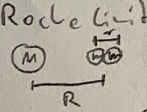
	size [km]	density $\left[\frac{kg}{m^3}\right]$
star	~10^6	$10^3 - 10^5$
white dwarf	~10^4	10^9
neutron st.	~10	10^{17}
black holes	~5-10	$>10^{17}$

$1{,}44\,M_\odot \Rightarrow$ Neutron star

$2{,}2\,M_\odot \Rightarrow$ black hole

$I_{star} \cdot \omega_{star} = I_{pulsar} \cdot \omega_{pulsar} \Rightarrow P_{pulsar} = \frac{r_{pulsar}^2}{r_{star}^2}$ | ≈ 1ms z.B.

Roche Limit at which torn apart

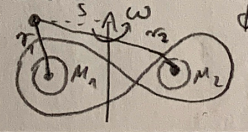


$\Delta F = GMm\left[\frac{1}{(R-r)^2} - \frac{1}{(R+r)^2}\right]$

Taylor $\frac{r}{R} \ll 1$

$\approx 4G\frac{Mmr}{R^3}$ $R_{Roche} = \sqrt[3]{\frac{16r^3M}{m}}$

$R_{Roche} \approx 2{,}5 \cdot R_{planet}$ of equal Mass



$\phi = -\frac{GM_1}{r_1} - \frac{GM_2}{r_2} - \frac{1}{2}\omega^2 s^2$

21cm - Hydrogen neutral radio line

Milky Way $D = 170.000 - 200.000\,ly$

$\sim 250 \cdot 10^9$ Stars $M \approx 1{,}5 \cdot 10^{12}\,M_\odot$

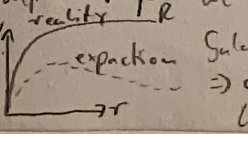
Stars: Population I • high metallicity, young 2-4% Elements heavier than He, in galactic plane

Population II • low metallicity, old, outside Plane

Sun: $r_{sun} = 8{,}1\,kpc$; $v = \frac{0{,}0053''}{a} \hat{=} 220\,\frac{km}{s}$

to center MW. $P_0 = \frac{2\pi}{\omega_0} \approx 2{,}5 \cdot 10^8$ years | $\omega_0 = 25\,\frac{km}{s\,kpc}$

Kepler: $v = \sqrt{\frac{GM}{R}}$ we expect: $v \sim \frac{1}{\sqrt{R}}$



Galaxy Center after Black Hole ($\sim 10^5 - 10^9\,M_\odot$) Mass: $10^{-3} - 10^{-6}$

Galaxy brightness $\sim 10^5 - 10^{15}\,L_\odot$

Galaxy Classification

Ellipticals E_0 E_2 E_5 E_7 S0

Spirals Sa, Sb, Sc

transition: lenticular no arms

barred SBa, SBb, SBc

60% spirals, 20% S0 & E, 10% irregular

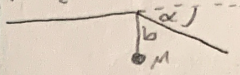
Stellar Structure

virial theorem

$0 = W + 2K$ W potential E K-kinetic E.

$W = -\alpha\frac{GM^2}{r_h}$ M-total mass, r_h-half mass radius, α-density profil factor

$K = \frac{1}{2}M\langle v^2 \rangle$ $M = \frac{\langle v^2 \rangle r_h}{\alpha G}$



$\alpha = \frac{4GM}{c^2 b}$

CMB - Cosmic microwave Background

$\lambda = 7{,}35\,cm$ - Telescope | $T_0 = 2{,}7255 \pm 0{,}0006\,K$

$\varepsilon_\gamma = \alpha T^4 = 0{,}2606\,MeV\,m^{-3}$ universe Temp.

$n_\gamma = 411/cm^3$ $\lambda = 2\,mm$ CMB

early Universe: $H + \gamma \rightleftharpoons p + e^- + \nu$ stops when $T_{well} < 13{,}6\,eV$

$\Omega = \rho/\rho_c$ ρ_c-critical density

$\Omega \approx 1 \Rightarrow$ flat Universe | $\rho_c = \frac{3h^2}{8\pi G} = 10^{-26}\,\frac{kg}{m^3}$

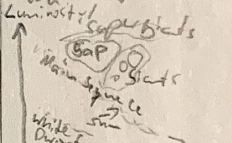
Dark Energy 72% / Atoms 4,6% / Dark Matter 23%

$E = h \cdot f$ relativistic Impuls: $p = \frac{m_0 v \cdot \gamma}{}$

$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ $p^2c^2 = -m_0^2c^4 + (mc^2)^2$

$E_{kin} = m_0c^2 \cdot [\gamma - 1]$ $E^2 = (pc)^2 + (m_0c^2)^2$

$E = \gamma m_0 c^2$



Hertzsprung-Russell-Diag.

Hayashi-Linie: Sequenzlinie | hydrostatisches Gleichgewicht

$^1H + {}^1H \to {}^2H + e^+ + \nu_e$ | $^1H + {}^2H \to {}^3He + \gamma$

99,75% $^2H + {}^1H \to {}^3He + \gamma$

26,73 MeV $^3He + {}^3He \to {}^4He + 2\,{}^1H$

carbon (CNO) circle

$^{12}C + {}^1H \to {}^{13}N + \gamma$

$^{13}N \to {}^{13}C + {}^1H$ $^{13}N \to {}^{13}C + e^+ + \nu_e$

Atomic H: studied via 21cm-α Line $n: 1 \to n: 2$

Flux

$F = \frac{L}{4\pi r^2}\ \left[\frac{W}{m^2}\right]$

Luminosity L [W]

$m_E = 5{,}97 \cdot 10^{24}\,kg$

$R_E = 6{,}376\,km$

$1\,AU = 1{,}496 \cdot 10^8\,km$

$M_\odot = 1{,}99 \cdot 10^{30}\,kg$

$R_\odot = 7 \cdot 10^5\,km$

$L_\odot = 3{,}85 \cdot 10^{26}\,W$

$1\,pc = 3{,}086 \cdot 10^{16}\,m = 3{,}26\,ly$

$1\,ly = 9{,}461 \cdot 10^{15}\,m$

$u = 1{,}660539 \cdot 10^{-27}\,kg$

$m_{He} = 4{,}0026\,u$

$m_H = 1{,}008\,u$

$k_B = 1{,}38 \cdot 10^{-23}\,J/K$

$\sigma = 5{,}670373 \cdot 10^{-8}\,\frac{W}{m^2K^4}$ Stefan-Boltzmann const.