Theoretische Physik 1

Gallileo Transformation

$\vec{x}' = A\vec{x} + \vec{a}t + \vec{b}$ $\Rightarrow$ bilden Gruppe

$t' = t + \tau$

$A\vec{\Phi} = e^{\vec{\Phi}T} = e^{\Phi T_1 + \Phi T_2 + \Phi T_3}$

$T_1 = \begin{pmatrix} 0&1&0\\ -1&0&0\\ 0&0&0 \end{pmatrix}$ $T_3 = \begin{pmatrix} 0&0&-1\\ 0&0&0\\ 1&0&0 \end{pmatrix}$

$T_2 = \begin{pmatrix} 0&0&0\\ 0&0&1\\ 0&-1&0 \end{pmatrix}$ Rotations Matrix

$A^T A = \mathbb{1}$ $T^T = -T$ $\det(A) = \pm 1$

$A \cdot \vec{v} = \vec{v}$

$R_x = \begin{pmatrix} 1 & & \\ & \cos & -\sin \\ & \sin & \cos \end{pmatrix}$ $R_z = \begin{pmatrix} \cos & -\sin & \\ \sin & \cos & \\ & & 1 \end{pmatrix}$

$R_y = \begin{pmatrix} \cos & & \sin \\ & 1 & \\ -\sin & & \cos \end{pmatrix}$

Koordinatensysteme:

gleichm. beschleunigt $\vec{x} = \vec{x} - \frac{b}{2} t^2 \vec{e}_3$

$\ddot{\vec{x}} = \ddot{\vec{x}} - b\vec{e}_3$ } Trägheit

Rotiertes Koordinatensyst. $D_{ij} = \vec{e}_i \vec{e}_j$

$x_i' = \sum_j D_{ij}(t) x_i$ $\vec{\omega}(t)$ drehachse und Drehgeschwindigkeit

$\dot{\vec{e}}_j'(t) = \vec{\omega}(t) \times \vec{e}_j(t)$

$\dot{x}'(t) = \sum_j \dot{D}_{ij} x_j + \sum_j D_{ij}(t) \dot{x}_j$

$\dot{\vec{x}}(t) = \sum \dot{x}\, \vec{e}'(t) + \sum x' \dot{\vec{e}}'(t) = \sum \dot{x}' \vec{e}' + \vec{\omega} \times \vec{x}$

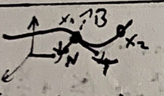
$m \frac{d'^2 \vec{x}}{dt^2} = \underbrace{-2m\vec{\omega} \times \frac{d'\vec{x}}{dt}}_{\text{Corriolis}} \underbrace{- m\vec{\omega}(\vec{\omega} \times \vec{x})}_{\text{zentral}} - m\dot{\vec{\omega}} \times \vec{x} + \vec{F}$ ext. Kraft

$m\ddot{\vec{x}}' = \vec{F} - 2m\vec{\omega} \times \vec{v}' - m\dot{\vec{\omega}} \times x' - m\vec{\omega} \times (\vec{\omega} \times \vec{x}')$

$\frac{d'\vec{x}}{dt} = \sum \dot{x}'(t)\, \vec{e}'(t)$

$m\ddot{\vec{x}} = \vec{K}$

Newton



Tangente
Normale
Binormale

$s(t) = \int_{t_0}^{t} |\vec{v}|\, dt$ $\vec{e}_T = \frac{d\vec{x}}{ds}$

$\left|\frac{d^2\vec{x}}{ds^2}\right| = \frac{1}{R}$ $\vec{e}_B = \vec{e}_T \times \vec{e}_N$ $\frac{d\vec{x}}{dt} = v\vec{e}_T$

$\ddot{\vec{x}} = \dot{v}\vec{e}_T + \frac{v^2}{R}\vec{e}_N$

$\sum_i K_i \frac{\partial f_i}{\partial q_k} = \left[\frac{d}{dt}\frac{\partial}{\partial \dot{q}_k} T(q,\dot{q},t) - \frac{\partial}{\partial q_k} T(q,\dot{q},t)\right]$

Potential

$K_i = -\frac{\partial}{\partial x_i} V(\vec{x},t) = -\nabla V(\vec{x},t)$

Kräftefrei: $V = c$ Konst. $V = -\sum x_i b_i$

Feder: $V = \frac{1}{2}kx^2$ Coulomb: $V = -\frac{\alpha}{r}$

$\vec{\nabla} \times \vec{K} = 0 \Rightarrow$ konservativ $\oint \vec{K}\, d\vec{s} = \int_F \mathrm{rot}\, \vec{K}\, d\vec{F} = 0$

Lagrange Funktion

$L(q,\dot{q},t) = T(q,\dot{q},t) - V(q,t)$ $\frac{d}{dt}\frac{\partial L}{\partial \dot{q}} - \frac{\partial L}{\partial q} = 0$

Holonome Zwangsbedingung: $\varphi(x,y,z,t) = c$ $\frac{\partial \varphi}{\partial t} = 0$

Lagrange 1. Art:

$m\ddot{\vec{x}} = \vec{K} + \lambda \vec{\nabla} \varphi(\vec{x},t)$ $\varphi(\vec{x},t) = \mathrm{const.}$

Spezielle kinetische Energie

$L' = L + \frac{d}{dt}(f(q,t))$ invariant ??

Kartesisch: $\frac{1}{2}m(\dot{x}^2 + \dot{y}^2 + \dot{z}^2)$

Polar: $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2)$

Kugel: $\frac{1}{2}m(r^2[\dot{\theta}^2 + \dot{\varphi}^2 \sin\theta] + \dot{r}^2)$

Zylinder: $\frac{1}{2}m(\dot{r}^2 + r^2\dot{\varphi}^2 + \dot{z}^2)$

Starrer Körper $E_{kin} = \frac{1}{2}mV^2 + \frac{1}{2}I\omega^2$

Mehrteilige Systeme

$m_i \ddot{x}_i = K_i + Z_i$ $Z_i = \sum_{f=1}^{r} \lambda_f \frac{\partial \varphi_f}{\partial x_i}$

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) - \frac{\partial T}{\partial q} = Q$ $f = 1 \dots r$ Freiheitsgrad, $k = 1 \dots 3N$ N-Teilchen

$Q_k = \sum_i K_i \frac{\partial x_i}{\partial q_i} + \sum_f \lambda_f \frac{\partial \varphi_f}{\partial q_k}$

$\Rightarrow$ geeignete Koord. Wahl

$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{q}}\right) = \frac{\partial T}{\partial q}$

Energie erhalten, falls:

$E = T + V = \mathrm{const.}$ falls $\frac{\partial V}{\partial t} = 0$

3p: $\frac{1}{2}m\dot{x}^2 = E - V(x)$ $\frac{dx}{dt} = \sqrt{\frac{2}{m}(E - V(x))}$

$T = \sqrt{2m} \int_{x_1}^{x_2} \frac{dx}{\sqrt{E-V}}$

Impuls: $\vec{p} = \sum_{i=1}^{N} m_i \dot{\vec{x}}^i$ $\vec{p}^i = m_i \dot{x}^i$ $\frac{d\vec{p}}{dt} = 0$

Impulserhaltung: $V(\vec{x},t) = V(\vec{x}',t)$

Mehrteilchen:

Schwerpunkt: $\vec{R} = \frac{1}{M}\sum_{i=1}^{N} m_i \vec{x}^i$ $M\ddot{\vec{R}} = 0$

$M = \sum_{i=1}^{N} m_i$ ges. Masse

$\frac{d\vec{L}}{dt} = 0$ Drehimpulserhaltung

Zweikörperproblem: reduzierte Masse

$L = \frac{\mu}{2}\dot{\vec{r}}^2 - V(r)$ $r = |\vec{r}|$ $\mu = \frac{m_1 m_2}{m_1 + m_2}$

Zentralfeld $V = V(r)$ $\vec{K} = -\frac{\vec{x}}{|\vec{x}|}\frac{dV}{dr}$

$E = T + V$ – Drehimpulserhaltung

geeignetes Koordinatensystem:

$\Rightarrow$ Zyl. Koord. mit $z = 0$

$\Rightarrow L = \frac{m}{2}(\dot{r}^2 + r^2\dot{\varphi}^2) - V(r)$

$t = \pm \int \frac{dr}{\sqrt{\frac{2}{m}(E - V(r)) - \frac{l^2}{m^2 r^2}}}$

$\varphi = \pm \int \frac{l}{r^2} \frac{dr}{\sqrt{2m(E - V(r)) - \frac{l^2}{r^2}}}$

finite Bewegung n Umläufe

$\Rightarrow \Delta\varphi = 2\pi m$

$\Rightarrow n\Delta\varphi = 2\pi m$

$\Delta\varphi = 2\pi \frac{m}{n}$

falls:

$\Delta\varphi = 2\int_{r_{min}}^{r_{max}} \frac{l}{r^2} \frac{dr}{\sqrt{2m(E-V) - \frac{l^2}{r^2}}} = 2\pi z$

z is rational

Zentralfeld:

$E = V(r) + \frac{1}{2}m(r\dot{\varphi})^2 + \frac{1}{2}m\dot{r}^2$

L erhalten $\Rightarrow L = mr^2\dot{\varphi}$

$V_{eff} = V(r) + \frac{1}{2}\frac{L^2}{mr^2}$ $E_{rot} = \frac{l^2}{2I}$

Anziehender Fall:

$V_{eff} = \frac{l^2}{2mr^2} - \frac{\alpha}{r}$ $\varphi = \pm\arccos\left(\frac{x}{x_0}\right) + \varphi_0$

$p = \frac{l^2}{m\alpha}$ $\varepsilon = \sqrt{1 + 2E\frac{l^2}{m\alpha^2}}$

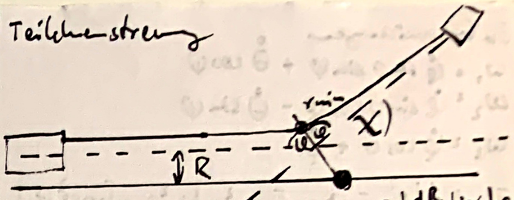
$r(\varphi) = \frac{p}{1 + \varepsilon\cos(\varphi - \varphi_0)}$

Abstoßender Fall:

$V_{eff} = \frac{l^2}{2mr^2} + \frac{\alpha}{r}$ | $r(\varphi) = \frac{p}{-1 + \varepsilon\cos(\varphi - \varphi_0 - \pi)}$

$p = \frac{l^2}{m\alpha} > 0$ $\varepsilon = \sqrt{1 + 2E\frac{l^2}{m\alpha^2}}$

Teilchenstreuung



$E = \frac{\mu}{2}v_\infty^2$ $l = \mu R v_\infty^2$ $dN = nR\left|\frac{dR}{d\chi}\right| d\chi\, d\varphi$

$d\sigma = \frac{dN}{n} = R\left|\frac{dR}{d\chi}\right|\frac{d\Omega}{\sin\chi}$ $d\Omega = \sin\chi\, d\chi\, d\varphi$

$\sigma_{tot} = \int d\sigma = \int \frac{d\sigma}{d\Omega} d\Omega$

$\frac{p}{r} = 1 + \varepsilon\cos\varphi$ $d\sigma = \left(\frac{\alpha}{4E}\right)^2 \frac{1}{\sin^4\left(\frac{\chi}{2}\right)} d\Omega$

homogene Felder:

k-Grad der Homogenität

$\sum_{a=1}^{3N} x_a \frac{\partial}{\partial x_a} V = k V(\vec{x}^1, \vec{x}^2, \dots \vec{x}^N)$

Virialsatz: $\bar{T} = \frac{k}{2}\bar{V}$

Der starre Körper

$M = \int \rho\, dV$ 6 Freiheitsgrade 3 Ort, 3 Winkel

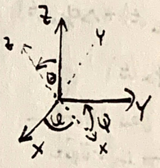
$\vec{v} = \vec{V} + \vec{\omega} \times \vec{r}$ (Geschw., Schwerpunkt, Rotation) $\vec{\omega}' = \vec{\omega}$ $\vec{V}' = \vec{V} + \vec{\omega} \times \vec{r}'$

anderes Koord. $\Rightarrow$ gleiches ω

$(\vec{\omega} \times \vec{r})^2 = \vec{\omega}^2\vec{r}^2 - (\vec{\omega}\vec{r})^2$

$T = \frac{M}{2}\vec{V}^2 + \frac{1}{2}I_{ik}\omega_i\omega_k$

$L = \frac{M}{2}\vec{V}^2 + \frac{1}{2}I_{ik}\omega_i\omega_k - V(x,y,z,\varphi,\theta,\psi)$



$T_{rot} = \frac{1}{2}(I_1\omega_1^2 + I_2\omega_2^2 + I_3\omega_3^2)$

1. Drehung um z
2. Drehung um x
3. Drehung um z'

$I = m\begin{pmatrix} r_2^2 + r_3^2 & -r_1 r_2 & -r_1 r_3 \\ -r_1 r_2 & r_1^2 + r_3^2 & -r_2 r_3 \\ -r_1 r_3 & -r_2 r_3 & r_1^2 + r_2^2 \end{pmatrix}$ Massenpunkt

Symmetrischer Tensor: $I_{ij} = I_{ji}$

$\vec{L} = I_k \vec{\omega}_k$ $I_k = \begin{pmatrix} I_1 & & \\ & I_2 & \\ & & I_3 \end{pmatrix}$

Hauptträgheitsmomente für HT-Achsen

$I_{xx} = \rho \int_V y^2 + z^2\, dV$ Trägheitsmoment

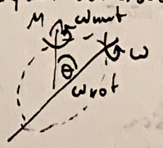
Punkt: mr^2	Steinerscher Satz
Hohlzylinder: mr^2	Verschiebung um d
Zylinder: $\frac{1}{2}mr^2$	aus Schwerpunkt
dünner Stab $\frac{1}{12}ml^2$	$I' = I + md^2$
Kugel $\frac{2}{5}mr^2$	

$M_i = I_{ij}\omega_j$ Drehimpuls $M_1 = I_1\omega_1$

im Allg.: $\vec{M} \nparallel \vec{\omega}$ außer für HT Achsen

Kugel: $I_1 = I_2 = I_3$ Rotator $I_1 = I_2$ $I_3 = 0$

Sym. Kreisel: $I_1 = I_2 = I$ $I_3 \neq 0$



$\omega_{nut} = \frac{|\vec{M}|}{I}$

$\omega_{rot} = \frac{M_3}{I_3} - \frac{M_3}{I} = \left(\frac{1}{I_3} - \frac{1}{I}\right)|\vec{M}|\cos\theta$

$\vec{D} = \sum_i \vec{r}_i \times \vec{K}_i$

$\frac{d}{dt}(I_{ik}\omega_k) = D_i$ $\frac{d}{dt}(\vec{M}) = \vec{D}$

Eulergleichungen

$\omega_1 = \dot\varphi \sin\theta \sin\psi + \dot\theta \cos\psi$

$\omega_2 = \dot\varphi \sin\theta \cos\psi - \dot\theta \sin\psi$

$\omega_3 = \dot\varphi \cos\theta + \dot\psi$

$E_{rot} = \frac{1}{2}\left[\dot\varphi^2((I_1\sin^2\psi + I_2\cos^2\psi)\sin^2\theta + I_3\cos^2\theta) + \dot\theta^2(I_1\cos^2\psi + I_2\sin^2\psi) + \dot\psi I_3 + 2\dot\varphi\dot\theta(I_1 - I_2)\sin\theta\sin\psi\cos\psi + 2\dot\varphi\dot\psi I_3\cos\theta\right]$

Für sym. Kreisel:

$E_{rot} = \frac{1}{2}(\dot\theta^2 + \dot\varphi^2 \sin\theta) + \frac{I_3}{2}(\dot\psi + \dot\varphi\cos\theta)^2$

Eulergleichungen   μ ges Masse

$\mu(\dot V_1 + \omega_2 V_3 - \omega_3 V_2) = K_1$

$\mu(\dot V_2 + \omega_3 V_1 - \omega_1 V_3) = K_2$

$\mu(\dot V_3 + \omega_1 V_2 - \omega_2 V_1) = K_3$

$I_1\dot\omega_1 + (I_3 - I_2)\omega_2\omega_3 = D_1$

$I_2\dot\omega_2 + (I_1 - I_3)\omega_3\omega_1 = D_2$

$I_3\dot\omega_3 + (I_2 - I_1)\omega_1\omega_2 = D_3$

Prinzip der kleinsten Wirkung

$S = \int_{t_a}^{t_e} L(q_1 \dots q_f, \dot q_1 \dots \dot q_f)\, dt$

S minimal (extremal)

=> es folgt Lagrange

Noether Theorem

$t' = t + \tau(t)$ $q_a'(t') = q_a(t) + \Delta q_a(t)$

$\delta q_a(t) = \Delta q_a(t) - \tau(t)\dot q_a(t)$

Noether: zu jeder Symmetrie des Systems → eine Erhaltungsgröße

Zeit → Energie   Ort → Impuls

Sym. Drehung → Drehimpuls

$\frac{\partial L}{\partial \dot q_a} = P_a$ verallg. Impulse

$H = \sum_{a=1}^{f} P_a \dot q_a - L$ Hamiltonfunktion

$\Delta S = 0$ $H(t)\tau(t) - \sum_{a=1}^{f} P_a(t)\delta q_a(t) = \text{const.}$

Energieerhaltung:

$H(t) = \sum_{a=1}^{f} P_a \dot q_a - L(q, \dot q) = \text{const.}$

Hamiltonische Bewegungsgleichung

$P_a = \frac{\partial L}{\partial \dot q_a}$ $H(q_1, \dots q_f, P_1, \dots P_f, t) = \sum_{a=1}^{f}\{P_a \dot q_a - L(q_1 \dots q_f, \dot q_1 \dots \dot q_f, t)\}$

$\frac{\partial H}{\partial p_a} = \dot q_a$ $\frac{\partial H}{\partial q_a} = -\dot p_a$ | $\frac{dH}{dt} = \frac{\partial H}{\partial t}$ => H erhalten, falls nicht exp. zeitabhängig

$S = \int L\, dt = \int \left\{\sum_{a=1}^{f} P_a\, dq_a - H\, dt\right\}$

$p\, q \Longrightarrow P, Q$

kanonische Transformation

$\dot q = \frac{\partial H}{\partial p}$, $\dot p = -\frac{\partial H}{\partial q}$ => $\dot Q = \frac{\partial H'}{\partial P_a}$, $\dot P = -\frac{\partial H'}{\partial Q}$

$\sum_{a=1}^{f} p_a \dot q_a - H = \sum_{a=1}^{f} P_a \dot Q_a - H' + \frac{\partial F}{\partial t}$

$dF = \sum_{a=1}^{f} p_a\, dq_a - \sum_{a=1}^{f} P_a\, dQ_a + (H - H')\,dt$

$H' = H + \frac{\partial F_1}{\partial t}$

Erzeugende Funkt.   Kanonische Trafo

$F_1(q, Q) = F$ $p = \frac{\partial F_1}{\partial q}$ $P = -\frac{\partial F_1}{\partial Q}$

$F_2(q, P) = F + \sum QP$ $p = \frac{\partial F_2}{\partial q}$ $Q = \frac{\partial F_2}{\partial P}$

$F_3(p, Q) = F - \sum qp$ $q = -\frac{\partial F_3}{\partial p}$ $P = -\frac{\partial F_3}{\partial Q}$

$F_4(p, P) = F + \sum(QP - qp)$ $q = -\frac{\partial F_4}{\partial p}$ $Q = \frac{\partial F_4}{\partial P}$

Routh'sche Funktion

$R(q, p, \xi, \dot\xi) = p\dot q(q, p, \xi, \dot\xi) - L(q, \dot q, \xi, \dot\xi)$

$\dot q = \frac{\partial R}{\partial p}$ $\dot p = -\frac{\partial R}{\partial q}$ $\frac{\partial R}{\partial \xi} = \frac{d}{dt}\left(\frac{\partial R}{\partial \dot\xi}\right)$

Poissonklammern

$\{g, f\} := \sum_{a=1}^{f}\left(\frac{\partial g}{\partial p_a}\frac{\partial f}{\partial q_a} - \frac{\partial g}{\partial q_a}\frac{\partial f}{\partial p_a}\right)$

$\frac{df}{dt} = \frac{\partial f}{\partial t} + \{H, f\}$

(i) $\{f, g\} = -\{g, f\}$

(ii) $\{c_1 f_1 + c_2 f_2, g\} = c_1\{f_1, g\} + c_2\{f_2, g\}$

(iii) $\{f_1 f_2, g\} = f_1\{f_2, g\} + \{f_1, g\} f_2$

(iv) $\{f, \{g, h\}\} + \{g, \{h, f\}\} + \{h, \{f, g\}\} = 0$

$\dot p = \{H, p\}$ $\dot q = \{H, q\}$

$\delta q = Q - q = \epsilon\{G, q\}$ $\delta p = P - p = \epsilon\{G, p\}$

$\delta H = H' - H = \epsilon\{G, H\}$

Ziel: $H' = 0$ $Q = \text{const.}$ $P = \text{const.}$

HJGl. $H(q, p, t) + \frac{\partial F_2}{\partial t} = 0$

$H + \frac{\partial S}{\partial t} = 0$

spezielle Relativität:

$\vec x \to \vec x'$ $\vec x = \begin{pmatrix} x \\ y \\ z \\ ct \end{pmatrix}$ $\vec x' = \begin{pmatrix} x' \\ y' \\ z' \\ ct' \end{pmatrix}$

→ v in z Richtung

$x'^\mu = \Lambda^\mu{}_\nu x^\nu$ $\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \begin{pmatrix} \Lambda^0{}_0 & \Lambda^0{}_1 & & \\ \Lambda^1{}_0 & \Lambda^1{}_1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

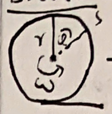
$\Lambda^1{}_1 = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\begin{pmatrix} ct' \\ x' \\ y' \\ z' \end{pmatrix} = \Lambda^1{}_1\begin{pmatrix} 1 & -\frac{v}{c} & & \\ -\frac{v}{c} & 1 & & \\ & & 1 & \\ & & & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix}$

$ct' = \frac{ct - \frac{v}{c}x}{\sqrt{1 - \frac{v^2}{c^2}}}$ $y' = y$ $z' = z$

$x' = \frac{x - vt}{\sqrt{1 - \frac{v^2}{c^2}}}$ $\tau' = \frac{\tau}{\sqrt{1 - \frac{v^2}{c^2}}}$

$l' = l\sqrt{1 - \frac{v^2}{c^2}}$ $E = \frac{mc^2}{\sqrt{1 - \dot{\vec x}^2/c^2}}$

Stuff



$\omega = 2\pi \cdot f$

$V = r \cdot \omega$

$s = \varphi \cdot r$

Drehung: $\left.\begin{array}{l}\vec D = (\vec M) = \vec r \times \vec F \\ D = (M) = r \cdot F\end{array}\right\}$ Drehmoment

$\vec L = \vec r \times \vec p$ $\frac{d\vec L}{dt} = (\vec M) = \vec D$

$\vec L = I\vec\omega$

$L = mr^2\omega$ $\vec F_{zentral} = f \cdot \frac{\vec x}{|\vec r|}$

$\dot{\vec p} = \vec F$

Integrale:

Länge: $L(\alpha) = \int_a^b \|\dot\alpha(t)\|\, dt$ α Kurve

Skalar-Kurvenint. $\int_c f\, ds = \int_a^b f(\alpha(t)) \cdot \|\dot\alpha(t)\|\, dt$

Kurvenintg. $\int_\alpha \vec V\, dx = \int_a^b \langle \vec V(\alpha(t)), \dot\alpha(t)\rangle\, dt.$

Oberflächenintg.

skalar $\int f(x)\, d\sigma = \iint f(\varphi(u,v))\, \|\vec\varphi_u \times \vec\varphi_v\|\, du\, dv$

vekt. $\int f(x)\, d\sigma$ $\vec\varphi_u = \frac{\partial \varphi}{\partial u}$

$= \iint f(\varphi(u,v)) \cdot (\vec\varphi_u \times \vec\varphi_v)\, du\, dv.$

Gauß Intg. $\int_G \mathrm{div}\, f(x)\, dx = \oint_{\partial G} f(x)\, d\sigma$

$\delta_{ij} = \begin{cases} 1 & i = j \\ 0 & j \neq i \end{cases}$ $\int_G \mathrm{rot}\, v\, d\sigma = \int_{\partial G} v\, dx$

$\varepsilon_{ijk} = \begin{cases} 1 & \text{gerade Perm.} \\ -1 & \text{ungerade Perm.} \\ 0 & \text{2 gleich} \end{cases}$ | $e^{ix} = \cos(x) + i\sin(x)$; $\sin = \frac{1}{2i}(e^{ix} - e^{-ix})$; $\cos = \frac{1}{2}(e^{ix} + e^{-ix})$

$\int \ln(x)\, dx = x\ln x - x$

$\int \sqrt{a^2 - x^2}\, dx = \frac{a^2}{2}\operatorname{asin}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 - x^2}$

$\int \sqrt{a^2 + x^2}\, dx = \frac{a^2}{2}\operatorname{asinh}\left(\frac{x}{a}\right) + \frac{x}{2}\sqrt{a^2 + x^2}$

$\int \sin^2 x\, dx = \frac{1}{2}(x - \sin x\cos x)$ | $\int \tan x\, dx = -\ln|\cos x|$

$\int \cos^2 x\, dx = \frac{1}{2}(x + \sin x\cos x)$ | $\int \frac{1}{\sqrt{1 - x^2}}\, dx = \operatorname{asin} x$

$\int e^{-x^2}\, dx = \frac{\sqrt{\pi}}{2}\operatorname{erf} x$ $\int \frac{u'(x)}{u(x)}\, dx = \ln|u(x)|$

$\int \sin^n(x)\, dx = \frac{n-1}{n}\int \sin^{n-2}(x)\, dx - \frac{1}{n}\cos(x)\sin^{n-1}(x)$

$\int \cos^n(x)\, dx = \frac{n-1}{n}\int \cos^{n-2}(x)\, dx + \frac{1}{n}\sin(x)\cos^{n-1}(x)$

DGLs: $f(x,y) = g(x)h(y)$ $\int \frac{1}{h(y)}\, dy = \int g(x)\, dx$

$P(x,y)\, dx + Q(x,y)\, dy = 0$

$\Rightarrow \frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$ $\frac{\partial \Phi}{\partial x} = P$ $\frac{\partial \Phi}{\partial y} = Q$

$\Phi(x,y) = \Phi(x_0, y_0) + \int_{x_0}^{x} P\, dx + \int_{y_0}^{y} Q\, dy$

$1 + \tan^2 x = \frac{1}{\cos^2}$

$\sin(x \pm y) = \sin x\cos y \pm \cos x\sin y$

$\cos(x \pm y) = \cos x\cos y \mp \sin x\sin y$

$\sin(2x) = 2\sin x\cos x$

$\cos(2x) = \cos^2 x - \sin^2 x$

$\sin x + \sin y = 2\sin\frac{x+y}{2}\cos\frac{x-y}{2}$

$\sin x\cos x = \frac{1}{2}\sin(2x)$