Theo 2 - Probeklausur

$Q = \int_V d^3x\, \rho(\vec{x})$ Ladungsdichte

$I = \int_F \vec{j}\, d\vec{f}$ Stromdichte

$\vec{F} = -\vec{F}' = k \frac{q q' (\vec{x}-\vec{x}')}{|\vec{x}-\vec{x}'|^3}$ | Gauss: $k = 1$, SI: $k = \frac{1}{4\pi\varepsilon_0}$

$\vec{E}(\vec{x}) = \sum \frac{q_i (\vec{x}-\vec{x}_i)}{4\pi\varepsilon_0 |\vec{x}-\vec{x}_i|^3}$ | $\vec{E} = \int d^3y \frac{\rho(\vec{y})(\vec{x}-\vec{y})}{4\pi\varepsilon_0 |\vec{x}-\vec{y}|^3}$

Statik: $\nabla \vec{E} = 0$ $\nabla \times \vec{E} = \vec{0}$ $\oint \vec{E} d\vec{s} = 0$

Ladungsdichte: $\nabla \vec{E} = \frac{\rho}{\varepsilon_0}$ $\vec{E} = -\nabla\phi$

$\Rightarrow -\nabla^2 \phi = \frac{\rho(\vec{x})}{\varepsilon_0}$ $\phi = \int d^3y \frac{\rho(\vec{y})}{4\pi\varepsilon_0 |\vec{x}-\vec{y}|}$

$W = \frac{\varepsilon_0}{2} \int d^3x\, |\vec{E}(\vec{x})|^2$ Arbeit

E-Feld geladene Kugel $E(r) = \frac{Q}{4\pi\varepsilon_0} \begin{cases} \frac{r}{R^3} & r < R \\ \frac{1}{r^2} & r > R \end{cases}$

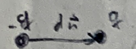
Ladungsverteilung: $\rho(\vec{x}) = \sigma(\vec{x}_n)\, \delta(\vec{x}_\perp)$

Kondensator: $U = \frac{xQ}{\varepsilon_0 A}$ $W = \frac{\varepsilon_0 A}{2} \int_0^d \frac{Q^2}{\varepsilon_0 A^2} dx$

$C = \frac{Q}{U}$ | $W = \frac{1}{2} C U^2$

Dipol

$\phi(\vec{x}) = \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{|\vec{x} - \frac{1}{2} d\vec{u}|} - \frac{1}{|\vec{x} + \frac{1}{2} d\vec{u}|} \right)$



$\vec{p} = q\, d\vec{u}$ Dipolmoment

$\phi(\vec{x}) = \frac{\vec{p}\vec{x}}{4\pi\varepsilon_0 |\vec{x}|^3} = -\vec{p}\nabla \frac{1}{4\pi\varepsilon_0 |\vec{x}|}$

$\vec{F}_{dipol} = (\vec{p}\nabla)\vec{E}$ $\vec{M}_{dipol} = \vec{p} \times \vec{E}$

Monopol $= q = \int d^3x'\, \rho(\vec{x}')$

Dipolmoment: $\vec{p} = \int d^3x'\, \vec{x}'\, \rho(\vec{x}')$

Quadrupolmoment: $Q_{ij} = \int d^3x'\, \rho(\vec{x}') (3x'_i x'_j - \vec{x}'^2 \delta_{ij})$

$4\pi\varepsilon_0 \phi \approx \frac{q}{|\vec{x}|} + \frac{\vec{x}\vec{p}}{|\vec{x}|^3} + \frac{1}{2}\sum Q_{ij} \frac{x_i x_j}{|\vec{x}|^5}$

Randproblem

$\phi = \frac{1}{4\pi\varepsilon_0} \int_V d^3x' \frac{\rho(\vec{x}')}{|\vec{x}-\vec{x}'|} + \frac{1}{4\pi} \oint_{\partial V} df' \left(\frac{1}{|\vec{x}-\vec{x}'|} \frac{\partial\phi}{\partial n'} - \phi(\vec{x}') \frac{\partial}{\partial n'} \frac{1}{|\vec{x}-\vec{x}'|} \right)$

Dirichlet: $\psi|_{\partial V} = 0 \Rightarrow \psi(\vec{x}) = 0$

Neumann $\psi(\vec{x}) = \text{const}$ $\frac{\partial\psi}{\partial n}\big|_{\partial V} = 0$

Greensche Funktion

$\Delta_x G(\vec{x}, \vec{x}') = -\delta^{(3)}(\vec{x}-\vec{x}')$

$G(\vec{x}, \vec{x}') = \frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{x}'|} + f(\vec{x}, \vec{x}')$

Dirichlet:

$\phi(\vec{x}) = \frac{1}{\varepsilon_0} \int_V d^3x'\, \rho(\vec{x}') G_D(\vec{x}, \vec{x}') - \oint_{\partial V} df'\, \omega(\vec{x}') \frac{\partial G_D}{\partial n'}$

$\phi(\vec{x})|_{\partial V} = \omega(\vec{x})$

Neumann geg: $-\frac{\partial\phi}{\partial n}\big|_{\partial V} = E_\perp = \nu(\vec{x})$

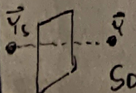
mit $\oint_{\partial V} df' F(\vec{x}') = 1$ $\frac{\partial G_N(\vec{x}, \vec{x}')}{\partial n'} = -F(\vec{x}')$

$\phi(\vec{x}) = \frac{1}{\varepsilon_0} \int_V d^3x'\, \rho(\vec{x}') G_N(\vec{x}, \vec{x}') - \oint_{\partial V} df'\, G_N(\vec{x}, \vec{x}')\, \nu$

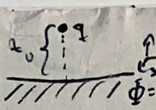
$-\nu = \frac{\partial\phi(\vec{x}')}{\partial n'}$

Greensche Funktion

$\vec{y}_s = (y_1, y_2, -y_3)$



$G_D(\vec{x}, \vec{y}) = \frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{y}|} - \frac{1}{4\pi} \frac{1}{|\vec{x}-\vec{y}_s|}$



$\phi(\vec{x}) = \frac{1}{\varepsilon_0} \int_H d^3x'\, \rho(\vec{x}') G_D(\vec{x}, \vec{x}') - \oint_{\partial H} df \cdot 0 \cdot \frac{\partial G_D}{\partial n}$

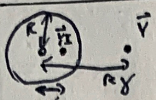
$\phi = 0$

induzierte Ladung

$\sigma = \varepsilon_0 E_z$

$q = \int_{\partial H} df\, \sigma = -q$

$= \frac{q}{4\pi\varepsilon_0} \left(\frac{1}{|\vec{x} - z_0 \vec{e}_z|} - \frac{1}{|\vec{x} + z_0 \vec{e}_z|} \right)$



$\gamma = \frac{|\vec{y}|}{R}$ $\vec{y}_I = \frac{R^2}{|\vec{y}|^2} \vec{y}$

$G_D = \frac{1}{4\pi|\vec{x}-\vec{y}|} - \frac{R}{|\vec{y}|} \frac{1}{4\pi|\vec{x} - \frac{R^2}{|\vec{y}|^2}\vec{y}|}$

Außenraum Kugel Green: (Neumann)

$G_N(\vec{x}, \vec{y}) = \frac{1}{4\pi|\vec{x}-\vec{y}|} + \frac{R}{4\pi|\vec{y}||\vec{x}-\vec{y}_I|} + \frac{1}{4\pi R} \log \frac{|\vec{y}||\vec{x}-\vec{y}_I|\, *}{|\vec{x}||\vec{y}| - \vec{x}\vec{y}}$

$* + \vec{y} \cdot (\vec{x} - \vec{y}_I)$

Seperation der Variablen

$\Delta\phi = \left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \right) \phi = 0$

Ansatz z.B. $\phi(x, y, z) = \phi(x, y) \cdot \phi(z)$

Seperationsansatz Kugelkoordinaten:

$\phi = R(r) \cdot Y(\theta, \varphi)$ | $\Delta = \frac{\partial^2}{\partial r^2} + \frac{2}{r}\frac{\partial}{\partial r} - \frac{1}{r^2} \vec{L}^2$

$\vec{L}^2 := \left(\frac{\partial}{\partial\theta} \right)^2 + \cot\theta \frac{\partial}{\partial\theta} + \frac{1}{\sin^2\theta} \left(\frac{\partial}{\partial\varphi} \right)^2$

$L_z = -i \frac{\partial}{\partial\varphi}$ $m = 0$ $P_l(u) = \frac{1}{2^l l!} \left(\frac{d}{du} \right)^l (u^2 - 1)^l$

allg. $P_l^m(u) = \frac{(-1)^m}{2^l l!} (1-u^2)^{m/2} \left(\frac{d}{du} \right)^{l+m} (u^2-1)^l$

Kugelflächenfunktion

$Y_{l,m}(\theta, \varphi) = \sqrt{(2l+1) \frac{(l-m)!}{(l+m)!}}\, P_l^m(\cos\theta)\, e^{im\varphi}$

erfüllen $\vec{L}^2 Y_{lm} = l(l+1) Y_{lm}$ | $L_z Y_{lm} = m Y_{lm}$

$Y_{0,0} = 1$

$Y_{1,0} = \sqrt{3}\cos\theta = \sqrt{3} \frac{z}{r}$

$Y_{1,\pm1} = \mp\sqrt{\frac{3}{2}} \sin\theta\, e^{\pm i\varphi}$

$Y_{2,0} = \sqrt{\frac{5}{4}} (3\cos^2\theta - 1) = \sqrt{\frac{5}{4}} \frac{2z^2 - x^2 - y^2}{r^2}$

$Y_{2,\pm1} = -\sqrt{\frac{15}{2}} \sin\theta\cos\theta\, e^{\pm i\varphi}$

$= -\sqrt{\frac{15}{2}} \frac{z(x \pm iy)}{r^2}$

$Y_{2,\pm2} = \sqrt{\frac{15}{8}} \sin^2\theta\, e^{\pm i 2\varphi} = \sqrt{\frac{15}{8}} \frac{(x \pm iy)^2}{r^2}$

$\int_a^b dx\, U_n^*(x) U_m(x) = \delta_{nm}$

$F(\theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} Y_{l,m}(\theta, \varphi)\, c_{l,m}$

$c_{l,m} = \frac{1}{4\pi} \int d\theta\, d\varphi\, \sin\theta\, Y_{l,m}^*(\theta, \varphi) F(\theta, \varphi)$

$\phi(r, \theta, \varphi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} R_{l,m}(r)\, Y_{l,m}(\theta, \varphi)$

Rad. Gleichung: $\frac{1}{r^2} \frac{d}{dr} \left(r^2 \frac{dR_{lm}(r)}{dr} \right) - \frac{l(l+1)}{r^2} R_{lm} = 0$

$\phi = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} (A_{lm} r^l + B_{lm} r^{-(l+1)})\, Y_{lm}(\theta, \varphi)$

Symmetrie in $\varphi \Rightarrow m = 0$

$\phi(r, \theta) = \sum_{l=0}^{\infty} \sqrt{2l+1} \left[A_l r^l + B_l r^{-l-1} \right] P_l(\cos\theta)$

falls $\phi = 0$ für $r \to \infty$ $\Rightarrow A_{lm} = 0$

Geladene Kugeloberfläche: $\sigma(\theta) = \sum_{l=0}^{\infty} \sqrt{2l+1}\, \sigma_l P_l(\cos\theta)$

Multipol in Kugelkoord.

für großes r: $\phi(r, \theta, \varphi) = \sum_{l,m} \frac{Q_{lm}}{4\pi\varepsilon_0} \frac{1}{(2l+1) r^{l+1}} Y_{lm}$

$B_{lm} = \frac{Q_{lm}}{4\pi\varepsilon_0} \frac{1}{2l+1}$

$Q_{lm} = \int d^2\Omega\, dr\, r^{l+2}\, Y_{lm}^*(\theta, \varphi)\, \rho(r, \theta, \varphi)$

Magnetostatik

Kontinuität:

$\vec{j}(\vec{x}, t) = \rho(\vec{x}, t)\, \vec{v}(\vec{x}, t)$ $\frac{\partial\rho}{\partial t} + \nabla\vec{j} = 0$

Biot-Savart: $\vec{F}_{12} = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1}\oint_{C_2} \frac{d\vec{x}_1 \times (d\vec{x}_2 \times \vec{x}_{12})}{|\vec{x}_{12}|^3}$

$\varepsilon_0 \mu_0 = \frac{1}{c^2}$

$\vec{F}_{12} = -\frac{\mu_0 I_1 I_2}{4\pi} \oint_{C_1}\oint_{C_2} (d\vec{x}_1 \cdot d\vec{x}_2) \frac{\vec{x}_{12}}{|\vec{x}_{12}|^3}$

$\vec{B}(\vec{x}) = \mu_0 I \oint_C \frac{d\vec{y} \times (\vec{x}-\vec{y})}{4\pi|\vec{x}-\vec{y}|^3}$ $\vec{F} = I \oint_C d\vec{x}\, \vec{B}(\vec{x})$

$\vec{B}(\vec{x}) = \nabla_x \times \frac{\mu_0}{4\pi} \int d^3y \frac{\vec{j}(\vec{y})}{|\vec{x}-\vec{y}|} \Rightarrow \nabla\vec{B} = 0$

$\int_F d\vec{f}\, (\nabla \times \vec{B}) = \oint_{\partial F} d\vec{s}\, \vec{B} = \mu_0 I$ $\nabla \times \vec{B}(\vec{x}) = \mu_0 \vec{j}(\vec{x})$

Vektorpotenzial: $\vec{B} = \nabla \times \vec{A}$

$\vec{A} = \mu_0 \int d^3y \frac{\vec{j}(\vec{y})}{|\vec{x}-\vec{y}|}$

Eichtrafo: $\vec{A}'(\vec{x}) = \vec{A}(\vec{x}) + \nabla\Lambda(\vec{x})$

Coulomb Eich: $\nabla \vec{A}(\vec{x}) = 0$

Axial Eich: $\vec{u} \cdot \vec{A}(\vec{x}) = 0$

$\Delta \vec{A}(\vec{x}) = -\mu_0 \vec{j}(\vec{x})$

$\vec{F} = \int d^3x\, \vec{j} \times \vec{B}$

$\vec{M} = \int d^3x\, \vec{x} \times (\vec{j} \times \vec{B})$

Elektro/Magnetostatik in Schmutz

Mikrostatik:

$\nabla\vec{E} = \frac{\rho}{\varepsilon_0}$ $\nabla\vec{B} = 0$ $\nabla \times \vec{E} = \vec{0}$ $\nabla \times \vec{B} = \mu_0 \vec{j}$

Dipoldichte: $\vec{P}(\vec{x}) = \varepsilon_0 \chi \vec{E} + O(\vec{E}^2)$

$\vec{D}(\vec{x}) = \varepsilon_0 \vec{E}(\vec{x}) + \vec{P}(\vec{x})$ $\nabla\vec{D} = \rho_{frei}$ $\nabla \times \vec{E} = 0$

$\vec{D}(\vec{x}) = \varepsilon \vec{E}(\vec{x})$ $\varepsilon = \varepsilon_0 \varepsilon_r$ $\varepsilon_r = 1 + \chi_e$

Grenzfläche: Gauss Kästchen $\vec{n}(\vec{D} - \vec{D}') = 0$

Stokes Fläche $\vec{t}(\vec{E} - \vec{E}') = 0$

$D_\perp = D'_\perp$ $E_\perp = \frac{\varepsilon'}{\varepsilon} E'_\perp$ $E_\parallel = E'_\parallel$ $D_\parallel = \frac{\varepsilon}{\varepsilon'} D'_\parallel$

$\vec{B}(\vec{x}) = \mu_0 (\vec{H}(\vec{x}) + \vec{M}(\vec{x}))$ $\vec{M} = \chi_m \vec{H}$

$\vec{B}(\vec{x}) = \mu \vec{H}(\vec{x})$ $\mu_r = 1 + \chi_m$ $\nabla \times \vec{H} = \vec{j}_{frei}$

$B_\perp = B'_\perp$ $H_\perp = \frac{\mu'_r}{\mu_r} H'_\perp$ $H_\parallel = H'_\parallel$ $B_\parallel = \frac{\mu_r}{\mu'_r} B'_\parallel$

Randwertprobleme Magn. $\Delta\vec{A} = -\mu_r \mu_0 \vec{j}_{frei}$

$\Delta\phi_m = \nabla\vec{M}(\vec{x})$ $\phi_m(\vec{x}) = -\frac{1}{4\pi} \nabla \int d^3y \frac{\vec{M}(\vec{y})}{|\vec{x}-\vec{y}|}$

Kurvenintegral: $\int f\,ds = \int_a^b f(\vec\gamma(t))\cdot\|\dot{\vec\gamma}(t)\|\,dt$

$\int \vec f(\vec x)\,d\vec x = \int_a^b \vec f(\vec\gamma(t))\,\dot{\vec\gamma}(t)\,dt$ Oberflächenintegral:

$\iint_F f(\vec x)\,d\sigma = \iint_F f(\vec\varphi(u,v))\cdot\|\vec\varphi_u\times\vec\varphi_v\|\,d(u,v)$

$\iint_F \vec f(\vec x)\,d\vec\sigma = \iint_F \vec f(\vec\varphi(u,v))\cdot(\vec\varphi_u\times\vec\varphi_v)\,d(u,v)$

Volumenint: $\iiint_V f(\vec x)\,dV \cdot \iiint_V \vec f(\vec\xi(u,v,w))\,\vec N\,dV$

$\vec N = \left(\frac{\partial\vec\xi}{\partial u}\times\frac{\partial\vec\xi}{\partial v}\right)\cdot\frac{\partial\vec\xi}{\partial w}$ Nabla

Zylinderkoor. $\text{grad}\,\phi = \vec e_r\frac{\partial\phi}{\partial r} + \vec e_z\frac{\partial\phi}{\partial z} + \vec e_\varphi\frac{1}{r}\frac{\partial\phi}{\partial\varphi}$

$\text{div}\,\vec F = \frac1r\frac{\partial}{\partial r}(rF_r) + \frac1r\frac{\partial F_\varphi}{\partial\varphi} + \frac{\partial F_z}{\partial z}$

$\text{rot}\,\vec F = \left[\frac1r\frac{\partial F_z}{\partial\varphi} - \frac{\partial F_\varphi}{\partial z}\right]\vec e_r + \left[\frac{\partial F_r}{\partial z} - \frac{\partial F_z}{\partial r}\right]\vec e_\varphi + \frac1r\left[\frac{\partial}{\partial r}(rF_\varphi) - \frac{\partial F_r}{\partial\varphi}\right]\vec e_z$

Kugelkoor. $\text{grad}\,\phi = \vec e_r\frac{\partial\phi}{\partial r} + \vec e_\theta\frac1r\frac{\partial\phi}{\partial\theta} + \vec e_\varphi\frac{1}{r\sin\theta}\frac{\partial\phi}{\partial\varphi}$

$\text{div}\,\vec F = \frac1{r^2}\frac{\partial}{\partial r}(r^2F_r) + \frac1{r\sin\theta}\frac{\partial}{\partial\theta}(F_\theta\sin\theta) + \frac1{r\sin\theta}\frac{\partial F_\varphi}{\partial\varphi}$

$\text{rot}\,\vec F = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(F_\varphi\sin\theta) - \frac{\partial F_\theta}{\partial\varphi}\right]\vec e_r + \left[\frac1{r\sin\theta}\frac{\partial F_r}{\partial\varphi} - \frac1r\frac{\partial}{\partial r}(rF_\varphi)\right]\vec e_\theta + \frac1r\left[\frac{\partial}{\partial r}(rF_\theta) - \frac{\partial F_r}{\partial\theta}\right]\vec e_\varphi$

Integralsätze Gauß $\int_V dV\,\text{div}\,\vec F(\vec x) = \oint_{\partial V}\vec F\,d\vec\sigma$

Stokes $\int_F d\vec f\,\text{rot}\,\vec F(\vec x) = \oint_{\partial F}\vec F\,d\vec x$

Green: 1. $\int_V dV(\varphi\Delta\psi + (\nabla\psi)(\nabla\varphi)) = \oint_{\partial V}\varphi\frac{\partial\psi}{\partial n}d\sigma$

2. $\int_V dV(\varphi\Delta\psi - \psi\Delta\varphi) = \oint_{\partial V}d\sigma\left(\varphi\frac{\partial\psi}{\partial n} - \psi\frac{\partial\varphi}{\partial n}\right)$

$\frac{\partial\psi}{\partial n} = \nabla\psi\,\vec n$ $\int_{\vec a}^{\vec b}\nabla f = f(\vec b) - f(\vec a)$

Krummlinige Koord. $g_{ij} = \frac{\partial\vec x}{\partial u_i}\times\frac{\partial\vec x}{\partial u_j}$

$\vec e_i = \frac{\frac{\partial\vec x}{\partial u_i}}{\left|\frac{\partial\vec x}{\partial u_i}\right|}$ $(ds)^2 = (\Delta\vec x)^2 = g_{ij}\Delta u_i\Delta u_j$

$\Delta V = g_u g_v g_w\,\Delta u\,\Delta v\,\Delta w$

$g_u = \sqrt{g_{uu}}$

$\text{grad}\,\varphi = \frac1{g_u}\frac{\partial\varphi}{\partial u}\vec e_u + \frac1{g_v}\frac{\partial\varphi}{\partial v}\vec e_v + \frac1{g_w}\frac{\partial\varphi}{\partial w}\vec e_w$

$\text{div}\,\vec A = \frac{1}{g_ug_vg_w}\left[\frac{\partial}{\partial u}(g_vg_wA_u) + \frac{\partial}{\partial v}(g_ug_wA_v) + \frac{\partial}{\partial w}(g_ug_vA_w)\right]$

$\begin{vmatrix}\frac{\vec e_u}{g_vg_w} & \frac{\vec e_v}{g_ug_w} & \frac{\vec e_w}{g_ug_v}\\ \frac{\partial}{\partial u}&\frac{\partial}{\partial v}&\frac{\partial}{\partial w}\\ g_uA_u & g_vA_v & g_wA_w\end{vmatrix} = \text{rot}\,\vec A$

$\Delta\varphi = \frac1{g_ug_vg_w}\left[\frac{\partial}{\partial u}\left(\frac{g_vg_w}{g_u}\frac{\partial\varphi}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{g_ug_w}{g_v}\frac{\partial\varphi}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{g_ug_v}{g_w}\frac{\partial\varphi}{\partial w}\right)\right]$

$\vec A(\vec B\times\vec C) = \vec B(\vec C\times\vec A) = \vec C(\vec A\times\vec B)$ $\varepsilon_{ijk}a_ib_jc_k\vec e_k$

$\vec A\times(\vec B\times\vec C) = \vec B\cdot(\vec A\cdot\vec C) - \vec C(\vec A\cdot\vec B)$ $\varepsilon_{ijk}\varepsilon_{klm} = \delta_{il}\delta_{jm} - \delta_{im}\delta_{jl}$

$\vec a\times\vec b = \varepsilon_{ijk}a_ib_j\vec e_k$

$\vec a\cdot\vec b = \delta_{ij}a_ib_j\vec e_i$

$\varepsilon_{ijk}\varepsilon_{lmn} = \begin{vmatrix}\delta_{il}&\delta_{im}&\delta_{in}\\ \delta_{jl}&\delta_{jm}&\delta_{jn}\\ \delta_{kl}&\delta_{km}&\delta_{kn}\end{vmatrix}$

$\nabla(fg) = f(\nabla g) + g(\nabla f)$

$\nabla(\vec A\vec B) = \vec A\times(\nabla\times\vec B) + \vec B\times(\nabla\times\vec A) + (\vec A\nabla)\vec B + (\vec B\nabla)\vec A$

$\nabla(\vec A\times\vec B) = \vec B(\nabla\times\vec A) - \vec A(\nabla\times\vec B)$

$\nabla(f\vec A) = f(\nabla\vec A) - \vec A\times(\nabla f)$

$\nabla\times(\vec A\times\vec B) = (\vec B\nabla)\vec A - (\vec A\nabla)\vec B + \vec A(\nabla\vec B) - \vec B(\nabla\vec A)$

$\nabla\cdot(\nabla\times\vec A) = 0$ $\nabla\times(\nabla f) = 0$

$\nabla\times(\nabla\times\vec A) = \nabla(\nabla\vec A) - \nabla^2\vec A$

Taylor-Entwicklung Felder

$\frac{1}{|\vec r-\vec r'|} = \sum_{n=0}^\infty \frac1{n!}(\vec r'\cdot\nabla_{\vec r})^n\frac{1}{|\vec r-\vec r'|}\Big|_{\vec r=0}$

$\frac{1}{|\vec x-\vec r'|} = \alpha\left[\frac1{|\vec r'|} + \frac{\vec r\vec r'}{|\vec r'|^3} + \frac12\frac{3(\vec r\vec r')^2 - \vec r^2\vec r'^2}{\vec r'^5}\right]$

Relativistische Formulierung E-M-Feld

$x^\mu = (ct,x,y,z)$ $x^0 = ct$ $x^i = \vec e_i\vec x$

$x_\mu = \eta_{\mu\nu}x^\nu$ $\eta_{\mu\nu} = \text{diag}(1,-1,-1,-1)$

$(ds)^2 = c^2(dt)^2 - (d\vec x)^2$ $ds = \sqrt{\dot x^\mu\dot x_\mu}\,d\tau$

$S_m = -mc\int_a^b d\tau\sqrt{\dot x^\mu\dot x_\mu} = -mc^2\int_{t_1}^{t_2}dt\sqrt{1-\frac{\dot{\vec x}^2}{c^2}}$

$x'^\mu = \Lambda^\mu{}_\nu x^\nu + a^\mu$ $a^\mu = \text{const.}$

$\Lambda^\mu{}_\varrho\,\eta_{\mu\nu}\Lambda^\nu{}_\kappa = \eta_{\varrho\kappa}$ | $\frac{\partial L}{\partial x^i} = 0$ $\frac{\partial L}{\partial\dot x^i} = \frac{m\dot x^i}{\sqrt{1-\frac{\dot{\vec x}^2}{c^2}}} = p^i$

$\frac{\partial L}{\partial x^i} = \frac{d}{dt}\frac{\partial L}{\partial\dot x^i} \Rightarrow \frac{d}{dt}p^i = 0$

Vierentensoren:

$F'^{\mu\nu} = \Lambda^\mu{}_\varrho\Lambda^\nu{}_\kappa F^{\varrho\kappa}$ | $\Lambda'^{\mu\nu\varrho} = \Lambda^\mu{}_\alpha\Lambda^\nu{}_\beta\Lambda^\varrho{}_\gamma\Lambda^{\alpha\beta\gamma}$

$F_\mu{}^\nu = \eta_{\mu\kappa}F^{\kappa\nu}$ $F_{\mu\nu} = \eta_{\mu\kappa}\eta_{\nu\varrho}F^{\kappa\varrho}$

$\eta^{\mu\nu} = \begin{pmatrix}1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}$ $\eta_{\mu\nu} = \begin{pmatrix}1&&&\\&-1&&\\&&-1&\\&&&-1\end{pmatrix}$ $\delta^\mu_\nu = \begin{pmatrix}1&&&\\&1&&\\&&1&\\&&&1\end{pmatrix}$

$\delta^\mu_\nu = \eta^{\mu\varrho}\eta_{\varrho\nu}$ $\delta^\mu_\nu\eta^{\nu\varrho} = \eta^{\mu\varrho}$ $\varepsilon^{0123} = +1$ $\varepsilon_{0123} = -1$

$\varepsilon^{\mu\nu\varrho\kappa}\varepsilon_{\mu\nu\varrho\kappa} = -24$ $\varepsilon^{\mu\nu\varrho\kappa}\varepsilon_{\alpha\nu\varrho\kappa} = -6\delta^\mu_\alpha$

$\varepsilon^{\mu\nu\alpha\beta}\varepsilon_{\varrho\kappa\alpha\beta} = -2(\delta^\mu_\varrho\delta^\nu_\kappa - \delta^\nu_\varrho\delta^\mu_\kappa)$

$A^\mu(\vec x) = (\phi(\vec x),\vec A(\vec x))$ $A_\mu(\vec x) = (\phi(\vec x), -\vec A(\vec x))$

Lorentz Trafo $A'^\mu = \Lambda^\mu{}_\nu A^\nu$ $A'_\mu = \eta_{\mu\varrho}\Lambda^\varrho{}_\nu A^\nu$

Lagrange Teilchen in E Feld:

$L(\vec x,\dot{\vec x},t) = -mc^2\sqrt{1-\frac{\dot{\vec x}^2}{c^2}} + \frac ec\vec A(\vec x,t)\dot{\vec x} - e\phi(\vec x,t)$

$\vec p = \vec p_{mech} + \frac ec\vec A$ $\vec p_{mech} = \frac{m\dot{\vec x}}{\sqrt{1-\frac{\dot{\vec x}^2}{c^2}}}$

$H = c\sqrt{m^2c^2 + (\vec p - \frac ec\vec A)^2} + e\phi$

$\vec E = -\nabla\phi - \frac1c\frac{\partial\vec A}{\partial t}$ $\vec B = \text{rot}\,\vec A$ $\dot{\vec p}_{mech} = e\vec E + \frac ec\dot{\vec x}\times\vec B$

Eichinvariant: $A'_\mu(\vec x,t) = A_\mu - \frac{\partial\Lambda(\vec x,t)}{\partial x^\mu}$

$\vec A' = \vec A + \nabla\Lambda$ $\phi' = \phi - \frac1c\frac{\partial\Lambda}{\partial t}$

$\sin(x) = \sum(-1)^n\frac{x^{2n+1}}{(2n+1)!}$ | $\sin(x) = \frac1{2i}(e^{ix} - e^{-ix})$

$\cos(x) = \sum(-1)^n\frac{x^{2n}}{(2n)!}$ | $\cos(x) = \frac12(e^{ix}+e^{-ix})$

$e^{ix} = \cos(x) + i\sin(x)$

$\sin(\alpha\pm\beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta$

$\int\frac1{1+x^2} = \arctan x$

$\sin(2\alpha) = 2\sin\alpha\cos\alpha$

$\int\frac1{\sqrt{1-x^2}} = \arcsin x$

$\int x\sin x = \sin x - x\cos x$

DGL

$P(x,y)dx + Q(x,y)dy = 0$

$\int\frac1{\cos^2} = \tan$

$\frac{dy}{dx} = -\frac PQ$ | $\frac{\partial P}{\partial y} = \frac{\partial Q}{\partial x}$

$\phi = \phi(x_0,y_0) + \int_{x_0}^x P(x,y)\,dx + \int_{y_0}^y Q(x_0,y)\,dy$

$\frac{\partial\phi}{\partial x} = P$ $\frac{\partial\phi}{\partial y} = Q$

$\int\sin^n = \frac{n-1}{n}\int\sin^{n-2}dx - \frac1n\cos\sin^{n-1}$

$\int\cos^n = \frac{n-1}{n}\int\cos^{n-2}dx + \frac1n\sin\cos^{n-1}$

Drehmatrix $R_x = \begin{pmatrix}1&&\\&\cos&-\sin\\&\sin&\cos\end{pmatrix}$

$R_z = \begin{pmatrix}\cos&-\sin&\\ \sin&\cos&\\&&1\end{pmatrix}$ $R_y = \begin{pmatrix}\cos&&\sin\\&1&\\-\sin&&\cos\end{pmatrix}$

Lagrange 1. $m\ddot{\vec x} = \vec K + \lambda\nabla\varphi(\vec x,t)$

E_{kin}: $\frac12m(r^2[\dot\theta^2 + \dot\varphi^2\sin^2\theta] + \dot r^2)$

$\frac12m(\dot r^2 + r^2\dot\varphi^2 + \dot z^2)$

$L = T(q,\dot q,t) - V(q,t)$ $\frac{d}{dt}\frac{\partial L}{\partial\dot q} - \frac{\partial L}{\partial q} = 0$

Zentralfeld: $V_{eff} = V(r) + \frac12\frac{L^2}{mr^2}$

kleinste Wirkung

$S = \int_{t_1}^{t_2}L(q_i,\dot q_i)\,dt$ S_{min} (extremal)

Hamilton $\frac{\partial L}{\partial\dot q_i} = p_i$

$H = \sum p_i\dot q_i - L$ Hamiltonfkt.

"const." ⇒ Energieerhaltung

$\frac{\partial H}{\partial p_i} = \dot q_i$ $\frac{\partial H}{\partial q_i} = -\dot p_i$ } Bewegungs-Gleichungen

kanonische Trafo

$\dot q = \frac{\partial H}{\partial p}$ ⇒ $\dot Q_i = \frac{\partial H'}{\partial P_i}$

$\dot p = -\frac{\partial H}{\partial q}$ $\dot P_i = -\frac{\partial H'}{\partial Q}$

$\{f,g\}_{q,p} = \sum\left(\frac{\partial f}{\partial q_i}\frac{\partial g}{\partial p_i} - \frac{\partial f}{\partial p_i}\frac{\partial g}{\partial q_i}\right)$

$F_1 = F$ $p = \frac{\partial F_1}{\partial q}$ $P = -\frac{\partial F_1}{\partial Q}$

$F_2 = F + \sum Q_iP_i$ $p = \frac{\partial F_2}{\partial q}$ $Q = \frac{\partial F_2}{\partial P}$

$F_3 = F - \sum q_ip_i$ $q = -\frac{\partial F_3}{\partial p}$ $P = -\frac{\partial F_3}{\partial Q}$

$F_4 = F + \sum Q_iP_i - q_ip_i$ $q = -\frac{\partial F_4}{\partial p}$ $Q = \frac{\partial F_4}{\partial P}$